Acoustic intensity measurement in a narrow duct by a two-sensor method

著者	琵琶 哲志
journal or publication title	Review of scientific instruments
volume	78
number	8
page range	086110
year	2007
URL	http://hdl.handle.net/10097/46684

doi: 10.1063/1.2768929

Acoustic intensity measurement in a narrow duct by a two-sensor method

Tetsushi Biwa
Department of Mechanical Systems and Design, Tohoku University, Sendai 980-0013, Japan

Yusuke Tashiro and Hiroshi Nomura
Department of Crystalline Materials Science, Nagoya University, Nagoya 464-8603, Japan

Yuki Ueda
Tokyo University of Agriculture and Technology, Koganei, Tokyo 184-8588, Japan

Taichi Yazaki
Department of Physics, Aichi University of Education, Kariya 448-8542, Japan

(Received 21 May 2007; accepted 15 July 2007; published online 20 August 2007)

The use of two pressure sensors [Fusco *et al.*, J. Acoust. Soc. Am. **91**, 2229 (1992)] makes it possible to determine the acoustic intensity of a gas column in a duct, but the application of this method was limited to wide ducts. In this letter, the formulation of the method is modified to include narrow ducts where the duct radius is as small as the viscous boundary layer thickness of the gas. The validity of this method is shown by comparison with the direct measurements of the pressure and velocity. © *2007 American Institute of Physics.* [DOI: 10.1063/1.2768929]

Acoustic intensity I represents a time-averaged energy flux sustained by coupled oscillations of pressure and velocity of a gas. For a sound wave in a duct, the acoustic intensity I is expressed by

$$I=\langle PV\rangle, \tag{1}$$

where P and V are the acoustic pressure and cross-sectional average of axial acoustic particle velocity and angular brackets represent time average. Recent measurements of I have been contributing to deepening the experimental understanding of acoustics in ducts, particularly those involving thermoacoustic energy conversions and energy dissipations.

Two methods have been widely used in the measurement of I. One is the direct method simultaneously measuring P and V using a small pressure sensor and a laser Doppler velocimeter (LDV). This method was first reported in 1998 (Ref. 1) and has been employed for the measurements of the acoustic power of thermoacoustic heat engines[2–6] and the quality factor of a resonator.[7]

The second method is called a two-sensor method. In this method, P and V in Eq. (1) are deduced from the difference and the sum of the two pressures located side by side on the duct wall.[8] The simplicity of the measurements is a great advantage of the two-sensor method over the direct method involving LDV. This method has been successfully used in the study of thermoacoustic phenomena,[9–11] and we have also confirmed the validity of this method. However, the application of this method was limited in "wide" ducts where the duct size such as the radius r_0 for a circular tube is much longer than the viscous boundary layer thickness δ of the gas.[12]

In this letter, we show from the comparison with the direct method that the conventional two-sensor method becomes inapplicable for a narrow tube having $r_0/\delta\sim 1$ and introduce a more general formulation of the two-sensor method that is applicable regardless of the magnitude of r_0/δ. This method would become a good experimental tool in thermoacoustics and in the study of acoustic waves in various porous media.

Figure 1 shows the schematic illustration of the present experimental setup to test the two-sensor method when $r_0/\delta=r_0\sqrt{\omega/2\nu}$ (Ref. 12) is close to 1. A Pyrex glass tube with internal radius of 2 mm was filled with atmospheric air at room temperature (284 K). The gas column in the tube was driven sinusoidally at $f=2.0$ Hz by an acoustic driver at the end of the tube. Thus, the nondimensional parameter $r_0/\delta=1.3$ was achieved in this experiment.

The acoustic pressure $P(x,t)=p(x)e^{i\omega t}$ $(\omega=2\pi f)$ was measured by a series of small pressure transducers flush mounted on the tube wall, where the axial coordinate x was taken from the driver end of the tube $(x=0)$ to the other end $(x=4.3$ m). The amplitude $|p(x)|$ and the phase relative to $P(4.2)$, $\Theta(x)=\arg[p(x)/p(4.2)]$, were determined from the spectra obtained with a multichannel spectrum analyzer.

We closed the end of the tube with a rigid plate and found that $\Theta(x)$ monotonically decreased with increasing x, and its total decrease was very small $[\Theta(0.1)=2.6°]$. The measuring uncertainty of I by the two-sensor method tends to become large when the acoustic wave has a high standing-wave ratio.[8] Hence, we replaced the rigid plate at the end with a rubber balloon, to increase the traveling wave component. The measured Θ and $|p|$ are shown in Figs. 2(a) and 2(b) by solid circles (•). It is shown that $\Theta(0.1)$ reached 54.5° and $d\Theta/dx$ became large enough to decrease the error within the size of the symbols shown in Fig 2.

We determine the acoustic intensity I from the measured pressure shown in Figs. 2(a) and 2(b). We choose a pair of pressures separated by the distance $\Delta x=0.7$ m and denote the pressure closer to the driver as p_A and the other as p_B. For a plane and monofrequency acoustic wave, the acoustic intensity I (Ref. 8) is expressed as

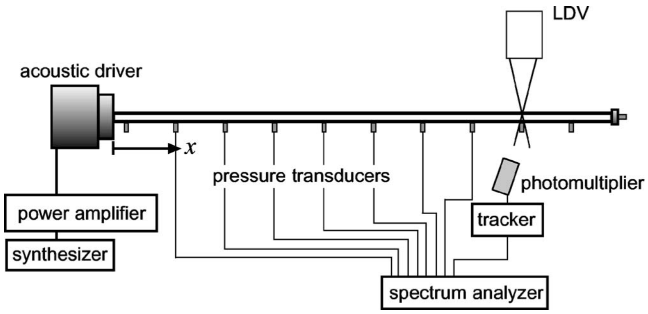

FIG. 1. Schematic illustration of a present experimental setup. The axial coordinate x is directed from the acoustic driver to a closed end.

$$I = \frac{1}{8\omega\rho}\{\mathrm{Im}[H](|p_A|^2 - |p_B|^2) + 2\,\mathrm{Re}[H]|p_A||p_B|\sin\theta\}, \tag{2}$$

using

$$H = \frac{kF}{\cos(\tilde{k}\Delta x/2)\sin(k\Delta x/2)}, \tag{3}$$

where ρ is the mean density of the gas, Re[ ] and Im[ ] represent the real and imaginary parts, and $\theta = \arg[p_A/p_B]$ represents the phase lead of p_A relative to p_B. Also, k represents the complex wave number and F is a complex factor relating the cross-sectional averaged velocity with the pressure gradient in the momentum equation,[13,14]

$$V = \frac{iF}{\omega\rho}\frac{dP}{dx}. \tag{4}$$

In contrast to the conventional two-sensor method,[8] we propose to use k and F given by

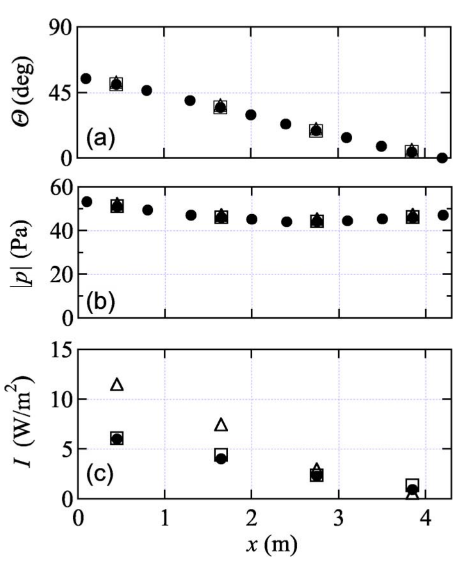

FIG. 2. Acoustic field in a tube with $r_0/\delta = 1.3$; (a) the phase Θ of the acoustic pressure $p(x)$ relative to $p(4.2)$, (b) pressure amplitude $|p|$, and (c) acoustic intensity I. Data shown by the solid circles (●) denote the results of the direct measurements. Open triangles (△) represent the data obtained by the modified two-sensor method with the use of k and F, whereas open squares (□) by the conventional two-sensor method.

$$k = -ik_0\sqrt{\frac{J_0(i^{3/2}\sqrt{2}r_0/\delta)}{J_2(i^{3/2}\sqrt{2}r_0/\delta)}}\sqrt{\gamma + (\gamma-1)\frac{J_2(i^{3/2}\sqrt{2\sigma}r_0/\delta)}{J_0(i^{3/2}\sqrt{2\sigma}r_0/\delta)}} \tag{5}$$

and

$$F = 1 - \frac{2J_1(i^{3/2}\sqrt{2}r_0/\delta)}{i^{3/2}(\sqrt{2}r_0/\delta)J_0(i^{3/2}\sqrt{2}r_0/\delta)}, \tag{6}$$

where J_n is the nth order complex Bessel function,[14,15] k_0 is the wave number in free space, and σ and γ denote the Prandtl number and the specific heat ratio. The obtained I in the use of k and F is plotted by open squares (□) in Fig. 2(c). For comparison, we also determined I by the conventional two-sensor method proposed by Fusco *et al.*,[8] and plotted it by open triangles (△). The derivation of this method is done by replacing k and F in Eqs. (5) and (6) with their approximate solutions obtained when the boundary-layer approximation is appropriate. Figure 2(c) shows a significant difference between I by these two methods; the conventional method gives 85% larger I at $x = 0.45$ m than that determined by the present method.

In order to show the validity of the method that we developed, we determined I by the direct method. We measured the acoustic particle velocity $U(x,t)$ on the central axis using a LDV with cigarette smoke as seeding particles. The relative phase of U was determined with reference to the pressure P at the same x. The observed velocities ($|U| < 0.4$ m/s) were all low enough to ensure laminar flow. Hence, on the basis of the laminar oscillating flow theory, the radial average velocity $V(x,t) = v(x)e^{i\omega t}$ was determined from the measured U and the theoretical factor $\Gamma = |\Gamma|e^{i\Phi}$ as $V = U/\Gamma$. Here, $|\Gamma| = 1.99$ and $\Phi = 4.1$ (Ref. 7) when $r_0/\delta = 1.3$. The acoustic intensity is then determined from Eq. (1) as

$$I = \frac{1}{2}|p||v|\cos\phi, \tag{7}$$

where $\phi = \arg[v/p]$. The acoustic intensity I by the direct method is further plotted in Fig. 2(c) by solid circles (•). We see that I obtained by the modified two-sensor method agrees with that by the direct method. Thus, we conclude that the conventional two-sensor method is inapplicable when $r_0/\delta = 1.3$, and the present method by the use of k and F in Eqs. (5) and (6) should be applied.

The complex wave number k in Eq. (5) is theoretically derived by Tijdeman[15] and is experimentally verified including the narrow tube region from $r_0/\delta = 10^{-2}$ to 10 very recently.[16] As is shown in these literatures, k obtained under the boundary-layer approximation deviates from the exact k below $r_0/\delta \sim 4$. However, in the present experiment, the use of the boundary-layer limit does not cause serious difference, even though r_0/δ is as low as 1.3. As shown in Figs. 2(a) and 2(b), the phase Θ and the amplitude $|p|$ estimated by the two-sensor methods (△ and □) fall onto the data obtained by the direct measurements. Such agreement is attributable to the fact that Δx is much smaller than the wavelength in this experiment.

The large discrepancy of I originates from the complex factor $F = |F|e^{i\Psi}$. We plotted the absolute value $|F|$ and the argument Ψ in Figs. 3(a) and 3(b), together with those obtained under the boundary-layer approximation. The differ-

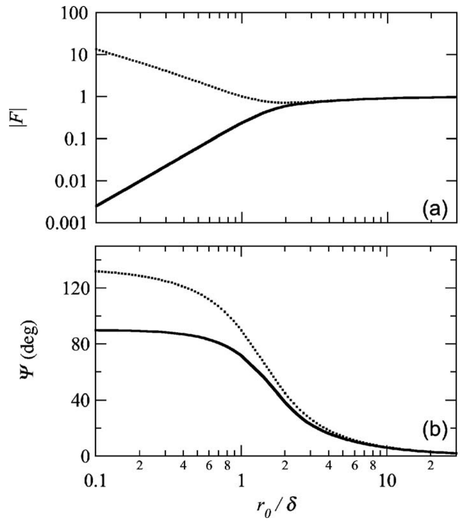

FIG. 3. The complex factor $F=|F|e^{i\Psi}$ as a function of the ratio r_0/δ. Dotted curves represent F obtained under the boundary-layer approximation.

ence of $|F|$ rapidly grows below $r_0/\delta\sim 4$, and the difference of Ψ becomes a few degrees below $r_0/\delta=10$. When $r_0/\delta=1.3$, the ratio of their absolute values reaches 2.2 and the phase difference becomes 12°, respectively. These differences are reflected to the large deviation of I in Fig. 2(c).

The apparent simplicity is a great advantage of the two-sensor method over the direct method, but its application was limited to the wide ducts where the boundary-layer approximation is appropriate. In this work, we extended the applicability of this method to "narrow" tube regions and verified it by the direct method. The modified two-sensor method would offer a quick and accurate determination of the acoustic intensity in ducts and porous media.

[1] T. Yazaki and A. Tominaga, Proc. R. Soc. London, Ser. A **454**, 2113 (1998).
[2] T. Yazaki, A. Iwata, T. Maekawa, and A. Tominaga, Phys. Rev. Lett. **81**, 3128 (1998).
[3] H. Bailliet, P. Lotton, M. Bruneau, V. Gusev, J. C. Valiere, and B. Ganzengel, Appl. Acoust. **60**, 1 (2000).
[4] T. Yazaki, T. Biwa, and A. Tominaga, Appl. Phys. Lett. **80**, 157 (2002).
[5] Y. Ueda, T. Biwa, U. Mizutani, and T. Yazaki, Appl. Phys. Lett. **81**, 5252 (2002); J. Acoust. Soc. Am. **115**, 1134 (2004).
[6] T. Biwa, Y. Tashiro, U. Mizutani, M. Kozuka, and T. Yazaki, Phys. Rev. E **69**, 066304 (2004).
[7] T. Biwa, Y. Ueda, H. Nomura, U. Mizutani, and T. Yazaki, Phys. Rev. E **72**, 026601 (2005).
[8] A. M. Fusco, W. C. Ward, and G. W. Swift, J. Acoust. Soc. Am. **91**, 2229 (1992).
[9] G. W. Swift, D. L. Gardner, and S. Backhaus, J. Acoust. Soc. Am. **105**, 711 (1999).
[10] G. Petculescu and L. A. Wilen, ARLO **3**, 71 (2002).
[11] D. L. Gardner and G. W. Swift, J. Acoust. Soc. Am. **114**, 1905 (2003).
[12] The viscous boundary-layer thickness δ is given by $\sqrt{2\nu/\omega}$, where ν is the kinematic viscosity of the gas.
[13] G. W. Swift, *Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators* (Acoustical Society of America, Sewickley, PA, 2002).
[14] A. Tominaga, Cryogenics **35**, 427 (1995); *Fundamental Thermoacoustics* (Uchida Roukakuho Publishing, Tokyo, Japan, 1998).
[15] H. Tijdeman, J. Sound and Vib. **39**, 1 (1975).
[16] T. Yazaki, Proc. R. Soc. London, Ser. A (to be published).