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Thermoacoustic heat pumping effect in a Gifford–McMahon refrigerator

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The heat exchange process between the working helium gas and the regenerator materials in a Gifford–McMahon refrigerator is studied from the viewpoint of thermoacoustic phenomena, by measuring the cooling power as a function of the phase angle between pressure and displacement oscillations of the working gas. It is found that the optimum phase angle maximizing the cooling power dramatically increases when the operating temperature decreases below 20 K. This behavior represents the progress of the irreversible heat exchange process due to the reduction of the thermal diffusivity of helium gas. [DOI: 10.1063/1.1517730]

I. INTRODUCTION

The expansion work done by an oscillating piston and a displacer had been believed to play the most important role for the refrigeration[1,2] in Stirling and Gifford–McMahon (GM) refrigerators. However, we are now well aware that the performance of the recent pulse-tube refrigerators has reached a level comparable to those of conventional regenerative refrigerators in spite of their operation without mechanical moving parts at the cold end.[3–5] This has posed the need for deeper understanding of regenerative refrigerators from the viewpoint of thermoacoustic phenomena.[6–8]

Ceperley[9] pointed out that the phase angle between pressure and displacement of gas parcels in acoustic traveling waves is the same as that in the regenerator in Stirling engines, and proposed a thermoacoustic Stirling engine, where the traveling wave instead of mechanical pistons executes a Stirling cycle. Experimental and theoretical studies on thermoacoustics[6–8] have revealed that in such heat engines axial heat flow from the cold to hot end is produced through the transverse heat exchange between the acoustically oscillating gas and surrounding walls. Theoretical analyses on the thermal behavior of fluid have been given for the gas passage with simple geometries.[7,8,10] However, the heat exchange process in the regenerators filled with screen meshes and spherical particles is still unclear, even though those regenerators are widely used for practical refrigerators.

In this work, we measured the cooling power of a GM refrigerator as a function of the phase angle θ between the oscillating pressure $P=p_0e^{i\omega t}$ and displacement $\xi=\xi_0e^{i(\omega t-\theta)}$ of the working gas, and analyzed in terms of thermoacoustic phenomena. We found that the heat exchange process between the spherical lead particles with an averaged diameter of 0.2 mm and helium gas becomes irreversible with decreasing temperature. On the basis of the present results, we propose an efficient operating method to enhance the cooling performance of regenerative refrigerators.

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The thermoacoustic heat pumping effect originates from displacement and pressure-induced entropy oscillations of gas parcels;[6–8,10] the gas absorbs entropy from the passage wall at the cold end, and releases it at the hot end. As a result, the bucket brigade of the entropy produces the net entropy and hence heat flows. If the local equilibrium is always kept in a cross section of the gas passage, the gas entropy oscillates in phase with P, and, therefore, the optimum phase angle between P and ξ for maximum net heat flow is 90°.[5,8–11]

When the heat exchange process is irreversible,[6,7] thermal relaxation does take place in the cross section of the gas passage. Consequently, the gas entropy averaged over the cross section lags behind P due to a finite thermal relaxation time.[8] Here the relaxation time is given by $\tau=r_0^2/2\alpha$ and is closely related to a thermal penetration depth δ through the equation $\omega\tau=(r_0/\delta)^2$, where r_0 is the characteristic transverse length of the gas passage, and α is the thermal diffusivity of the gas. Thus, the optimum phase angle θ for the cooling power ought to depend on the heat exchange process between the gas and the passage walls. In the present work, we controlled the phase lead θ of P relative to ξ to compensate for the phase delay caused by τ.

II. EXPERIMENT

We employed a second-stage GM refrigerator (AISIN, GA-08A), as schematically shown in Fig. 1. A moving stroke of the regenerator is 24 mm and inner diameters of the first and second stage cylinders are 55 and 26 mm, respectively. We filled the first and second regenerators with Cu meshes and Pb particles of 0.2 mm diameter, respectively, in the same way as in commercially available GM refrigerators.

The pressure and temperature oscillations of the working helium gas at the second-stage expansion space are measured using a pressure transducer through a thin stainless steel tube and a Au(Fe)–Chromel thermocouple, respectively. The heat input fed through by an electrical heater around the cold end is used as the cooling power at the resulting temperature after the system reached a stationary state. The hot end of the second regenerator, monitored by a Si diode thermometer, was kept at 85 K throughout the experiments by using a

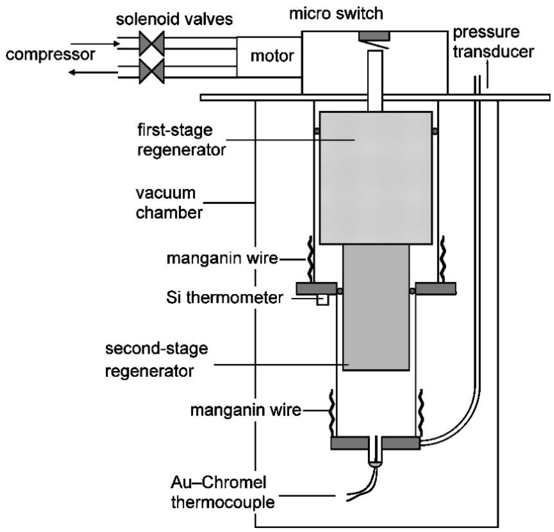

FIG. 1. Schematic illustration of the present two stage GM refrigerator.

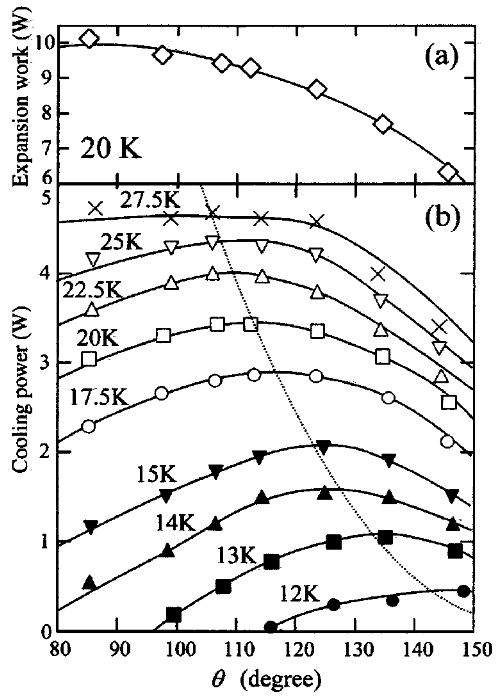

FIG. 2. θ dependence of the expansion work (a) and the cooling power (b) of the present refrigerator. The dotted curve represents the temperature dependence of the optimum phase angle. Error of the data does not exceed the size of the markers.

temperature controller (Lake Shore 330). A vacuum chamber was evacuated better than 10^{-5} Torr before the operation.

Other modifications are made to operate the present GM refrigerator with an arbitrary phase angle θ between the pressure variation P and the sinusoidal motion of the displacer; (1) two mechanically driven pressure valves are replaced with two solenoid valves driven by four electronic timers and (2) a microswitch is installed in the upper volume of the displacer to switch the timers on when the position of the displacer reaches its upper end. Accordingly, two solenoid valves are periodically switched on and off at specific time intervals after the triggered signal of the microswitch. We experimentally observed that the pressure variations were shifted in phase angle without any sizable changes in wave form by choosing suitable parameter sets of the electronic timers. We safely regarded the displacement ξ of the working helium gas at the cold end as that of the displacer determined from the signal of the microswitch. By ignoring a slight amount of higher harmonics in P, the phase angle θ is now represented as

$$\theta = \tan^{-1}\left\{ \frac{1}{\omega} \frac{\oint P \frac{d\xi}{dt} dt}{\oint P d\xi} \right\}. \tag{1}$$

We determined θ by inserting the measured pressure oscillations into Eq. (1).

III. RESULTS AND DISCUSSION

One expects the cooling power to reach the largest value at $\theta = 90°$, if the oscillating helium gas isothermally exchanges heat with Pb particles of 0.2 mm diameter. The phase angle θ is closely related to the expansion work of the displacer through the relation

$$(\omega/2\pi)A \oint P \; d\xi = 0.5\omega A p_0 \xi_0 \sin\theta, \tag{2}$$

where A is the cross sectional area. We operated the present GM refrigerator under several sets of time intervals, and estimated the expansion work by inserting measured P and ξ into the left-hand side of Eq. (2). Figure 2(a) shows the expansion work thus deduced as a function of θ when the operating temperature and the operating frequency were 20 K and 0.81 Hz, respectively. The expansion work is always maximized at $\theta = 90°$. As θ is increased above 90°, the expansion work decreases to about half of its maximum value. Therefore, both the optimum phase angle and the maximum expansion work should be simultaneously obtained when θ is 90°, as long as the isothermal heat exchange is executed in the regenerator.

The temperature dependence of the cooling power was also measured as a function of θ. We plotted it in the temperature range from 12 to 27.5 K in Fig. 2(b). The cooling power of the present GM refrigerator clearly shows a maximum centered at about 100°, when the operating temperature is above 20 K. This assures an almost isothermal heat exchange process within the regenerator in this temperature range. However, as the temperature decreases below 20 K, the optimum phase angle dramatically increases and reaches 150° at 12 K, whereas the magnitude of the expansion work is reduced to almost half of its maximum. This means that the expansion work no longer directly reflects the cooling power below 20 K. The shift in the optimum phase angle toward higher angles is certainly attributed to an increase of the relaxation time for thermal equilibrium.

We plotted the temperature dependence of the thermal diffusivity α of helium gas under 1.2 MPa (see Fig. 3).[12] Obviously, α decreases with decreasing temperature, and shows a minimum at around 10 K, which is lower than the

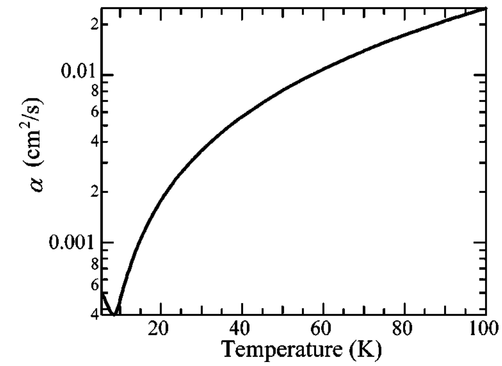

FIG. 3. Temperature dependence of the thermal diffusivity α of helium gas at 1.2 MPa (Ref. 12), which is close to the mean pressure during the measurement.

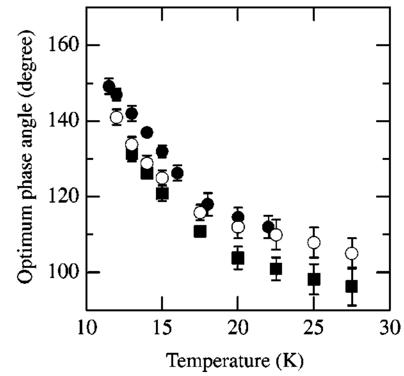

FIG. 4. Temperature dependence of the optimum phase angle for 1.20 Hz (solid circles), 0.81 Hz (open circles) and 0.61 Hz (solid squares).

lowest temperature reached by the present GM refrigerator. The monotonous reduction of α with decreasing temperature down to 10 K leads to an increase in the relaxation time τ, since the geometry of Pb particles is certainly temperature independent. Therefore, we are led to conclude that the irreversible heat exchange process at low temperatures is definitely brought about by the reduction in α of helium gas. Thus the manifestation of the irreversible heat exchange due to a poor thermal contact is inevitably induced in refrigerators working below 20 K.

Next we studied the operating frequency dependence of the optimum phase angle in the present GM refrigerator, which is shown in Fig. 4. Error bars indicate the uncertainty in reading the optimum phase angle from the plot of the cooling power versus θ. The optimum phase angle was displaced toward a higher angle with an increase in the frequency from 0.61, 0.81–1.2 Hz over the entire temperature range studied in this experiment. The observed frequency dependence of the optimum phase angle further strengthens the argument above, since the phase delay due to the finite relaxation time τ should certainly become larger when the frequency is increased. We also found that the cooling power obtained was higher with the higher frequency f at 20 K, though the lowest temperature of 11.2 K was attained at the frequency of $f=0.81$ Hz. Therefore, a higher operating frequency f is effective to enhance the cooling power, when the working temperature is high enough to assure the almost isothermal heat exchange process.

As we demonstrated above, the θ dependence of the cooling power represents well the regenerator performance. If the optimum phase angle is 90°, and hence the isothermal heat exchange is assured in the regenerator, one can operate a refrigerator in a higher frequency to achieve a larger cooling power. When the irreversibility in the heat exchange process becomes profound due to a rapid decrease in the thermal diffusivity of the helium gas, one is still able to extract the maximum cooling power by choosing the optimum phase angle θ at a given temperature. Also, we can control the operating temperature by shifting θ away from the optimum value, instead of feeding extra heat through an electrical heater. By controlling the phase angle between P and ξ and the frequency, one can operate GM refrigerators more effectively at low temperatures. This method should be applicable to other refrigerators which are driven by a thermoacoustic heat pumping effect.

IV. SUMMARY

In summary, we demonstrated that the performance of a GM refrigerator can be well understood from the viewpoint of thermoacoustic phenomena by measuring the phase angle dependence of the cooling power of a GM refrigerator. We found that the optimum phase angle increases with decreasing temperature. This finding can be attributed to the progress of the irreversible heat exchange process due to an increase in τ. We conclude that a choice of regenerator particles smaller than 0.2 mm in diameter and use of an optimum phase angle at a given temperature should be important to extract the maximum cooling power of the regenerative refrigerators working below about 20 K.

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