Ion-Energy Distribution in a Plasma under a Diverging Magnetic Field

著者	畠山　力三
journal or publication title	Journal of Applied Physics
volume	45
number	1
page range	85-88
year	1974
URL	http://hdl.handle.net/10097/46568

doi: 10.1063/1.1663023

Ion-energy distribution in a plasma under a diverging magnetic field

R. Hatakeyama, N. Sato, Y. Tsunoda, H. Sugai, and Y. Hatta

Department of Electronic Engineering, Tohoku University, Sendai, Japan
(Received 3 August 1973; in final form 21 September 1973)

A distribution function of ion energy parallel to a magnetic field is measured to be deformed along a diverging magnetic field in a single-ended Q machine. The deformation is well explained by the orbit theory for a collisionless plasma.

I. INTRODUCTION

It is often an important and basic problem in plasma physics to measure energy distributions of electrons and ions, especially in a collisionless plasma. In order to discuss some details of plasma waves, the energy distributions have been measured by several authors[1-3] both with and without a uniform magnetic field. Under a nonuniform magnetic field, the motion of charged particles is characterized by an exchange between kinetic energies parallel and perpendicular to the magnetic field,[4] and thus the energy distributions parallel and perpendicular to the magnetic field should be deformed along the magnetic field.

This paper reports measurements of an ion energy distribution parallel to a magnetic field in a single-ended Q machine[5] under a diverging magnetic field. The deformation of the distribution function measured along the plasma column is consistent with the prediction based on the orbit theory for a collisionless plasma. Section II presents the measured results, which are discussed theoretically in Sec. III. Section IV contains conclusions.

II. EXPERIMENT

A. Apparatus

The plasma, about 4 cm in diameter, is produced by surface ionization of sodium atoms on a tungsten plate of about 2000 °K and is confined by a diverging magnetic field, as shown in Fig. 1. The hot plate is situated at a position where the axial magnetic field B is uniform and is kept at a constant value B_0 ($=3.6$ kG). Beyond a distance of 30 cm or so from the hot plate, B decreases gradually along the plasma flow. The machine is operated under a so-called "electron-rich" condition.[2,5] The plasma density $n \approx 10^8$ cm^{-3} and the electron temperature $T_e = 2000-3000$ °K are measured by Langmuir probes. The background gas pressure is 5×10^{-6} Torr. Thus, collision mean free paths of charged particles with both the same and different kinds of particles are longer than the plasma column.

The plasma is terminated at a movable stainless-steel target of 5-cm diameter, biased negatively enough to reflect electrons. The target has a hole covered with a grid (made of 45-μm-diam wires spaced 100 μm) at the center. Ions passed through this grid are picked up by the collector. An energy analyzer[1-3] consists of the grid and the collector (see Fig. 1). The separation between them is 1 mm. Their diameter, 8mm, is larger than the ion Larmor radius, 0.8–2.0 mm. The ion energy distribution parallel to the magnetic field is given by the derivative of the collector current I_c with respect to the applied collector voltage V_c. The derivative dI_c/dV_c is obtained by superposing a small oscillating voltage (5 kHz, 20 mV peak to peak) on the dc collector voltage.

B. Ion current density

At first, by moving a plane Langmuir probe (2×2 mm) biased at −20 V the density of ion currents flowing downstream, J_{is}, is measured along the plasma column at the center of the plasma cross section. The plane of the probe faces the hot plate. The ion currents picked up by the back of the probe was confirmed to be negligibly small. The results are shown for four different diverging magnetic fields in Fig. 2. Those profiles of ion currents are also obtained by moving the energy analyzer biased to pick up ion saturation currents. As seen from Fig. 2, the normalized ion current density $J_{is}/J_{is,0}$ has almost the same axial profile as the corresponding configuration of B, where $J_{is,0}$, the ion current at $R(=B/B_0)=1$, is almost independent of the magnetic configuration. Even if B is increased for a fixed configuration of B, the measured profile of J_{is} does not change. Thus, in this experiment the collisional diffusion across the magnetic field can be neglected. For higher densities ($\gtrsim 10^9$ cm^{-3}) or lower values of B ($\lesssim 1$ kG), however, the collisions affect the profile of J_{is}. Hence, by measuring the profiles of J_{is} for various densities under various magnetic fields, we can roughly set the region where the collisional effects are negligibly small.

The plasma potential is also measured along the plasma column. The weak axial electric field is found

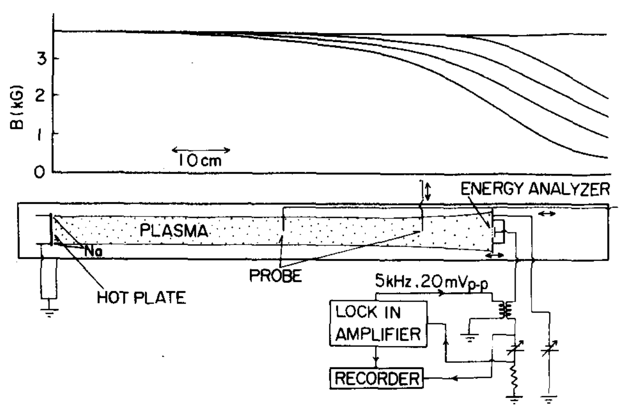

FIG. 1. Schematic diagram of experimental apparatus. The magnetic field (upper figure) diverges so gradually along the plasma flow that the magnetic moment is conserved in the experiment.

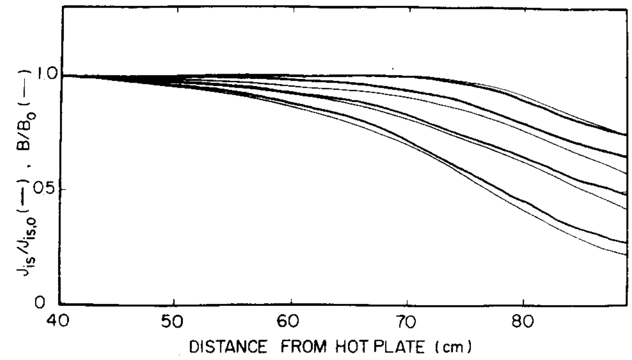

FIG. 2. Axial profiles of normalized ion current density flowing downstream, $J_{is}/J_{is,0}$, where $J_{is,0}$ is the value of J_{is} at $B=B_0$. Corresponding magnetic configurations B/B_0 are also shown, because the theory [Eq. (5) in Sec. III] predicts the relation $J_{is}/J_{is,0}=R$.

to be in the direction of ion acceleration. When B is uniform, its value is 10^{-3} V/cm. This value increases up to 6×10^{-3} V/cm in a diverging magnetic field.

C. Ion energy distribution

The ion energy distribution parallel to the magnetic field is measured as a function of the collector voltage applied with respect to the hot plate. The measurements are made at various values of R by varying the magnetic configuration for a given position of the energy analyzer or by moving the analyzer for a given magnetic configuration. If the magnetic field is uniform, the distribution function $F_i(V_c)$ ($\propto -dI_c/dV_c$) is almost the same along the plasma column, as shown in Fig. 3.

Typical results under the diverging magnetic field are demonstrated in Figs. 4 and 5. Figure 4 shows the results obtained for various magnetic configurations at a fixed distance 120 cm from the hot plate. It is easily seen from this figure that $F_i(V_c)$ is deformed as the magnetic field diverges. The same deformation of the distribution function is also measured along the magnetic field for a fixed magnetic configuration by moving the analyzer axially, as shown in Fig. 5. Almost the same shape of the distribution function is obtained at the same value of R for a different position of the analyzer.

With a decrease in R, (a) the peak value of $F_i(V_c)$ decreases, (b) its shape becomes fat, and (c) the peak position shifts toward a higher energy. Result (a) is attributed to the decrement of the current density and the broadening of $F_i(V_c)$ along the plasma flow. We estimate the broadening of $F_i(V_c)$ by the voltage at which $F_i(V_c)$ is e^{-1} times its peak value on the side of the higher energy (where e is the natural logarithm base). In Fig. 6(a) the difference of this value from the peak position, ΔV_{ce}, as a function of R is plotted. The relative shift of the peak position, ΔV_{cp}, is shown also as a function of R in Fig. 6 (b), where the shift is measured as a difference from the peak position at $B=B_0$. It is found from Fig. 6 that ΔV_{ce} and ΔV_{cp} increase monotonically as the magnetic field diverges.

III. THEORETICAL DISCUSSIONS

We try to explain the measured results by basing them

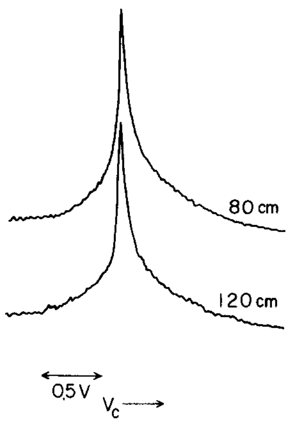

FIG. 3. Ion energy distributions parallel to the magnetic field, $F_i(V_c)$ ($\propto -dI_c/dV_c$), measured at 80 and 120 cm from the hot plate under a uniform magnetic field.

on the orbit theory[6] for ions in a diverging magnetic field. Ions emitted from the hot plate of temperature T are assumed to have an energy distribution function given by

$$f_{i0}(v_{\parallel 0}, v_{\perp 0}^2) = (2n_0/\pi^{3/2}a^3)\exp[-(v_{\parallel 0}^2 + v_{\perp 0}^2)/a^2], \quad \text{for } v_{\parallel 0} \geq 0$$
$$= 0, \quad \text{for } v_{\parallel 0} < 0, \tag{1}$$

where $a^2 = 2\kappa T/M$, $n_0 = \int\int f_{i0} 2\pi v_{\perp 0} dv_{\perp 0} dv_{\parallel 0}$, and $\parallel$ and $\perp$ imply, respectively, parallel and perpendicular to the magnetic field which is uniform around the hot plate. The factor 2 in Eq. (1) comes about because the distribution is half-Maxwellian for $v_{\parallel 0}$. It is necessary to know the distribution function $f_i(v_\parallel, v_\perp^2)$ at a point where the magnetic field is B and the plasma potential is $-V_s$ (<0) with respect to the hot plate. Ions are accelerated through the electron sheath in front of the hot plate and the weak electric field along the plasma column. Under a gradually diverging magnetic field, their motion satisfies the conservation of magnetic moment μ ($=Mv_\perp^2/2B$) in addition to the conservation of particle energy $[M(v_\parallel^2 + v_\perp^2)/2 - eV_s]$. On the other hand, according to Liouville's theorem the distribution function is preserved along particle trajectories in phase space in a collisionless system; i.e.,

$$f_i(v_\parallel, v_\perp^2) = f_{i0}(v_{\parallel 0}, v_{\perp 0}^2). \tag{2}$$

By using the conservations of ion energy and magnetic moment μ, the right-hand side of this equation is expressed by the quantities at a point where the magnetic field is B. Thus, we get

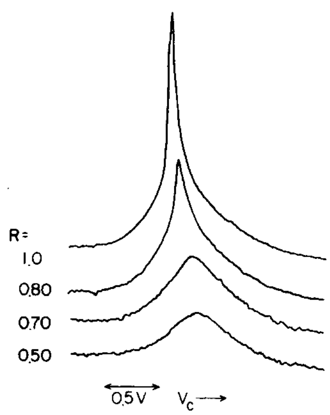

FIG. 4. Ion energy distributions $F_i(V_c)$ measured for various values of R at 120 cm from the hot plate.

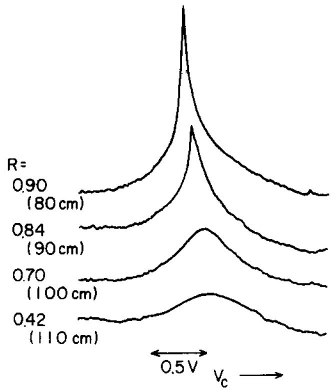

FIG. 5. Ion energy distributions $F_i(V_c)$ measured for various values of R (various positions from the hot plate) at a fixed magnetic configuration.

$$f_i(v_\parallel, v_\perp^2) = (2n/\pi^{3/2}a^3A)\exp[-(v_\parallel^2 + v_\perp^2)/a^2], \quad \text{for } v_\parallel \geq v'$$
$$= 0, \quad \text{for } v_\parallel < v', \quad (3)$$

where

$$A = 1 - \text{erf}[(eV_s/\kappa T)^{1/2}] - (1-R)^{1/2}\exp[eV_sR/\kappa T(1-R)] \times\{1 - \text{erf}[eV_s/\kappa T(1-R)]\}^{1/2},$$

$$n = An_0\exp(eV_s/\kappa T),$$

and

$$v' = \{2[eV_s + \mu(B_0 - B)]/M\}^{1/2}.$$

It is easily understood that there is no ion whose parallel velocity is smaller than v' because $v_\parallel = \{v_{\parallel 0}^2 + 2[eV_s + \mu(B_0 - B)]/M\}^{1/2}$. The expression of A is derived from the relation $n = \int\int f_i 2\pi v_\perp dv_\perp dv_\parallel$.

We are interested in the energy distribution $F_i(v_\parallel) = \int f_i 2\pi v_\perp dv_\perp$. Because $v_{\parallel 0}^2 = v_\parallel^2 + (R-1)v_\perp^2/R - 2eV_s/M \geq 0$, the integral is restricted to the region $0 \leq v_\perp \leq [(v_\parallel^2 - 2eV_s/M)R/(1-R)]^{1/2}$. By using the relation $n = An_0 \times \exp(eV_s/\kappa T)$ and replacing $\frac{1}{2}Mv_\parallel^2$ by eV,

$$F_i(V) = (2n_0/\pi^{1/2}a)\exp[-e(V - V_s)/\kappa T] - \exp[-e(V-V_s)/\kappa T(1-R)], \quad \text{for } V \geq V_s$$
$$= 0, \quad \text{for } V < V_s. \quad (4)$$

The ion current density $J_{is} = e\int_0^\infty v_\parallel F_i(v_\parallel)\,dv_\parallel$ is given by

$$J_{is}/J_{is,0} = R, \quad (5)$$

where $J_{is,0}$ is the value at $R = 1$. This relation is well confirmed by the measurements that J_{is} has almost the same axial profile as B. Thus, the theory is reasonable for our experimental conditions.

Characteristics of the energy analyzer are well known.[1-3] The ion current I_c picked up by the collector of surface area S (biased at V_c with respect to the hot plate) is given by

$$I_c = Se\int_{v_{\min}}^\infty dv_\parallel v_\parallel F_i(v_\parallel), \quad \text{for } V_c \geq -V_s$$
$$= Se\int_0^\infty dv_\parallel v_\parallel F_i(v_\parallel), \quad \text{for } V_c < -V_s, \quad (6)$$

where $v_{\min} = \{2e[V_c - (-V_s)]/M\}^{1/2}$. Substituting Eq. (4) into Eq. (6), we get

$$I_c = (Sen_0a/\pi^{1/2})\{\exp(-eV_c/\kappa T) - (1-R)\exp[-eV_c/\kappa T(1-R)]\}, \quad \text{for } V_c \geq 0$$
$$= Sen_0aR/\pi^{1/2}, \quad \text{for } V_c < 0, \quad (7)$$

the derivative of which is given by

$$\frac{dI_c}{dV_c} = \left(-\frac{Se^2}{M}\right)\left(\frac{2n_0}{\pi^{1/2}a}\right)\left[\exp\left(-\frac{eV_c}{\kappa T}\right) - \exp\left(-\frac{eV_c}{\kappa T(1-R)}\right)\right], \quad \text{for } V_c \geq 0$$
$$= 0 \quad \text{for } V_c < 0. \quad (8)$$

Comparing Eq. (8) with Eq. (4) we notice that the distribution function of the kinetic energy, $F_i(V)$, coincides with the derivative of the collector current with respect to the collector voltage if V_c is replaced by $V - V_s$, except for the numerical constant $-Se^2/M$. In the experiment $-V_s$ is decomposed into the sheath potential of approximately -1.0 V in front of the hot plate and into the small potential drop along the plasma flow. It is seen from Eq. (4) that the potential drop $-V_s$ displaces the energy distribution function to the higher energy, keeping its shape unchanged. We are not interested in this displacement due to the electric acceleration. Equation (8) means that the analyzer used gives only the deformation of $F_i(V)$ due to the magnetic configuration. This is because the collector voltage is applied with respect to the hot plate when the plasma potential is lower than the potential of the hot plate and the electric field is in the direction of ion acceleration.

The predicted deformation of the distribution function under a diverging magnetic field is shown in Fig. 7, corresponding to the measurements shown in Figs. 4 and 5. A qualitative agreement is found between the measured and predicted deformations. The measured

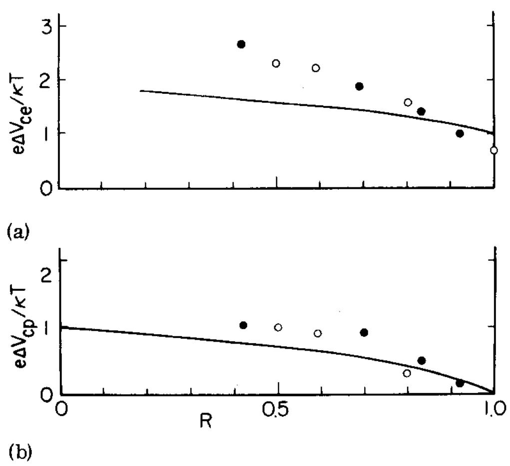

FIG. 6. Measured and predicted values of (a) broadening, ΔV_{ce}, and (b) peak shift, ΔV_{cp}, of $F_i(V_c)$ as a function of R. The measured values are normalized by the temperature, 0.22 eV, which is determined by the higher-energy tail of $F_i(V_c)$ at $B = B_0$. Open circles are obtained by varying the magnetic configuration at 120 cm from the hot plate. Shaded circles are obtained by varying the position at a fixed magnetic configuration. Theoretical values (see Sec. III) are indicated by solid lines.

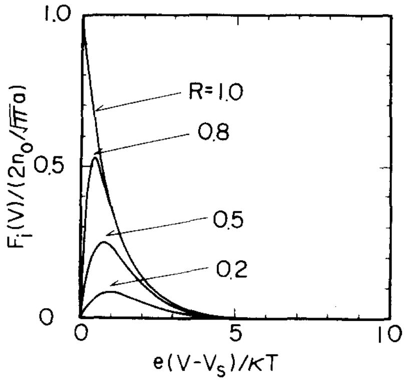

FIG. 7. Predicted deformations of ion distribution function $F_i(V)$ due to diverging magnetic field.

deformation on the side of the lower energy, however, is not explained by our theory because of its simple assumption. In our model a perfect truncated Maxwellian distribution is predicted at $B=B_0$, although an appreciable number of ions are measured to have kinetic energy lower than that giving the peak value of the distribution (see Figs. 3 and 4).

In order to check the broadening of $F_i(V)$, using Eq. (4) or (8), we calculate the difference ΔV_{ce} between the voltage giving the peak of the distribution and that giving the value e^{-1} times the peak value on the side of the higher energy. A solid line in Fig. 6(a) shows the calculated value of ΔV_{ce}. Equations (4) and (8) predict the peak shift of the distribution function due to the diverging magnetic field to be

$$\Delta V_{cp} = -(\kappa T/e)[(1-R)/R]\ln(1-R), \tag{9}$$

which is shown by a solid line in Fig. 6(b). It is found in Fig. 6 that the predicted values of the broadening and peak shift of $F_i(V)$ are reasonably consistent with the measured values.

IV. CONCLUSIONS

The ion energy distribution parallel to the magnetic field is measured in a collisionless plasma produced in a single-ended Q machine under a diverging magnetic field. The energy analyzer used gives only the deformation of the distribution function due to the diverging magnetic field in this experiment. The acceleration and broadening of the distribution due to the conversion of the perpendicular kinetic energy into the parallel energy is clearly observed in the experiment. The results are well explained by the orbit theory. If the real ion distribution function in front of the hot plate would be taken into account in our model, the measured deformation of the distribution function on the side of the lower energy could be explained and a better agreement between experiment and theory would be obtained.

Finally, our results are compatible with the gradual increases of the phase velocity and normalized damping distance (δ/λ, where λ is wavelength) of grid-excited waves[5,7] observed along the diverging magnetic field.[8]

ACKNOWLEDGMENTS

The authors wish to thank A. Sasaki for his collaboration in Q-machine experiments. They are also indebted to Y. Takahashi for his technical support.

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