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著者	琵琶　哲志
journal or publication title	Review of scientific instruments
volume	76
number	12
page range	124901-1-124901-5
year	2005
URL	http://hdl.handle.net/10097/46683

doi: 10.1063/1.2140489

Calibration of a thermocouple for measurement of oscillating temperature

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(Received 7 September 2005; accepted 31 October 2005; published online 14 December 2005)

We report on the dynamic calibration of a thermocouple for the measurement of the oscillating temperature. Temperature oscillation is induced in a gas-filled tube by a periodically forced pressure oscillation and measured by a thermocouple. The radial profile of the measured temperature oscillations is compared with the theoretical one, which is determined from the simultaneously measured pressure. A response function of the thermocouple is obtained from the difference in amplitude and phase angle between them by varying the diameter of the thermocouple, oscillating frequency, tube radius, and working gas. We can obtain a true temperature oscillation by using the response function given in this experiment. © *2005 American Institute of Physics.*
[DOI: 10.1063/1.2140489]

I. INTRODUCTION

An acoustic wave in a gas-filled tube is capable of converting acoustic heat flow, $Q=\rho_m T_m\langle SU\rangle$, into work flow, $I=\langle PU\rangle$, or vice versa through heat exchange with the tube wall,[1,2] where S, U, and P denote the oscillating entropy, velocity, and pressure of a gas, respectively, ρ_m and T_m a mean density and temperature,[3] and the angular brackets represent the time average over a period. The method for the accurate determination of the acoustic work flow I in the tube has been experimentally established through simultaneous measurements of P and U,[4] which has contributed to deepening the understanding of the thermoacoustics[5–8] and acoustics.[9] However, the method of measuring the heat flow Q in the tube has not been well established because of the difficulty in the direct measurement of oscillating entropy S.

We direct our attention to the total energy flow $H=Q+I$, rather than on Q itself. Fortunately, most of the gases, including inert ones like He and Ar and even air at an ambient temperature, can be well approximated as an ideal gas. The relation $H=\rho_m C_P\langle TU\rangle$ holds for an ideal gas, where C_P is an isobaric specific heat and T the oscillating temperature of the gas. Once H is derived by inserting measured T and U into this relation, Q is obviously deduced by subtracting the measured I from H. This is true only when a thermometer can measure accurately the temperature of a gas parcel at any instance without any delay. Otherwise, corrections are needed to determine accurately the value of H.

To the best of our knowledge, only a very limited number of works have been reported on the oscillating temperature measurement of the acoustic wave in a tube. Huelsz *et al.*[10] used a cold wire anemometer for the measurement in the fundamental mode (130 Hz) of a rectangular resonator filled with one-bar air. They stressed the need to employ a scale corrected accurately for the phase delay of the anemometer in response to temperature oscillations. We consider the need of corrections, not only for the delay in phase but also for that in the amplitude of temperature oscillations, both of which would depend on factors such as the size of the probe, the frequency of the acoustic wave, and thermal properties of a gas. It is, therefore, of critical importance to study a thermometer response to temperature oscillations in a gas by taking into account all possible factors affecting both the amplitude and phase in order to reliably determine H in a variety of acoustic systems.

In this paper, we report on the calibration of a thermocouple to measure oscillating temperatures of the gas-filled cylindrical tube, where the gases are forced by pressure oscillation. The radial distribution of oscillating temperatures was measured and compared with a solution of a laminar oscillating flow theory.[1,2] From the difference in these temperature oscillations, we derive a response function that serves as a correcting factor for both the amplitude and phase angle of the measured temperature. The response function is determined by varying the diameter of the thermocouple, the frequency of acoustic wave, the tube radius, and the working gas. Using the response function, we can reliably determine temperature oscillations and hence the heat flow Q.

II. THEORETICAL TREATMENT FOR TEMPERATURE OSCILLATIONS IN A TUBE

We employ a conventional complex number as acoustic variables to describe time-oscillatory quantities of gas parcel: for example, the pressure oscillations are expressed as $P=P_1 e^{i\omega t}$, where ω and P_1 are the angular frequency and amplitude of pressure oscillations. A mean value of the oscillating pressure is denoted as P_m. We first consider the acoustic plane wave in an ideal gas. In the adiabatic limit, the temperature oscillations T_S around the mean temperature T_m can be expressed in terms of P as

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$$T_s = \left(\frac{\partial T}{\partial P}\right)_s P = \frac{\gamma - 1}{\gamma}\frac{T_m}{P_m}P_1 e^{i\omega t}, \tag{1}$$

where the subscript s is attached to emphasize the adiabatic condition and γ is the ratio of isobaric to isochoric specific heats of a gas. In reality, any acoustic wave in a tube is subjected to the heat exchange with the tube wall. The temperature oscillation T under such arbitrary conditions is theoretically predicted from a laminar oscillating flow theory[1,2] and are given in the following form:

$$T = (1 - f_\alpha)T_s + g_\alpha \frac{U}{\omega}\frac{dT_w}{dx}, \tag{2}$$

where T_w is the temperature of the tube wall, x is an axial coordinate along the tube, and both $(1-f_\alpha)$ and g_α are functions involving only the radial coordinate r directed from the cylindrical axis to the wall, as described below.

The first term on the right-hand side in Eq. (2) represents temperature oscillations induced by pressure oscillations under the condition where the gas exchanges heat with the tube wall, and, hence, corresponds to the extension of Eq. (1) to a more general situation. The second term refers to temperature oscillations caused by the displacement of the gas along the tube wall with an axial temperature gradient dTw/dx. If T_w remains constant along x, Eq. (2) is obviously reduced to

$$\frac{T}{T_s} = [1 - f_\alpha(r)]. \tag{3}$$

The function $(1-f_\alpha)$ is now expressed simply in terms of a dimensionless radial temperature profile. For a cylindrical tube with a radius r_0, f_α is analytically given from a laminar oscillating flow theory[1,2] as

$$f_\alpha(r) = \frac{J_0\left(\sqrt{2}i^{3/2}r/\delta_\alpha\right)}{J_0\left(\sqrt{2}i^{3/2}r_0/\delta_\alpha\right)}, \tag{4}$$

where J_0 is the complex Bessel function of the first kind, r is a radial coordinate ranging from 0 to r_0, and δ_α is a thermal boundary layer thickness given as $\delta_\alpha = \sqrt{2\alpha/\omega}$, using the thermal diffusivity α of a gas.

The absolute value of $(1-f_\alpha)$, Abs$[1-f_\alpha]$, and its argument, Arg$[1-f_\alpha]$, represent the amplitude ratio of T to T_s and the phase lead of T relative to T_s and hence P, respectively. They are plotted in Figs. 1(a) and 1(b) as a function of the radial coordinate normalized with respect to r_0, under three conditions: $r_0/\delta_\alpha = 1000$, 5, and 0.5. In the case of $r_0/\delta_\alpha = 1000$, values of both Abs$[1-f_\alpha]$ and Arg$[1-f_\alpha]$ are indeed very close to unity and zero degrees, respectively, nearly throughout the cross section of the tube. Hence, the acoustic wave in a tube can be regarded as an adiabatic one when r_0 is far greater than δ_α. When $r_0/\delta_\alpha = 0.5$, we find Abs$[1-f_\alpha] \approx 0$ and Arg$[1-f_\alpha] \approx 90°$. This means that the acoustic wave is now described as an isothermal one, where temperature oscillations are lost and temperature is everywhere the same as the wall temperature T_w. In the intermediate case of $r_0/\delta_\alpha = 5$, values of Abs$[1-f_\alpha]$ and Arg$[1-f_\alpha]$ change significantly as a function of r due to imperfect thermal contact of the gas with the wall.

Before ending this section, we should emphasize the fact that, as long as T_w remains constant along x in the tube, the

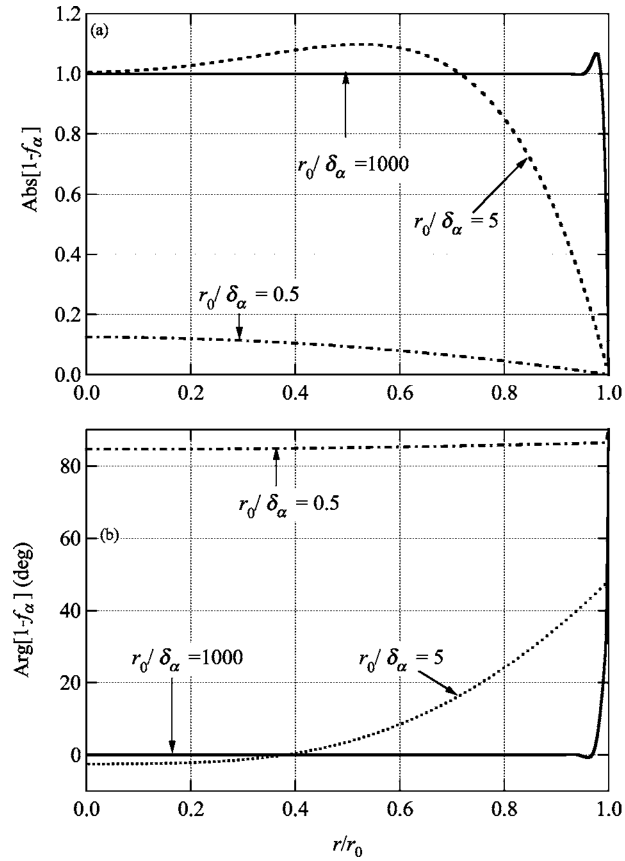

FIG. 1. Radial distributions of (a) Abs$[1-f_\alpha]$ and (b) Arg$[1-f_\alpha]$ for acoustic waves in a tube with different values of $r_0/\delta_\alpha = 1000$, 5, and 0.5.

radial distribution of the temperature ratio T/T_s is expressed as a function of only the parameter r_0/δ_α, which can be determined from experiments described below. In the present work, we have employed the theoretical radial temperature profile $(1-f_\alpha)$ as a reference to obtain a true temperature and discussed how accurately and reliably a correction is made for both the existing amplitude damping and phase delay in the response of a thermocouple to oscillating temperatures.

III. EXPERIMENTS

A. Experimental setup

The present experimental apparatus is schematically illustrated in Fig. 2. Three Pyrex glass tubes of length 0.6 m with different inner diameters of $2r_0 = 11.5, 21, 40$ mm were used as cylindrical waveguides. One end of the glass tube is rigidly closed with a solid plate, and the other end is con-

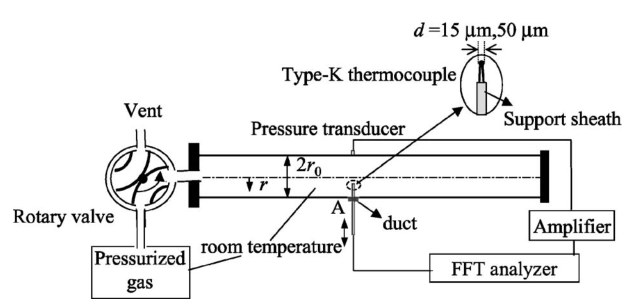

FIG. 2. Schematic illustrations of the present experimental setup.

TABLE I. Specific heat ratio γ and thermal diffusivity α of helium, argon, and nitrogen at 290 K.

Working gas	Specific heat ratio	Thermal diffusivity
Helium	5/3	180×10^{-6} m^2/s
Argon	5/3	17×10^{-6} m^2/s
Nitrogen	7/5	21×10^{-6} m^2/s

nected to the outlet of a rotary valve. Pressure oscillations are activated in the tube by designing the rotary valve such that it allows us to switch periodically either to a pressurized tank or a vent port. Its switching frequency f was varied from 0.5 to 18 Hz. The wall temperature was monitored and found to remain the same as the room temperature of 290 K throughout the experiments. Thus, we validated our basic assumption that the second term arising from the temperature gradient along the tube[11] can be ignored and, hence, only the first term in Eq. (2) is retained. Helium, nitrogen, and argon gases were employed as working gas. The physical properties of relevant gases at 290 K are summarized in Table I.

B. Temperature and pressure measurements

We used two chromel-alumel (K-type) thermocouples: one with the diameter of $d=15$ μm and the other with $d=50$ μm. The sensitivity of these thermocouples was experimentally confirmed as 39 μV/K. The thermocouple was covered with a support sheath made of a stainless-steel tube, but the junction was exposed to its surrounding working gas. The sheathed thermocouple was inserted into a glass tube through a narrow duct mounted on the wall, as marked with an A in Fig. 2. A small gap between the duct and the sheath was sealed with a rubber O ring, which enabled us to move smoothly the junction at any radial position and measure the radial distribution of temperature oscillations in the tube. The pressure at the same position as that where the junction of the thermocouple was located was also measured using a small pressure transducer mounted on the wall via a short duct.

Electrical signals from the pressure transducer and thermocouple were simultaneously recorded with a multichannel 24 bit spectrum analyzer. By using the power and phase spectra of their signals, we determined both the pressure oscillations $P=P_1e^{i\omega t}$ and temperature ones $T_{\rm ex}(r) = T_{\rm ex1}(r)e^{i\{\omega t+\phi(r)\}}$, where the suffix ex is attached to emphasize the experimentally derived temperature by the use of a thermometer. The temperature oscillations T_s for the adiabatic acoustic wave were determined by inserting measured values of T_m, P_m, and P into Eq. (1). In this way, we could determine experimentally the r dependence of the temperature ratio $T_{\rm ex}(r)/T_s$ to allow a direct comparison with the theoretical one $(1-f_\alpha)$ given in Eq. (3).

IV. RESULTS AND DISCUSSION

A. Response function of a thermocouple

The radial dependence of temperature oscillations was measured at $f=15$ Hz, $r_0=10.5$ mm, and helium as a working gas. Abs$[T_{\rm ex}/T_s]$ and Arg$[T_{\rm ex}/T_s]$, obtained with $d=15$ and 50 μm, are shown by solid circles and squares in Fig. 3 as a function of r normalized with respect to r_0. The measurements were made with P_1 from 0.5 to 5.0 kPa, and the temperature ratio $T_{\rm ex}/T_s$ was found to be independent of P_1. The theoretical profile $(1-f_\alpha)$ is shown in the form of solid curves in Fig. 3. We see that the r dependence of $T_{\rm ex}/T_s$ obtained with $d=15$ and 50 μm qualitatively reproduces that of $(1-f_\alpha)$. However, there exist quantitative disagreements in both the absolute value and argument between $T_{\rm ex}/T_s$ and $(1-f_\alpha)$: the observed amplitude damping and phase delay indicate that the thermocouples cannot reproduce the true temperature oscillation. The deviation of $T_{\rm ex}/T_s$ measured by the thermocouple with $d=50$ μm from $(1-f_\alpha)$ is larger than that with $d=15$ μm, so $T_{\rm ex}/T_s$ would approach $(1-f_\alpha)$ with decreasing the diameter d of the thermocouple. However, it becomes harder to handle a thermocouple wire when its diameter becomes thinner than 15 μm.

Instead of using a further thinner thermocouple, we tried to reproduce the theoretical profile $(1-f_\alpha)$ from the measured $T_{\rm ex}/T_s$. The value of Abs$[T_{\rm ex}/T_s]$ was multiplied by 1.33 ($\equiv 1/\Gamma$) and 4.78 while Arg$[T_{\rm ex}/T_s]$ was advanced by 37° ($\equiv -\theta$) and 75° for the data with $d=15$ μm and $d=50$ μm, respectively. The corrected Abs$[T_{\rm ex}/T_s]$ and Arg$[T_{\rm ex}/T_s]$, which are plotted by open circles and squares in Figs. 3, agree with $(1-f_\alpha)$ to within 5%. Therefore, the fol-

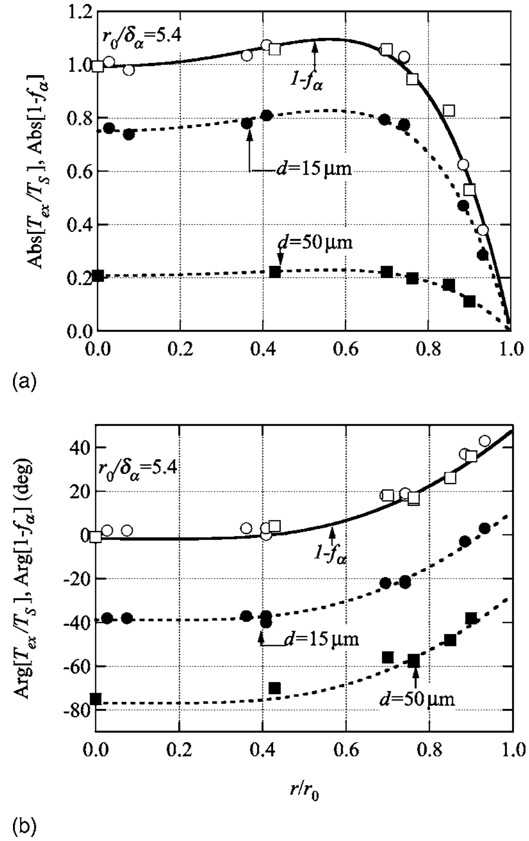

FIG. 3. Radial distributions of (a) the absolute value and (b) the argument of $T_{\rm ex}/T_s$ and $(1-f_\alpha)$ in the tube with $r_0/\delta_\alpha=5.4$. Abs$[1-f_\alpha]$ and Arg$[1-f_\alpha]$ are shown by solid curves. Solid circles and squares represent the data obtained with $d=15$ μm and 50 μm thermocouples, respectively. Open circles and squares are obtained after the correction described in the text. Uncertainties in determining the radial coordinate were smaller than the size of the symbols.

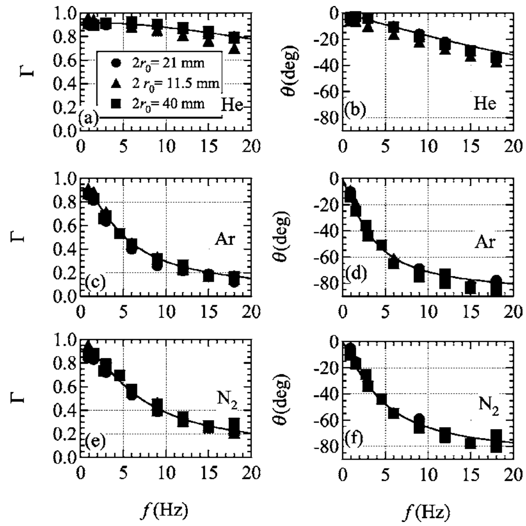

FIG. 4. Frequency dependences of $\Gamma(=\mathrm{Abs}[Z_{ex}])$ and $\theta(=\mathrm{Arg}[Z_{ex}])$ of acoustic waves in a tube filled with different working gases [(a) and (b) in helium, (c) and (d) in argon, (e) and (f) in nitrogen]. The data obtained by using different tube radii are found to fall onto master curves. Solid lines represent fitting lines using Eq. (6) with $\varepsilon=0.08$.

lowing relation is given between T_{ex}/T_s and $(1-f_\alpha)$ by using a complex number $Z_{ex}=\Gamma e^{i\theta}$:

$$\frac{1}{Z_{ex}}\frac{T_{ex}}{T_s} \cong (1-f_\alpha) = \frac{T}{T_s},$$

and, hence,

$$T_{ex} \cong Z_{ex} T.$$

We hereafter call Z_{ex} a response function of the thermocouple, which converts both the absolute value and argument of T_{ex} into those of true T. An ideal thermocouple with a perfect response to temperature oscillation T should possess $Z_{ex}=1$ ($\Gamma=1$ and $\theta=0$), but $\Gamma<1$ and $\theta<0$ in any thermocouple. Thus, the determination of the response function Z_{ex} enables us to deduce true temperature oscillations T from measured T_{ex} by using a thermocouple of a finite size.

To examine the behavior of Z_{ex} under different experimental conditions, we carried out repeatedly experiments by changing f, r_0 and working gas. We employed the thermocouple with $d=15$ μm rather than that with $d=50$ μm because of its better response to temperature oscillations. Figures 4(a) and 4(b) show $\mathrm{Abs}[Z_{ex}]=\Gamma$ and $\mathrm{Arg}[Z_{ex}]=\theta$ as a function of f for helium gas. Both Γ and θ are found to depend strongly on f. The value of Γ corresponding to the amplitude ratio of T_{ex} to T decreases with increasing f, while that of θ corresponding to the phase delay of T_{ex} relative to T becomes large. This means that the response of the thermocouple becomes worse with increasing f.

Figures 4(c)–4(f) show the results obtained when nitrogen and argon are used as working gas. Frequency dependences of Γ and θ in nitrogen and argon are qualitatively similar to those in helium, but they are quantitatively different from those in helium. The better response in helium than in nitrogen and argon can be attributed to the possession of a higher thermal diffusivity α in helium, as listed in Table I. As a working gas with a high α can easily diffuse heats around it, the thermocouple inside the gas can faster reproduce the

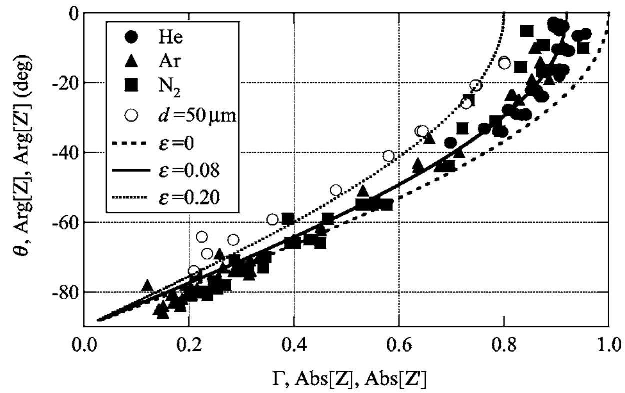

FIG. 5. The relation between $\Gamma(=\mathrm{Abs}[Z_{ex}])$ and $\theta(=\mathrm{Arg}[Z_{ex}])$ obtained under different conditions. Solid and open symbols were obtained by using $d=15$ μm and 50 μm, respectively. The dashed curve is drawn by using Eq. (5), while the solid and dotted curves by using Eq. (6) with $\varepsilon=0.08$ and 0.20, respectively.

temperature of helium than that of nitrogen and argon. The results obtained using $2r_0=11.5$ mm, 21 mm, and 40 mm are incorporated in Fig. 4. We found that both Γ and θ are independent of r_0.

B. Interpretation of Z_{ex}

Tagawa *et al.* expressed the response of a thermocouple using the response function Z in a first-order lag system,[12] which is given as

$$Z = \frac{1}{1+i\omega\tau}, \tag{5}$$

where τ is a time constant for a thermocouple to achieve thermal equilibrium with the surrounding gas. Equation (5) is plotted in Fig. 5 in the form of $\mathrm{Arg}[Z]=-\arctan(\omega\tau)$ against $\mathrm{Abs}[Z]=1/\sqrt{1+(\omega\tau)^2}$ as a dashed curve. The present data of Z_{ex} are incorporated in Fig. 5 for comparison. An agreement between Z_{ex} and $Z$is fairly good in the range $\Gamma<0.5$, but the discrepancy significantly appears above 0.5. In particular, in the limit of θ and $\mathrm{Arg}[Z]$ to 0, the value of $\mathrm{Abs}[Z]$ obviously approaches unity, whereas that of Γ is well suppressed below unity.

A whole set of Γ vs θ data may be phenomenologically described by the response function Z' in the following form:

$$Z' = \frac{1-\varepsilon}{1+i\omega\tau}, \tag{6}$$

where ε is a positive number. Now the absolute value of Eq. (6), i.e., $\mathrm{Abs}[Z']$ approaches $(1-\varepsilon)$ in the limit of $\mathrm{Arg}[Z']$ to 0. A solid curve in Fig. 5 represents Z' when $\varepsilon=0.08$. The agreement with Z_{ex} becomes almost perfect over a whole range of Γ. Moreover, we found that the value of ε does not sensitively depend on the gas species. The time constant τ is deduced to be 5, 35, and 47 ms for helium, nitrogen, and argon gases, respectively, by fitting $\mathrm{Abs}[Z']$ and $\mathrm{Arg}[Z']$ to the data in Fig. 4 (shown by solid curves).

Forney *et al.*[13] theoretically derived the response function in the case where the thermal diffusion along a thermocouple wire cannot be neglected. The additional parameter ε in Eq. (6) is similar to the term that they used to represent a degree of the heat leak through the wire. The parameter ε would depend strongly on the cross-sectional area of the thermocouple. The data of Γ and θ obtained by using the thermocouple with 50 μm in helium are included in Fig. 5 as open circles. The data are found to be well fitted to Eq. (6) with the choice of a larger value of ε equal to 0.25, as shown by a dotted line in Fig. 5. We believe the analysis above to ascertain the validity of Eq. (6) and the existence of heat leak in a thermocouple.

As is clear from the argument above, the response function of a thermocouple Z_{ex} is well described in terms of Z' given by Eq. (6), particularly when thermal diffusion along a thermocouple wire is non-negligible. An oscillating temperature T in an acoustic wave can be reliably determined from T_{ex} measured with a thermocouple, once the Γ and θ data in Z_{ex} are analyzed in terms of Eq. (6).

V. SUMMARY

We showed the dynamic calibration of a thermocouple to measure temperature oscillations induced by pressure oscillations in gas-filled tube. Radial distributions of temperature oscillations were measured and compared with the analytical solution derived from a laminar oscillating flow theory. We experimentally determined the response function of the thermocouple from the difference between them. The response function Z_{ex} is found to be described well by Eq. (6). The oscillating temperature T can be accurately determined by measuring T_{ex} with the thermocouple with its subsequent division by Z_{ex} at the frequency employed.

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