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著者	Ikeda Kiyohiro, Yamakawa Yuki, Desrues Jacques, Murota Kazuo
journal or publication title	Physical Review Letters < 'issue of Physical Review Letters
volume	100
number	19
page range	198001
year	2008
URL	http://hdl.handle.net/10097/52926

doi: 10.1103/PhysRevLett.100.198001

Bifurcations to Diversify Geometrical Patterns of Shear Bands on Granular Material

Kiyohiro Ikeda* and Yuki Yamakawa
Department of Civil and Environmental Engineering, Tohoku University, Sendai 980-8579, Japan

Jacques Desrues
Laboratoire 3S-R, UMR 5521 CNRS–Grenoble University, BP 53, 38000 Grenoble, France

Kazuo Murota
Graduate School of Information Science and Technology, University of Tokyo, Tokyo 113-8656, Japan
(Received 1 November 2007; published 14 May 2008)

The mechanism to diversify geometrical patterns on granular material was elucidated using a group-theoretic image analysis of patterned shear bands, with associated numerical bifurcation analysis. Pattern formation of granular materials took the course of the evolution of a diamondlike diffuse bifurcation breaking uniformity, followed by further bifurcation, mode jumping, and the formation and disappearance of shear bands through localization. A chaotic explosive increase of possible postbifurcation states was emphasized as a mechanism to diversify geometrical patterns.

DOI: 10.1103/PhysRevLett.100.198001 PACS numbers: 45.70.−n, 47.20.Ky, 47.54.−r, 83.10.Ff

Granular material displays diverse geometrical patterns [1–4]. In the shearing test of granular material, it has been a long standing paradox that "the patterns of shear bands [5] are so diversified that every test appears to be unique, even when conducted under identical conditions." The surprising aspect underscored in this case is that experimental efforts to produce homogeneous granular material specimens encounter unpredictable responses, which are designated later as the chaotic explosive increase of possible bifurcation states.

Since these specimens are fabricated to be uniform, it is logical to refer to a theoretical model of symmetry breaking from a uniform state. A group $O(2) \times O(2)$ [6] is used to express the symmetry of a rectangular cross section of a cubic or rectangular parallelepiped domain; diamond and oblique stripe patterns are produced through bifurcation of a system with this symmetry [7].

In fact, diamond and oblique stripe patterns have been observed in a sandbox experiment [8], which concurs with the theoretical prediction. Nevertheless, the mechanism of the formation of the complex geometrical pattern in Fig. 1, which is not periodic at the boundaries remains to be elucidated. The specimen starts from a homogeneous state but ends up with a nonhomogeneous one; an underlying link exists between these two states. One may raise the criticism that granular material specimens are not uniform because of initial heterogeneities that govern the orientation and spatial distribution of shear bands [9]. However, this criticism cannot answer a question: Can heterogeneity produce patterned shear bands?

To explain the mechanism of pattern formation in a granular material, the authors propose a hypothesis: "bifurcation with a diamondlike diffuse mode with spatially distributed continuous deformation occurs and breaks uniformity prior to formation of shear bands through localization." In this work, experimental, theoretical, and numerical results are set forth to test that hypothesis.

Image-analysis procedure.—In order to detect bifurcation of a uniform granular material specimen, a group-theoretic image-analysis procedure [10] was used. The scalar field of incremental strain invariant $\Delta\varepsilon(x, y)$ in a rectangular domain $\{(x, y) \mid 0 \leq x \leq l_x, 0 \leq y \leq l_y\}$ was expanded into a double Fourier series

FIG. 1 (color). A dry dense sand cubic specimen ($100 \times 100 \times 100$ mm^3) that was sheared by a triaxial test apparatus that can control three principal stresses independently. The specimen was surrounded by six rigid planes, and was deformed into a right-angled prism with confining pressure of 300 kPa.

$$\Delta\varepsilon = \sum_{n_x=0}\sum_{n_y=0}[A_{n_xn_y}\sin(2\pi n_x x/l_x)\cos(2\pi n_y y/l_y) + B_{n_xn_y}\sin(2\pi n_x x/l_x)\sin(2\pi n_y y/l_y) + C_{n_xn_y}\cos(2\pi n_x x/l_x)\cos(2\pi n_y y/l_y) + D_{n_xn_y}\cos(2\pi n_x x/l_x)\sin(2\pi n_y y/l_y)]. \quad (1)$$

We specifically address the wave number (n_x, n_y) $(n_x, n_y = 0, 1, 2, \ldots)$, which corresponds to a possible bifurcation mode. For this wave number, Eq. (1) can engender diamond and oblique stripe patterns

$$C_1\cos(2\pi n_x x/l_x + \alpha)\cos(2\pi n_y y/l_y + \beta)\text{: diamond}$$
$$C_2\cos[2\pi(n_x x/l_x \pm n_y y/l_y) + \alpha]\text{: oblique stripe}$$

for constants C_1, C_2, $-\pi \le \alpha < \pi$, and $-\pi \le \beta < \pi$.

Experiment.—We revive the experiment [11], in which characteristic shear band pattern was observed. A fine angular, siliceous sand (Hostun RF) specimen ($164.0 \times 173.0 \times 35.4$ mm^3) was tested by the plane strain compression apparatus. Deformation patterns on the rectangular cross section (164.0×173.0 mm^2) with the initial aspect ratio of 0.95 are investigated. The false relief stereophotogrammetry (FRS) [12] method was used to digitize the displacement fields of the cross section deforming under load. Photographs taken were numbered 1–9.

The progression of localization of incremental strain fields between two neighboring photographs was obtained (Fig. 2, top row). The specimen displayed the orientation of spatially distributed strain localization, which is weak and obscure. Two parallel oblique shear bands were observed during increments 3–5. During increments 5–8, some shear bands diminished gradually in favor of the emergence of two oblique shear bands in different directions. The shear bands thus formed complex geometrical patterns. Consequently, FRS visualized the strain fields; however, a bifurcation mode was not detected.

The original incremental fields of the shear strain invariant (Fig. 2, top row) were expanded into the double Fourier series [cf. Eq. (1)]. The history of the magnitude of the Fourier coefficient for each wave number was investigated (Fig. 3); as described previously, each wave number corresponds to a possible bifurcation mode. Strong magnitudes were detected for two wave numbers: $(n_x, n_y) = (3, 1)$ and (2,1). During increments 3–6, the magnitude for (3,1) increased sharply and was predominant among all modes. During increments 6–9, the magnitude for (2,1) increased stably and the modes for (3,1) and (2,1) were the largest in magnitude. At the final state, no such predominance was observed.

Histories of the decomposed strain fields for (3,1) and (2,1) (Fig. 2, second and third rows, respectively) displayed diamondlike and stripelike patterns, which appear to have physical necessity. During increments 3–6, the inclination

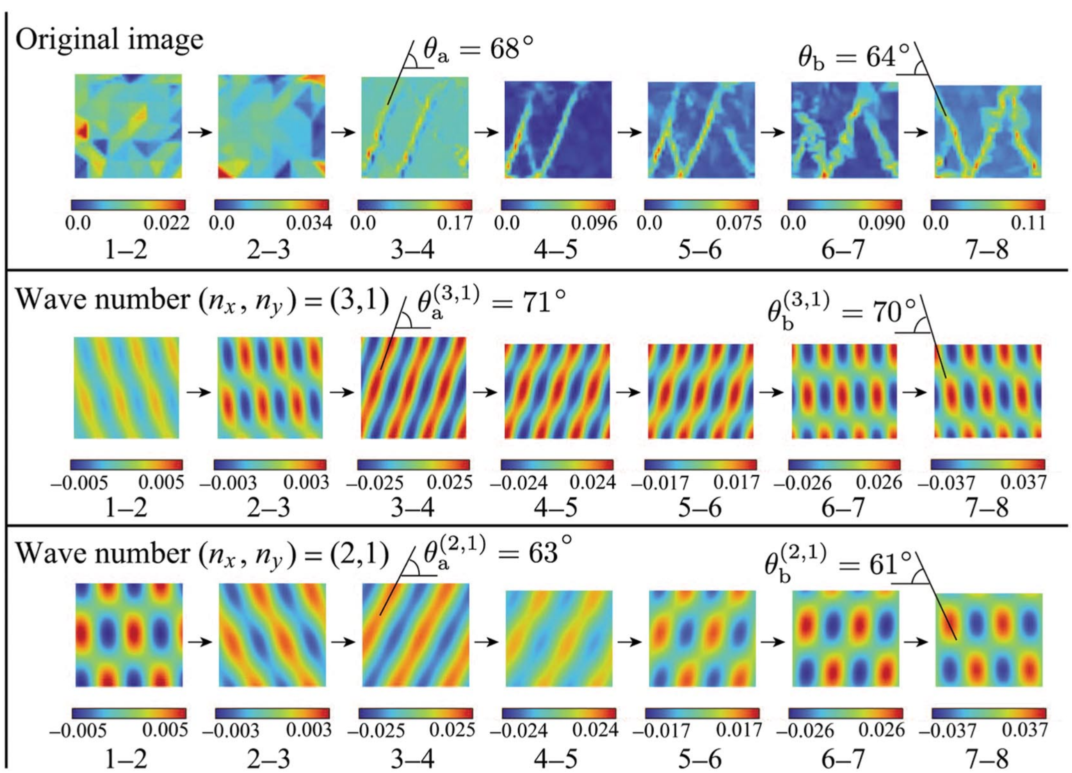

FIG. 2 (color). Contour views of incremental strain fields: top row, original images; second row, decomposed images for wave number (3,1); and third row, decomposed images for wave number (2,1). Points with large strain are colored red; those with small strain are colored blue. The strain stands for the second invariant of the logarithmic deviatoric strain that is nondimensional.

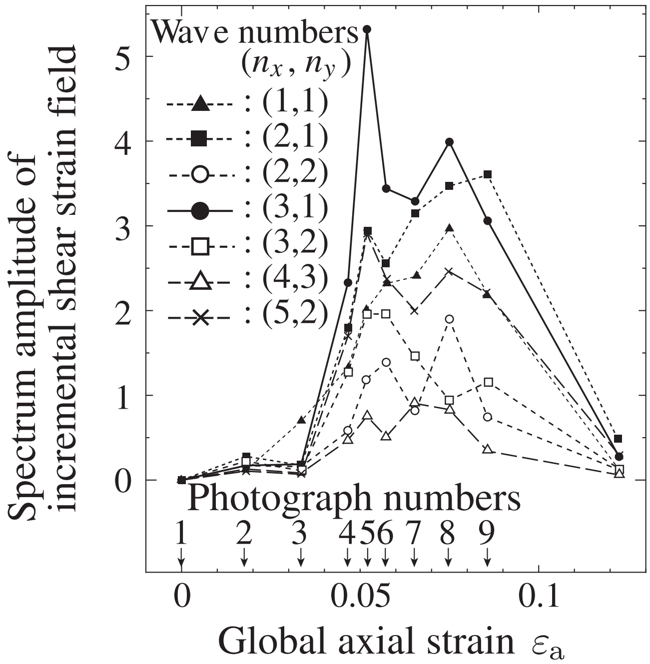

FIG. 3. History of intensity of decomposed strain fields between photographed points. Incremental shear strain fields were expanded into the double Fourier series and classified into distinct bifurcation modes. The intensity was defined as $(A^2_{n_x n_y} + B^2_{n_x n_y} + C^2_{n_x n_y} + D^2_{n_x n_y})^{1/2}$ [cf. Eq. (1)].

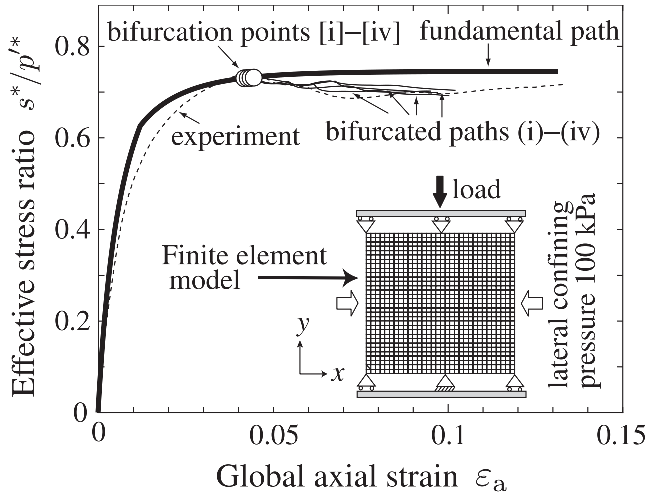

FIG. 4. Equilibrium paths of the finite element model of a rectangular uniform domain that is divided into 32 × 32 rectangular eight-nodes quadratic elements (3201 nodes). The Drucker–Prager model [18] was used with material properties: elastic bulk modulus, 12.50 MPa; elastic shear modulus, 5.77 MPa; critical stress ratio, 0.943; dilatancy factor, 0.299; internal friction angle, 35.0°; and dilatancy angle, 10.0°.

of shear bands was $\theta_a = 68°$ and is close to $\theta_a^{(3,1)} = 71°$ of localization patterns of (3,1); during increments 6–9, the inclination of shear bands was $\theta_b = 64°$ and is close to $\theta_b^{(2,1)} = 61°$ of localization patterns of (2,1). The change of inclination of shear bands, accordingly, can be explained as the change of the wave number of the predominant modes.

Based on this observation, we introduce an interpretation of the experimental behavior. During increments 2–3, a diffuse-mode bifurcation of the wave number (3,1) emanated, almost hidden, behind the predominant uniform compressive deformation [13]. Thereafter, the mode for (3,1) grew sharply to form diamond-pattern-like possible locations of localization. During increments 3–4, among these possible locations of localization, loading progressed in some locations and developed into shear bands, whereas unloading progressed in other locations. The shift of the predominant wave number from (3,1) to (2,1) rendered geometrical patterns complex; this shift is ascribed to recursive bifurcation and/or mode jumping [14]. The validity of this interpretation will be assessed in the sequel, based on numerical simulation.

Numerical simulation.—Elastoplastic finite-deformation bifurcation analysis [15] was conducted on the finite element model of a rectangular uniform domain with the initial aspect ratio of 0.95. Figure 4 shows computed loci of equilibria. From the fundamental equilibrium path (shown by thick solid line), associated with homogeneous deformation, four bifurcated equilibrium paths (thin solid lines) branched at four closely located bifurcation points [i]–[iv] (○). These bifurcated paths followed fairly well the experimental curve (dashed line).

As shown by the postbifurcation progress of strains on the bifurcated paths [Fig. 5(a) for (i) and (ii)], the bifurcation modes at the bifurcation points were spatially periodic, diamondlike, diffuse modes with different but nearly equivalent wave numbers. They generated spatially periodic strain-localized locations. Most of these locations underwent unloading; only a few of them underwent loading to engender shear bands with diverse and complex geometrical patterns.

Multiple possible states of equilibrium due to bifurcations are visible. Three different solutions [Fig. 5(b)] were oriented from the same bifurcation point [iii] with slightly different numerical conditions, such as the convergence criterion in numerical iterations and increments of equilibrium paths. The numerical equilibrium paths displayed zigzag up and down [16], which indicates possible further bifurcation and/or mode jumping. Such multiple equilibria should not be treated as a numerical problem, but should be treated as an essential difficulty of the problem in question. Because of the spatial periodicity of localized locations, several possible locations have a similar likelihood to develop into shear bands. The locations that develop into shear bands change case by case, possibly because of initial inhomogeneities in experiments. Therefore, the locations are accidental and unpredictable. This mechanism to diversify shear band patterns is designated as the "chaotic explosive increase of possible postbifurcation states." It resolves a paradox that has long puzzled the authors: "The patterns of shear bands are so diversified and geometrical and every test appears to be unique." The surprising aspect

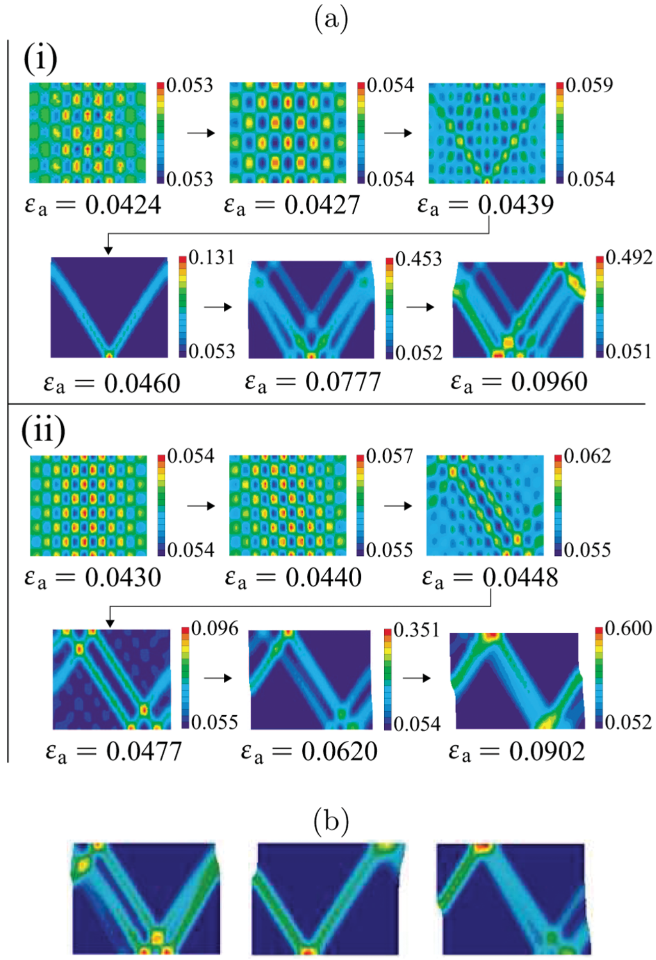

FIG. 5 (color). (a) Progress of the distribution of shear strains on bifurcated paths (i) and (ii). (b) Three different possible postbifurcation states emerging from the bifurcation point [iii].

is that experimental efforts to make specimens homogeneous produce chaotic, unpredictable responses.

Diffuse-mode bifurcation, recursive bifurcation, and/or mode jumping, followed by localization into shear bands, is the underlying chaotic mechanism forming geometrical patterns on granular material. It ends a long controversy related to the initial bifurcation that breaks uniformity: shear band mode bifurcation that spontaneously engenders a shear band(s) [17], or diffuse-mode bifurcation that causes distributed deformation initially and engenders shear bands later. Although shear band formation has heretofore attracted the most attention, diffuse-mode bifurcation is a catalyst that breaks uniformity, and via recursive bifurcation and/or mode jumping, engenders diversified shearing patterns. Future studies are expected to encode the mechanism of diffuse-mode bifurcation into the framework of the shear band analysis of granular material to overcome the difficulty of the chaotic explosive increase of possible postbifurcation states, then to elucidate the mechanical behavior of granular material.

*ikeda@civil.tohoku.ac.jp

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