## Dual Arm Coor di nation in Space Free－Fl yi ng Robot

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# Dual Arm Coordination in Space Free-Flying Robot 

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#### Abstract

This paper presents the control problem of multiple manipulators installed on a free-flying space robot. Firstly, kinematics and dynamics are studied and the Generalized Jacobian Matrix is formulated for the motion control of multi-arm system. Then, individual and coordinated control of dual manipulators is discussed. For the coordinated operation, a new method to control two arms simultaneously: one arm traces a given path, while the other arm works both to keep the satellite attitude and to optimize a total operation torque of the system, is developed. By means of it, an interesting torque optimum behaviour is observed and a practical target capture operation is exhibited by computer simulation.


## 1 Introduction

Space robotics is a new field. For a successful development of space projects, robotics and automation should be a key technology. Autonomous and dexterous space robots could reduce the workload of astronauts and increase operational efficiency in many missions. One major characteristic of space robots, which clearly distinguishes them from ground-operated ones, is the lack of a fixed base. Any motion of the manipulator arm will induce reaction forces and moments in the base, which disturb its position and attitude.

In order to compensate the reaction and keep the satellite attitude, on-board Reaction Wheels (RW) or

Control Moment Gyros (CMG) will be generally used $[1,2]$. Also, manipulators with redundant joints could be useful for this purpose [3,4]. However, compensation capability of such devices is limited. To cope with it, dual arm coordination is considerd to be effective.
This papers treats the control problem of multiple manipulators installed on a space robot. Firstly, the Generalized Jacobian Matrix (GJM) which guarantees proper motion control of free-flying manipulators, is formulated for the multi-arm system. Then, individual and coordinated motion control of dual manipulators for target capture operation is discussed. For the coordinated motion, a new control method of two arms: one arm traces a given path, while the other arm is operated both to keep the satellite attitude and to optimize a total operation torque, is developed on the basis of redundancy resolution technique. Torque optimization is essential to space robots, in which the size of actuators and electric power supply is limited. By means of it, a practical target capture scheme is proposed and its availability is examined through computer simulation.

## 2 Kinematics and Dynamics

### 2.1 Modeling of Space Free-Flying Robot with Multiple Arms

This paper deals with a mech anical link system which comprises a base satellite and a plural number of robot manipulators. The system freely flies in the inertial space, on which no external forces or moments are ex-
erted. The linear and angular momentum is conserved in it.
Let us suppose the system has l pieces of arms, which are numbered from 1 to $l$. Each link of the $k$-th arm are assumed to be rigid and numbered from a satellite side 1 to $n_{k}$. The $i$-th joint is between the $i$-th link and the $(i-1)$-th link. Angle of the $j$-th joint of the $k$-th arm is described as $\phi_{j}^{k}$, with upper-right suffix to indicate arm number and lower-right suffix for joint or link number.

Let us define the joint variables coordinates as

$$
\begin{equation*}
\phi=\left(\phi_{1}^{1}, . ., \phi_{n_{1}}^{1}, \phi_{1}^{2}, . ., \phi_{n_{2}}^{2}, . ., \phi_{1}^{l}, . ., \phi_{n_{l}}^{l}\right)^{T} \tag{1}
\end{equation*}
$$

The system is operated by the torque applied at each joint.

$$
\begin{equation*}
\tau=\left(\tau_{1}^{1}, . ., \tau_{n_{1}}^{1}, \tau_{1}^{2}, . ., \tau_{n_{2}}^{2}, . ., \tau_{1}^{l}, . ., \tau_{n_{l}}^{l}\right)^{T} \tag{2}
\end{equation*}
$$

Other symbols are defined as follows:
$\mathbf{r}_{i}^{k} \in R^{3}:$ position vector of the mass center of link $i^{k}$
$\mathbf{r}_{g} \in R^{3}$ : position vector of the mass center of the whole system
$\mathbf{p}_{j}^{k} \in R^{3}:$ position vector of joint $j^{k}$
$\mathbf{p}_{r}^{k} \in R^{3}:$ position vector of the $k$-th hand
$\mathbf{k}_{j}^{k} \in R^{3}$ : unit vector indicating a rotational axis of joint $j^{k}$
$\mathbf{v}_{0} \in R^{3}$ : linear velocity of the base satellite ( $\equiv \dot{\mathbf{r}}_{0}$ )
$\mathbf{v}_{\tau}^{k} \in R^{3}$ : linear velocity of the manipulator hand ( $\equiv$ $\left.\dot{\mathbf{p}}_{r}^{k}\right)$
$\omega_{i}^{k} \in R^{3}$ : angular velocity of link $i^{k}$
$m_{i}^{k}$ : mass of link $i^{k}$
$w$ : total mass of the system
$\mathbf{I}_{i}^{k} \in R^{3 \times 3}$ : inertia tensor of link $i^{k}$ with respect to its mass center
$\mathbf{E} \in R^{3 \times 3}: 3 \times 3$ identity matrix
where $i=0,1, \ldots, n, j=1,2, \ldots, n$ and $k=1,2, \ldots, l$. All vectors are described with respect to the inertial coordinate system.

### 2.2 The Generalized Jacobian Matrix

The Generalized Jacobian Matrix (GJM) concept was established for the motion control of space freeflying single-arm system [5,6]. This paper applies the GJM concept to the multi-arm system.
The linear velocity of hand of the $k$-th $\operatorname{arm} \mathbf{v}_{\tau}^{k}$ is represented by

$$
\begin{equation*}
\mathbf{v}_{r}^{k}=\mathbf{v}_{0}+\omega_{0} \times\left(\mathbf{p}_{r}^{k}-\mathbf{r}_{0}\right)+\sum_{i=1}^{n_{k}}\left\{\mathbf{k}_{i}^{k} \times\left(\mathbf{p}_{r}^{k}-\mathbf{p}_{i}^{k}\right)\right\} \dot{\phi}_{i}^{k} \tag{3}
\end{equation*}
$$

The angular velocity of hand of the $k$-th $\operatorname{arm} \omega_{n}^{k}$ is also represented by

$$
\begin{equation*}
\omega_{n}^{k}=\omega_{0}+\sum_{i=1}^{n_{k}} \mathbf{k}_{i}^{k} \dot{\phi}_{i}^{k} \tag{4}
\end{equation*}
$$

From eqs.(3) and (4), the task variables coordinates of hand at velocity-level $\boldsymbol{\nu}^{k}=\left(\mathbf{v}_{r}^{k^{T}}, \omega_{n}^{k^{T}}\right)^{T}$ is described with the variables $\mathbf{v}_{0}, \omega_{0}$ and $\dot{\phi}$ as follows.

$$
\nu^{k}=\mathbf{J}_{s}^{k}\left[\begin{array}{c}
\mathbf{v}_{0}  \tag{5}\\
\omega_{0}
\end{array}\right]+\mathbf{J}_{m}^{k} \dot{\phi}
$$

where

$$
\begin{gather*}
\mathbf{J}_{s}^{k} \equiv\left[\begin{array}{cc}
\mathbf{E} & -\tilde{\mathbf{p}}_{0 r}^{k} \\
0 & \mathbf{E}
\end{array}\right], \quad \mathbf{p}_{0 r}^{k} \equiv \mathbf{p}_{r}^{k}-\mathbf{r}_{0}^{k}  \tag{6}\\
\mathbf{J}_{m}^{k} \equiv\left[\begin{array}{ccc}
0 \ldots 0 & \mathbf{k}_{1}^{k} \times\left(\mathbf{p}_{r}^{k}-\mathbf{p}_{1}^{k}\right) & \mathbf{k}_{2}^{k} \times\left(\mathbf{p}_{r}^{k}-\mathbf{p}_{2}^{k}\right) \\
0 \ldots 0 & \mathbf{k}_{1}^{k} & \mathbf{k}_{2}^{k} \\
\ldots & \mathbf{k}_{n}^{k} \times\left(\mathbf{p}_{r}^{k}-\mathbf{p}_{n}^{k}\right) & 0 \ldots 0 \\
\ldots & \mathbf{k}_{n}^{k} & 0 \ldots 0
\end{array}\right]
\end{gather*}
$$

and an operator $\tilde{\mathbf{r}}$ for a vector $\mathbf{r}=(x, y, z)^{T}$ is defined as

$$
\tilde{\mathbf{r}} \equiv\left[\begin{array}{ccc}
0 & -z & y  \tag{8}\\
z & 0 & -x \\
-y & x & 0
\end{array}\right]
$$

Here, $\mathbf{J}_{s}^{k} \in R^{6 \times 6}$ and $\mathbf{J}_{m}^{k} \in R^{6 \times n}$ are Jacobian matrixes for satellite motion and manipulator motion, respectively. On the other hand, the total linear and angular momentum of the system $\mathbf{P}, \mathbf{L}$ is represented as follows.

$$
\begin{equation*}
\mathbf{P} \equiv m_{0} \dot{\mathbf{r}}_{0}+\sum_{k=1}^{l} \sum_{i=1}^{n_{k}} m_{i}^{k} \dot{\mathbf{r}}_{\mathfrak{i}}^{k} \tag{9}
\end{equation*}
$$

$\mathbf{L} \equiv \mathbf{I}_{0} \omega_{0}+\mathbf{r}_{0} \times m_{0} \dot{\mathbf{r}}_{0}+\sum_{k=1}^{l} \sum_{i=1}^{n_{k}}\left(\mathbf{I}_{i} \omega_{i}^{k}+\mathbf{r}_{i}^{k} \times m_{\dot{i}}^{k} \dot{\mathbf{r}}_{i}^{k}\right)$.

These equations are rewritten with the variables $v_{0}, \omega_{0}$ and $\dot{\phi}$,

$$
\begin{align*}
{\left[\begin{array}{l}
\mathbf{P} \\
\mathbf{L}
\end{array}\right] } & =\left[\begin{array}{cc}
w \mathbf{E} & -w \tilde{\mathbf{r}}_{g g} \\
w \tilde{\mathbf{r}}_{g} & \mathbf{I}_{\omega}
\end{array}\right]\left[\begin{array}{l}
\mathbf{v}_{0} \\
\omega_{0}
\end{array}\right]+\left[\begin{array}{c}
\mathbf{J}_{T w} \\
\mathbf{I}_{\phi}
\end{array}\right] \dot{\phi} \\
& \boxminus \mathbf{H}_{s}\left[\begin{array}{l}
\mathbf{v}_{0} \\
\omega_{0}
\end{array}\right]+\mathbf{H}_{m} \dot{\phi} \tag{11}
\end{align*}
$$

where

$$
\begin{gather*}
\mathbf{J}_{T i}^{k} \equiv\left[0, \ldots, \mathbf{0}, \mathbf{k}_{1}^{k} \times\left(\mathbf{r}_{i}^{k}-\mathbf{p}_{1}^{k}\right), \mathbf{k}_{2}^{k} \times\left(\mathbf{r}_{i}^{k}-\mathbf{p}_{2}^{k}\right), \ldots,\right. \\
\left.\ldots, \mathbf{k}_{i}^{k} \times\left(\mathbf{r}_{i}^{k}-\mathbf{p}_{i}^{k}\right), \mathbf{0}, \ldots, \mathbf{0}\right]  \tag{12}\\
\mathbf{J}_{R i}^{k} \equiv\left[0, \ldots, 0, \mathbf{k}_{1}^{k}, \mathbf{k}_{2}^{k}, \ldots, \mathbf{k}_{i}^{k}, 0, \ldots, 0\right] \tag{13}
\end{gather*}
$$

$$
\begin{equation*}
\mathbf{J}_{T w} \equiv \sum_{k=1}^{l} \sum_{i=1}^{n_{k}} m_{i}^{k} \mathbf{J}_{T i}^{k} \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{I}_{\omega} \equiv \sum_{k=1}^{1} \sum_{i=1}^{n_{k}}\left(\mathbf{I}_{i}^{k}-m_{i}^{k} \tilde{\mathbf{r}}_{i}^{k} \tilde{\mathbf{r}}_{0 i}^{k}\right)+\mathbf{I}_{0} \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{I}_{\phi} \equiv \sum_{k=1}^{l} \sum_{i=1}^{n}\left(\mathbf{I}_{i}^{k} \mathbf{J}_{R i}^{k}+m_{i}^{k} \tilde{\mathbf{r}}_{\mathfrak{i}}^{k} \mathbf{J}_{T i}^{k}\right) \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{r}_{0 g} \equiv \mathbf{r}_{g}-\mathbf{r}_{0}, \quad \mathbf{r}_{0 i} \equiv \mathbf{r}_{\mathbf{i}}-\mathbf{r}_{0} \tag{17}
\end{equation*}
$$

Now assuming that no external forces or moments exert on the system, the linear and angular momentum is conserved respectively. Providing total momentum is zero: $\left(\mathbf{P}^{T}, \mathbf{L}^{T}\right)^{T}=0$, eq.(11) can be solved for $\mathbf{v}_{0}$ and $\omega_{0}$,

$$
\begin{gather*}
\mathbf{v}_{0}=-\left\{\frac{1}{w} \mathbf{J}_{T w}+\tilde{\mathbf{r}}_{0 g} \mathbf{I}_{s}^{-1} \mathbf{I}_{m}\right\} \dot{\boldsymbol{\phi}} \equiv \mathbf{J}_{v} \dot{\phi}  \tag{18}\\
\omega_{0}=-\mathbf{I}_{s}^{-1} \mathbf{I}_{m} \dot{\phi} \equiv \mathbf{J}_{\omega} \dot{\phi} \tag{19}
\end{gather*}
$$

where

$$
\begin{equation*}
\mathbf{I}_{s} \equiv \mathbf{I}_{\omega}+w \tilde{\mathbf{r}}_{g} \tilde{\mathbf{r}}_{0 g} \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{I}_{m} \equiv \mathbf{I}_{\phi}-\tilde{\mathbf{r}}_{g} \mathbf{J}_{T w} \tag{21}
\end{equation*}
$$

$\mathrm{I}_{s} \in R^{3 \times 3}$ and $\mathrm{I}_{m} \in R^{3 \times n}$ are inertia matrixes of the satellite and manipulators, respectively.

Eliminating $\mathbf{v}_{0}$ and $\omega_{0}$ from (5) by substituting (18) and (19), the task coordinates $\nu$ is directly related to the joint coordinates $\dot{\phi}$.

$$
\begin{align*}
\nu^{k} & =\mathbf{J}_{s}^{k}\left[\begin{array}{c}
\mathbf{v}_{0} \\
\omega_{0}
\end{array}\right]+\mathbf{J}_{m}^{k} \dot{\phi} \\
& =\left\{\left[\begin{array}{c}
\mathbf{J}_{v}+\tilde{\mathbf{p}}_{0 \mathbf{r}}^{k} \mathbf{J}_{\omega} \\
\mathbf{J}_{\omega}
\end{array}\right]+\mathbf{J}_{m}^{k}\right\} \dot{\boldsymbol{\phi}} \\
& \equiv \mathbf{J}^{* k} \dot{\phi} \tag{22}
\end{align*}
$$

$\mathbf{J}^{* k} \in R^{6 \times n}$ is Generalized Jacobian Matrix of the $k$-th arm, which is an extended definition of the GJM in refs. [5,6] for multiple arm systems.
In the above discussion, an attitude control devices for the satellite is not considered, however such onboard devices generating inner moment as RW or CMG can be dealt as one of the installed arms.

### 2.3 Equation of Motion

The equation of motion of free-flying systems with no external forces or moments is formulated in previous works $[7,8]$, which can be applied to multi-arm systems.

$$
\begin{equation*}
\tau=\mathbf{H}^{*} \ddot{\phi}+\mathbf{C}^{*}(\phi, \dot{\phi}) \tag{23}
\end{equation*}
$$

where $\mathbf{H}^{*} \in R^{n \times n}$ is a generalized inertia matrix and $\mathrm{C}^{*}$ is a centrifugal and Coriolis term.

## 3 Motion Control

As for the discussion of motion control, a two-arm robot model as show in Fig. $\mathbf{1}$ is considered. Each arm having 6 Degrees-Of-Freedom, same in size and configuration, are symmetrically installed on the base satellite. Three axes Reaction Wheels are also installed on the base,
which are numbered as the 3rd to 5th arm, respectively. As a total, the model has 15 DOF: the joint variables coordinates are

$$
\begin{equation*}
\phi=\left(\phi_{1}^{1}, \ldots, \phi_{6}^{1}, \phi_{1}^{2}, . ., \phi_{6}^{2}, \phi^{3}, \phi^{4}, \phi^{5}\right)^{T} \in R^{15 \times 1} . \tag{24}
\end{equation*}
$$

In the following discussions, capture operation of a target, which is floating at still in a working area of the robot, will be demonstrated by the simulation. Where, the authors focus on the motion before grasping the target, but no attentions are paid for collision problem between the hand and the target or closed-loop kinematics that two manipulators hold the same object.

### 3.1 Resolved Acceleration Control of Each Arm

Let us consider a simultaneous but independent path tracking control of both hands, while attitude of the base satellite is maintained. Resolved Acceleration Control scheme is applied on the assumption that paths of both hands in the task space $\mathbf{x}_{d}^{k}, \boldsymbol{\nu}_{d}^{k}$ and $\dot{\boldsymbol{\nu}}_{d}^{k}(k=1,2)$ are prescribed. From eqs.(19) and (22), hand acceleration are resolved into joint space as

$$
\begin{align*}
\ddot{\boldsymbol{\phi}} & =\left[\begin{array}{c}
\mathbf{J}^{* 1} \\
\mathbf{J}^{* 2} \\
\mathbf{J}_{\omega}
\end{array}\right]^{-1}\left[\begin{array}{c}
\dot{\nu}_{d}^{1}-\dot{\mathbf{J}}^{* 1} \dot{\phi} \\
\dot{\nu}_{d}^{2}-\dot{\mathbf{J}}^{* 2} \dot{\phi} \\
\dot{\omega}_{0 d}-\dot{\mathbf{J}}_{\omega} \dot{\boldsymbol{\phi}}
\end{array}\right] \\
& =\mathbf{J}^{*-1}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right) \tag{25}
\end{align*}
$$

where
$\mathbf{J}^{*}=\left[\begin{array}{c}\mathbf{J}^{* 1} \\ \mathbf{J}^{* 2} \\ \mathbf{J}_{\omega}\end{array}\right] \in R^{15 \times 15}, \quad \dot{\boldsymbol{\nu}}_{d}=\left[\begin{array}{c}\dot{\boldsymbol{\nu}}_{d}^{1} \\ \dot{\boldsymbol{\nu}}_{d}^{2} \\ \dot{\boldsymbol{\omega}}_{0 d}\end{array}\right] \in R^{15 \times 1}$.
For this case $\mathbf{J}^{*}$ is a $15 \times 15$ square matrix, so its inversion always exists except at the singular configurations. Required torque for 12 manipulator joints and 3 wheels to follow the prescribed paths are calculated through the dynamic equation (23).

A simulated motion of individual path tracking for target capture is shown in Fig.2. Two arms are controlled to reach the target respectively, while the satellite attitude is maintained. The base is translated a
little bit by the reaction of the arm operation, nevertheless, both hands trace exactly on the prescribed paths by means of the GJM.


Fig. 1 Model of two-arm space robot


Fig. 2 Individual dual arm operation

### 3.2 Torque Optimization through Dual Arm Coordination

In this section, the authors develop a dual arm coordination control that one arm traces a given path while the other arm is operated to minimize a total operation torque of the system. The control method is based on the redundancy resolution technique with local torque optimization [9].
Phase I : Firstly, suppose that only Arm 1 is controlled to the target: the position and attitude of the hand is given as $\boldsymbol{\nu}_{d}^{1}=\left(\dot{\mathbf{v}}_{r d}^{1 T}, \dot{\omega}_{n d}^{1 T}\right)^{T}$ and the satel-
lite attitude is also controlled for $\dot{\boldsymbol{\omega}}_{0 d}=\mathbf{0}$, however Arm 2 works just for hand orientation maintenance as $\dot{\omega}_{n d}^{2}=0$. Here, 12 task variables are specified against 15 of the system DOF, so there is a 3 DOF redundancy. The general solution of inverse kinematics at acceleration is represented as follows.

$$
\begin{gather*}
\ddot{\phi}=\mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)+\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right) \zeta  \tag{26}\\
\mathbf{J}^{*}=\left[\begin{array}{c}
\mathbf{J}_{T}^{* 1} \\
\mathbf{J}_{R}^{* 1} \\
\mathbf{J}_{R}^{* 2} \\
\mathbf{J}_{\omega}
\end{array}\right], \quad \dot{\boldsymbol{\nu}}_{d}=\left[\begin{array}{c}
\dot{\mathbf{v}}_{r d}^{1} \\
\dot{\omega}_{n d}^{1} \\
\dot{\omega}_{n d}^{2} \\
\dot{\omega}_{0 d}
\end{array}\right] .
\end{gather*}
$$

Where $\mathbf{J}^{* \#}$ is a pseudoinverse of $\mathbf{J}^{*}$ and $\zeta \in R^{15 \times 1}$ is an arbitrary vector. Our objective is to minimize the norm of joint torque

$$
\begin{equation*}
\|\tau\| \tag{27}
\end{equation*}
$$

by means of the redundancy resolution.
By substituting the equations (23) and (26) into (27), the following expression is obtained.

$$
\begin{align*}
\|\tau\|= & \| \mathbf{H}^{*} \mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right) \\
& +\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right) \zeta+\mathbf{C}^{*}(\phi, \dot{\phi}) \| \tag{28}
\end{align*}
$$

where $\operatorname{rank}\left(\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right)\right)=3>0$. The problem finding $\zeta$ to minimize (28) in a least squares sense can be solved also by the generalized inverse, yielding

$$
\begin{equation*}
\zeta=-\left[\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right)\right]^{\#}\left\{\mathbf{H}^{*} \mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)+\mathbf{C}^{*}(\phi, \dot{\phi})\right\} . \tag{29}
\end{equation*}
$$

Substituting $\zeta$ into (26), the desired joint acceleration for the torque optimum control is given by

$$
\begin{align*}
\ddot{\phi}_{d}= & \mathbf{J}^{* \#}\left(\dot{\nu}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)-\left[\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right)\right]^{\#} \\
& \left\{\mathbf{H}^{*} \mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)+\mathbf{C}^{*}(\phi, \dot{\phi})\right\} \tag{30}
\end{align*}
$$

Also the desired operation torque for this control is obtained by substituting $\ddot{\phi}_{d}$ into (23).

$$
\begin{align*}
\tau= & \mathbf{H}^{*} \ddot{\phi}_{d}+\mathbf{C}^{*}(\phi, \dot{\phi}) \\
= & \left\{\mathbf{I}-\mathbf{H}^{*}\left[\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right)\right]^{\#}\right\} \\
& \left\{\mathbf{H}^{*} \mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)+\mathbf{C}^{*}(\phi, \dot{\phi})\right\} \\
= & \left\{\mathbf{I}-\mathbf{H}^{*}\left[\mathbf{H}^{*}\left(\mathbf{I}-\mathbf{J}^{* \#} \mathbf{J}^{*}\right)\right]^{\#}\right\} \tau_{0} \tag{31}
\end{align*}
$$

where $\tau_{0}$ is a torque vector in case $\zeta=0$ : this is an "acceleration-minimum" solution in a least square sense. Through eq.(30), $\tau_{0}$ is optimized into $\tau$ in a "torque-minimum" sense.

Fig. 3 (a) shows one of simulatations of torque optimum behaviour with satellite attitude maintenance obtained by eq.(31). We can find an interesting characteristic that one hand is operated to the target in front of the robot, at the same time, the other hand also moves automatically toward the target as a result of the torque optimization: a kind of symmetry in the motion of two arms is observed with respect to the center plane of the base satellite.


Fig. 3 (a) Coordinated dual arm operation (phasel)


Fig. 3 (b) Coordinated dual arm operation (phase2)

Phase II : Then, consider the operation of Arm 2 to the target after Arm 1 has reached. In this phase, Arm 2 is not necessary to operate with respect to the inertial frame but it's enough with respect to the error vector between tha hand and a grasping point: $\mathbf{e} \equiv \mathbf{p}_{t}-\mathbf{p}_{r}^{2}$. Then, the RAC scheme becomes as follows,

$$
\begin{gather*}
\ddot{\phi}=\mathbf{J}^{* \#}\left(\dot{\boldsymbol{\nu}}_{d}-\dot{\mathbf{J}}^{*} \dot{\phi}\right)+\left(\mathbf{I}-\mathbf{J}^{*} \# \mathbf{J}^{*}\right) \zeta  \tag{32}\\
\mathbf{J}^{*}=\left[\begin{array}{c}
\mathbf{J}_{T}^{* 1^{\prime}}-\mathbf{J}_{T}^{* 2} \\
\mathbf{J}_{R}^{* 1} \\
\mathbf{J}_{R}^{* 2} \\
\mathbf{J}_{\omega}
\end{array}\right], \quad \dot{\nu}_{d}=\left[\begin{array}{c}
\dot{\mathrm{e}} \\
\dot{\omega}_{n d}^{1} \\
\dot{\omega}_{n d}^{2} \\
\dot{\omega}_{0 d}
\end{array}\right] .
\end{gather*}
$$

Where $\mathbf{J}_{T}^{* 11^{\prime}}$ is the GJM for the grasping point on the target object held by Arm 1, in which inertia property of the object is included. We can get the torqueminimum solution for this problem through the same procedure as eqs.(28)-(31).
Fig. 3 (b) shows a simulation of the phase 2. Both arms work in cordination so as to minimize the operation torques.

## 4 Discussion

The comparison of the norm of torque $\|\tau\|$ between the operation of Fig. 2 and Fig.3, is shown in Fig.4. Where, the individual path tracking is presented by a dotted line for 9.0 seconds and the coordinated control is by a solid line from the beginning to 12.0 sec . for phase 1 and from 12.0 to 21.0 sec . for phase 2.
The normal torque of the coordinated control in phase 1 is always smaller than the individual control. In addition, the total energy consumption $\int(\tau \cdot \dot{\phi}) \mathrm{dt}$ of the coordinated control including phase 2 is calculated as only one-third of the individual control, nevertheless the coordinated control takes longer operation time.
The simulation study shows the effectiveness of the proposed control method for dual arm coordination, in the sense of minimizing the joint torque at each instance and also saving the total energy consumption throughout the operation.

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Fig. 4 Comparison of the required torque

