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Enhancement of spin injection from ferromagnetic metal into a two-dimensional electron gas using a tunnel barrier

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Using free electron approximation, we calculated the spin dependent tunnel conductance of ballistic ferromagnet / tunnel barrier / two-dimensional electron gas (FM/I/2DEG) junctions and FM/I/2DEG/I/FM double junctions for different barrier strengths. We find that a tunnel barrier improves spin injection considerably. For sufficiently strong barriers, it is predicted that the tunnel conductance ratio between spin up and spin down channels is, in first approximation, equal to the ratio between their Fermi velocities in the FM. For single junctions, this results in a significant current polarization ($\sim 10\%$). This corresponds to a relative resistance change of several percent between parallel and antiparallel magnetization of the two FM electrodes, respectively, for the double junction. In the weak barrier regime, the magnitude and sign of the current polarization are strongly dependent on the (controllable) electron density in the 2DEG.

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During the last decade, many attempts have been made to inject a spin-polarized current from a metallic ferromagnetic (FM) electrode into a semiconductor (SM). The great interest in this field is due to numerous potential applications of spin injection. In this respect, we mention explicitly the spin transistor, as proposed by Datta and Das,[1] based on spin-orbit coupling[2,3] in a two-dimensional electron gas (2DEG).[4] Promising results regarding spin injection into a SM have been reported,[5,6] but their interpretation is still under discussion.[7–10]

Recently, Schmidt *et al.*[11] pointed out that, for diffusive systems, the conductivity mismatch between FM and SM forms a major obstacle for spin injection. This problem can be avoided by using a ferromagnetic semiconductor instead, as has been shown experimentally by Friedling *et al.*[12] and Ohno *et al.*[13] However, an important advantage of metallic FM electrodes is their relatively high Curie temperature, making them indispensable for applications operating at room temperature. Assuming spin dependent tunneling probabilities, Rashba[14] showed that the introduction of a tunnel barrier (I) could provide a way to overcome the conductivity mismatch problem.[15] The underlying reason is that the largest resistance of the junction determines the degree of current polarization. A sufficiently strong tunnel barrier is therefore ideal for spin injection, if the tunnel conductance differs considerably for spin up and spin down electrons, respectively. In the present work we show quantitatively that this condition is indeed fulfilled for ballistic FM/I/2DEG junctions and FM/I/2DEG/I/FM double junctions. High magnetoconductance ratios have already been shown theoretically for carbon nanotube magnetic tunnel junctions by Mehrez *et al.*[16] Although the highest degree of spin polarizations is expected for strong barriers, it is found that the interface conductance of normal FM/2DEG contacts is also spin dependent. In the weak barrier regime, the degree of spin polarization shows a strong dependence on the electron density n_{2DEG} in the 2DEG. This is particulary interesting, since n_{2DEG} can be controlled by applying an external gate voltage.

Our treatment is based on an earlier work by Qi *et al.*,[17] who consider the tunnel magneto-resistance of FM/I/FM junctions. Later, a similar approach was also used by Zheng *et al.*[18] for FM/I/NM/I/FM (NM stands for normal metal) resonant junctions. In our case, two FM electrodes are connected to a 2DEG channel of length L by thin insulating layers which serve as tunnel barriers. In contrast to a NM, a 2DEG has low electron density. The junction is regarded as a two-dimensional system. The normal to the 2DEG plane is taken parallel to the z axis. Net current flows in the x direction. The 2DEG channel is assumed to be much longer than the inverse of the Fermi wave vector of the electrons, $L \gg 1/k^{sm}$, so that resonances in the 2DEG channel can be neglected. The tunnel barriers are approximated by two δ-type potentials of strength U_0 at $x=0$ and $x=L$, respectively. This method was first proposed by Blonder, Tinkham, and Klapwijk,[19] to the best of our knowledge, for normal metal-insulator-superconductor junctions. A two-band model is used to describe the electron dispersion relation in the FM electrodes.[17,18,20,21] This approach differs from the conventional Julliere model[22,23] because, in addition to the spin dependence of the density of states in the FM, spin dependent transmission probabilities are also taken into account.[17] In the free electron approximation, the single electron Hamiltonian is given by

$$H=\frac{-\hbar^2}{2m(x)}\frac{\partial^2}{\partial \mathbf{r}^2}+V(x)+U_0[\delta(x)+\delta(x-L)]-\mathbf{h}(x)\cdot\sigma_P . \tag{1}$$

Here, the electron mass $m(x)$ equals m_e, the free electron mass, for $x<0$, $x>L$ and m^*, the effective electron mass in the semiconductor, for $0<x<L$. The potential energy $V(x)$ is given by $V(x)=\Gamma'$ for $0<x<L$ and $V(x)=0$ elsewhere. For our calculations, we assumed[24,25] $m^*/m_e=0.04$. The fourth term of the Hamiltonian represents the internal exchange energy, where σ_P denotes the Pauli spin operator and $\mathbf{h}(x)$ the molecular field. In our case we consider the situation where $\mathbf{h}(x)=0$ in the 2DEG channel and $\mathbf{h}(x)=\pm h_0\cdot\hat{y}$ in the ferromagnetic electrodes, with the sign depending on the direction of magnetization of the electrode.

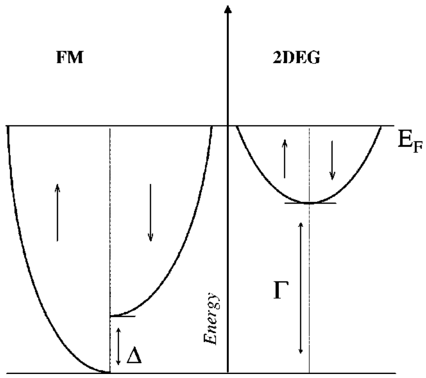

FIG. 1. Band structures of FM and 2DEG. In the FM electrodes the lower band edge is shifted up with energy Δ for the minority electrons.

A single FM/I/2DEG junction is treated first and the tunneling transmission coefficients for spin up and down electrons are calculated. The wave function of the electron satisfies the Schrödinger equation

$$H_\sigma \Psi_\sigma(\mathbf{r}) = E_\sigma \Psi_\sigma(\mathbf{r}), \tag{2}$$

where the subscript σ denotes the spin direction which can be either up ($\uparrow$) or down ($\downarrow$) relative to the applied magnetic field.[26] As depicted in Fig. 1 the energy eigenvalues are $E_\uparrow^{fm} = (\hbar \mathbf{k}_\uparrow^{fm})^2/2m_e$ and $E_\downarrow^{fm} = (\hbar \mathbf{k}_\downarrow^{fm})^2/2m_e + \Delta$ for $x<0$ and $E^{sm} = (\hbar \mathbf{k}^{sm})^2/2m^* + \Gamma$ for $0<x<L$. Here, the exchange energy $\Delta = 2h_0$ and Γ is the difference between the lower conduction-band edge of the 2DEG and of the FM. In the 2DEG, the density of states is constant for parabolic dispersion so that $(E_F - \Gamma) = \pi \hbar^2 n_{2DEG}/m^*$. The electron wave vector $\mathbf{k}_\sigma^i$ $(i = fm, sm)$ is defined by $\mathbf{k}_\sigma^i = (k_{\sigma,x}^i, k_{\sigma,y}^i)$. The solutions of the Schrödinger equation for spin $\uparrow$ in the two different regions are

$$\Psi_\uparrow^{fm}(\mathbf{r}) = \begin{pmatrix}1\\0\end{pmatrix} e^{i\mathbf{k}_\uparrow^{fm}\cdot \mathbf{r}} + r_\uparrow \begin{pmatrix}1\\0\end{pmatrix} e^{-i\mathbf{k}_\uparrow^{fm}\cdot \mathbf{r}}, \tag{3}$$

$$\Psi_\uparrow^{sm}(x) = t_\uparrow \begin{pmatrix}1\\0\end{pmatrix} e^{i\mathbf{k}_\uparrow^{sm}\cdot \mathbf{r}}, \tag{4}$$

with similar expressions for spin $\downarrow$. The coefficients $t_\uparrow$ ($t_\downarrow$) and $r_\uparrow$ ($r_\downarrow$) are determined by the boundary conditions. Since only electrons with energy approximately equal to the Fermi energy contribute to the net tunnel current, we take $E_\uparrow^{fm} = E_\downarrow^{fm} = E^{sm} = E_F$. The absolute values of the Fermi wave vectors can therefore be written as

$$k_\uparrow^{fm} = \frac{1}{\hbar}\sqrt{2m_e E_F}, \tag{5}$$

$$k_\downarrow^{fm} = \frac{1}{\hbar}\sqrt{2m_e(E_F - \Delta)}, \tag{6}$$

$$k^{sm} = \sqrt{2\pi n_{2DEG}}. \tag{7}$$

Since the tunnel barrier is assumed to be perfectly smooth and without diffusive scattering $k_{\sigma,y}$ conservation is required upon tunneling. Defining $\sin\phi = k_{\sigma,y}^{fm}/k_\sigma^{fm}$, where ϕ is the angle of incidence, the wave vectors in the x direction can be expressed as

$$k_{\sigma,x}^{fm}(\phi) = k_\sigma^{fm}\cos\phi, \tag{8}$$

$$k_{\sigma,x}^{sm}(\phi) = \sqrt{(k^{sm})^2 - [k_{\sigma,y}^{fm}(\phi)]^2}. \tag{9}$$

Now, we apply the boundary conditions for δ potentials[19] at $x=0$

$$\Psi_\sigma^{fm}(x=0,y) = \Psi_\sigma^{sm}(x=0,y), \tag{10}$$

$$\frac{\hbar^2}{2m_e}\left.\frac{\partial \Psi_\sigma^{fm}(\mathbf{r})}{\partial x}\right|_{x=0-} + U_0 \Psi_\sigma^{fm}(x=0,y) = \frac{\hbar^2}{2m^*}\left.\frac{\partial \Psi_\sigma^{sm}(\mathbf{r})}{\partial x}\right|_{x=0+}. \tag{11}$$

Combining wave functions and boundary conditions the derivation of the spin dependent transmission coefficients is straightforward

$$T_\sigma(\phi,Z) = \frac{v_{\sigma,x}^{sm}}{v_{\sigma,x}^{fm}}|t_\sigma|^2 = \frac{4v_{\sigma,x}^{sm}v_{\sigma,x}^{fm}}{4(v_\uparrow^{fm}Z)^2 + (v_{\sigma,x}^{sm} + v_{\sigma,x}^{fm})^2}. \tag{12}$$

We used the group velocities for parabolic dispersion $\mathbf{v}_\sigma^{fm} = \hbar \mathbf{k}_\sigma^{fm}/m_e$ and $\mathbf{v}_\sigma^{sm} = \hbar \mathbf{k}_\sigma^{sm}/m^*$. The dimensionless parameter[19] $Z = U_0/(\hbar v_\uparrow^{fm})$ has been introduced here in order to distinguish between the tunneling contact regime[27] $(Z/Z_{0,\sigma} \gg 1)$ and the low barrier regime $(Z/Z_{0,\sigma} \ll 1)$, with $Z_{0,\sigma} = 1/4(v^{sm}/v_\uparrow^{fm} + v_\sigma^{fm}/v_\uparrow^{fm})^2 \leqslant 1$. Note that $Z_{0,\sigma} = 1$ if $\Delta = \Gamma = 0$. In this case Eq. (12) reduces to the BTK result $T = 1/(Z^2+1)$. The total electrical current density through the single junction $J = J_\uparrow + J_\downarrow$ can be calculated using

$$J_\sigma = \frac{e^2 V}{(2\pi)^2}\int_{k_{\sigma,x}^{fm}>0} d^2\mathbf{k}_\sigma^{fm}\left(-\frac{\partial f_0(E_\sigma)}{\partial E_\sigma}\right) T_\sigma v_{\sigma,x}^{fm}, \tag{13}$$

which is valid for small bias voltages V. We introduced the Fermi-Dirac electron distribution at equilibrium $f_0(E_\sigma)$. For low temperatures T such that $k_B T \ll E_F$ we can use the approximation $-\partial f_0(E_\sigma)/\partial E_\sigma \cong \delta(E_\sigma - E_F)$. Now, the integral in Eq. (13) can be evaluated and the conductance per unit length for each spin channel, $G_\sigma = J_\sigma/V$, as a function of T_σ can be expressed as

$$G_\sigma = \frac{e^2 k_\sigma^{fm}}{h\pi}\int_0^{\phi_\sigma^c} d\phi\, T_\sigma(\phi,Z)\cos\phi, \tag{14}$$

where $\cos\phi = k_{\sigma,x}^{fm}/k_\sigma^{fm}$ and ϕ_σ^c denotes the critical angle of incidence.

For the typical case that $k_\sigma^{fm} \gg k_\sigma^{sm}$, due to the low electron density in the 2DEG compared to a metal, we find that $\phi_\sigma^c = \sin^{-1}(k_\sigma^{sm}/k_\sigma^{fm}) \approx k_\sigma^{sm}/k_\sigma^{fm} \ll 1$. This means that Eq. (14) can be further simplified by taking $\cos\phi \approx 1$ and re-scaling the integration limits

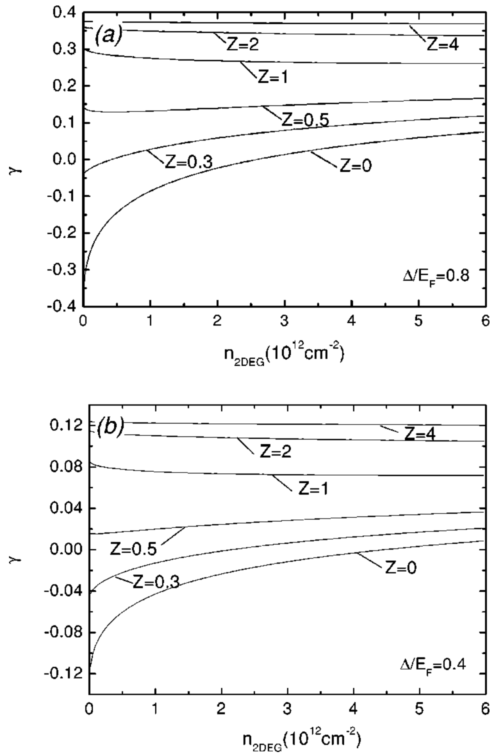

FIG. 2. Current polarization γ as a function of the electron density in the 2DEG, n_{2DEG}, for several values of the barrier strength Z. (a) Plotted for exchange energy $\Delta/E_F=0.8$ ($E_F=5$ eV). Maximum polarization is obtained for sufficiently strong barriers. Notice that the current polarization can have both directions. Since n_{2DEG} can be controlled by a gate electrode, the polarization direction can also be altered by varying the gate voltage. (b) $\Delta/E_F=0.4$.

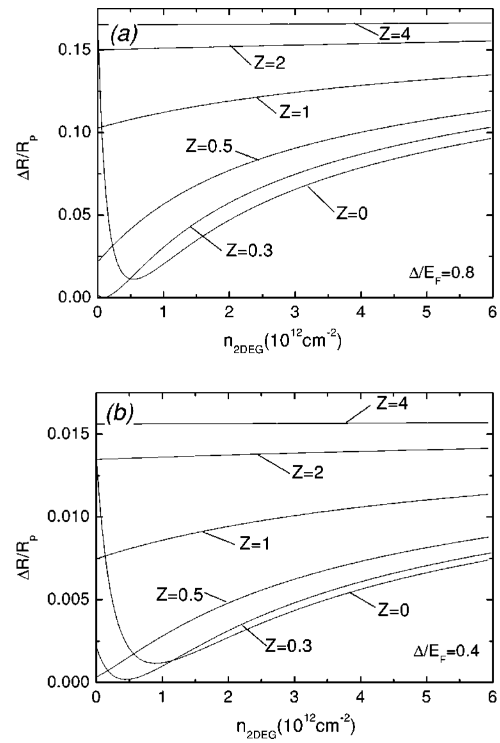

FIG. 3. The relative resistance change $\Delta R/R_p$ as a function of the electron density in the 2DEG, n_{2DEG} for various barrier strengths, Z. For InAs heterostructures typically 1×10^{12} cm^{-2} $<n_{2DEG}<3\times10^{12}$ cm^{-2}, so that a tunnel barrier increases $\Delta R/R_p$ substantially. (a) Exchange energy Δ/E_F fixed at 0.8 ($E_F=5$ eV) (b) $\Delta/E_F=0.4$.

$$G_\sigma \approx \frac{e^2 k^{sm}}{h\pi} T'_\sigma(Z), \tag{15}$$

with

$$T'_\sigma(Z)=\int_0^1 d\phi' \, T_\sigma(\phi',Z), \tag{16}$$

and $T_\sigma(\phi',Z)=T_\sigma(\phi,Z)|_{\phi=\phi'\phi^c_\sigma}$. In Eq. (16), $T'_\sigma(Z)$ denotes the probability that an electron with random angle of incidence smaller than ϕ^c_σ tunnels through the junction. In the strong barrier limit $T'_\sigma(Z)\approx(\pi/4)T_\sigma(0,Z)$ so that Eq. (15) reduces to the Landauer conductance formula for ballistic point contacts corrected by a factor $\pi/4$

$$G_\sigma \approx \frac{e^2 k^{sm}}{h\pi}\frac{\pi}{4} T_\sigma(\phi=0,Z). \tag{17}$$

The current polarization $\gamma=(J_\uparrow - J_\downarrow)/J$ can be written as

$$\gamma=\frac{r_G-1}{r_G+1}\approx\frac{r_T-1}{r_T+1}, \tag{18}$$

since, according to Eq. (15), $r_G=G_\uparrow/G_\downarrow$ is approximately equal to $r_T=T'_\uparrow/T'_\downarrow$. In Fig. 2(a), γ is plotted as a function of n_{2DEG} for different barrier strengths Z. For strong barriers, $Z/Z_{0,\sigma}\gg1$, it is clear from Eqs. (17) and (12) that $r_T\approx T_\uparrow(0,Z)/T_\downarrow(0,Z)\approx r_v$, with $r_v=v^{fm}_\uparrow/v^{fm}_\downarrow=1/\sqrt{1-\Delta/E_F}$, so that γ is independent of n_{2DEG}. For parabolic dispersion, r_v is equal to the ratio of the (three-dimensional) densities of states for spin $\uparrow$ and $\downarrow$, respectively, in the FM electrode. In the weak barrier regime, however, γ strongly depends on n_{2DEG}. For the ideal contact ($Z=0$) even the sign of γ, indicating the direction of the net injected magnetic moment, is determined by n_{2DEG}. We note that n_{2DEG} and therefore the spin polarization direction can be controlled using a gate on top of the 2DEG. Electron densities in InAs hetrostructures are typically in the range 1×10^{12} cm^{-2}$<n_{2DEG}<3\times10^{12}$ cm^{-2}. From Fig. 2 it is clear that within this range of n_{2DEG} the current polarization increases substantially with increasing barrier strength. This is related to the fact that the Fermi velocities in FM and 2DEG are of the same order of magnitude owing to the low effective mass in the 2DEG. This condition no longer holds for small values of n_{2DEG} so that $|\gamma(Z=0)|$ strongly increases when n_{2DEG} is decreased below 1×10^{12} cm^{-2} (see figure). The maximum absolute value of γ decreases strongly with decreasing exchange energy Δ, Fig. 2(b).

In order to detect spin injection into the 2DEG, a FM/I/2DEG/I/FM spin-valve junction can be used. The difference in resistance of the junction between antiparallel and parallel magnetization of the FM electrodes, $\Delta R=R_a-R_p$, is a measure for spin-injection. In order to calculate the total transmission through the double junction, multiple barrier reflections within the 2DEG have to be taken into account. The 2DEG is assumed to be ballistic and without spin relaxation. Expression (14) is also valid for the double junction if the single interface transmission is replaced by the transmission probability of double junction. For the conductance per unit length of the parallel magnetized junction $G_p = 1/R_p$ and of the anti-parallel magnetized junction $G_a = 1/R_a$ we obtain

$$G_p(Z) \approx \frac{e^2 k^{sm}}{h\pi} \int_0^1 d\phi' \left(\frac{T_\uparrow}{2-T_\uparrow} + \frac{T_\downarrow}{2-T_\downarrow} \right), \tag{19}$$

$$G_a(Z) \approx \frac{e^2 k^{sm}}{h\pi} 2 \int_0^1 d\phi' \frac{T_\uparrow T_\downarrow}{T_\uparrow + T_\downarrow - T_\uparrow T_\downarrow}, \tag{20}$$

using the same approximations as in Eq. (15). Here $T_\sigma = T_\sigma(\phi', Z)$, as defined above. For strong barriers $Z/Z_{0,\sigma} \gg 1$ it can be shown that $\Delta R/R_p$ can be approximated by

$$\frac{\Delta R}{R_p} \approx \frac{(r_v - 1)^2}{4 r_v}. \tag{21}$$

Therefore, $\Delta R/R_p$ is independent of n_{2DEG} for sufficiently strong barriers. In Fig. 3(a), $\Delta R/R_p$ is plotted as a function of n_{2DEG} for several values of Z with $\Delta/E_F = 0.8$. With increasing Z, $\Delta R/R_p$ approaches the constant value determined by Eq. (21). The particular value of Δ/E_F determines whether $\Delta R/R_p$ is large enough to be observable. Figure 3(b) shows the corresponding results for $\Delta/E_F = 0.4$. In order to obtain a signal exceeding 1%, the material dependent exchange energy Δ/E_F needs to be at least 0.3. The polarization of the current is, however, much larger. Note that Slonczewski,[20] who introduced the two-band model, uses $k_\uparrow^{fm} \simeq 1.09$ Å^{-1} and $k_\downarrow^{fm} = 0.42$ Å^{-1} for iron, based on experimental data, which accounts for $\Delta/E_F \simeq 0.8$.

In the low barrier regime, the degree of spin filtering is strongly dependent on n_{2DEG}. $\Delta R/R_p$ can be strongly suppressed to almost zero for certain electron densities. The value of n_{2DEG} at which the minimum is reached depends on both Δ and Z. An analytical expression was not obtained, but based on our numerical calculations it is found that $\Delta R/R_p$ can only reach a local minimum if $Z \lesssim 0.5$. As has been pointed out above, Γ can be controlled by using an external gate. In the weak barrier limit, this enables us to adjust $\Delta R/R_p$ from almost zero to a finite value by varying the gate voltage.

In summary, we calculated the spin dependent conductivity through ballistic FM/I/2DEG single junctions and FM/I/2DEG/FM double junctions quantitatively, based on a two-band model. Both the current polarization γ and the relative change in resistance $\Delta R/R_p$ improve with increasing barrier strength. In the weak barrier limit, a spin dependent interface conductance is still expected. In this regime, the magnitude and the sign of the current polarization dependent on the electron density in the 2DEG, n_{2DEG}. Both the absolute value of γ and $\Delta R/R_p$ improve for higher (material dependent) exchange energy Δ.

Note added. After submission, we have become aware of a recent publication by Grundler[28] about ballistic FM/2DEG junctions.

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