Postdetection selection diversity effects on digital FM land mobile radio

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Postdetection Selection Diversity Effects on Digital FM Land Mobile Radio

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Abstract—**The postdetection selection diversity effects on a binary digital FM system are theoretically analyzed in the fast Rayleigh fading signal environment encountered in the typical UHF or microwave land mobile radio channels. Both differential and discriminator detections are considered for demodulation of digital FM signal. The average error rate is presented by a simple closed form including both effects of Rayleigh envelope fading and random FM noise. A few examples of numerical results for minimum shift keying (MSK) are graphically presented.**

I. INTRODUCTION

RECENTLY DIGITAL modulation schemes have become of interest in the field of UHF or microwave land mobile radio communications [1]. Digital FM with a low modulation index, particularly minimum shift keying (MSK) with modulation index 0.5, is considered to be one of the most useful modulation schemes because of its constant envelope property and small radio frequency (RF) bandwidth requirement. For digital FM signals, there may be three typical demodulation schemes: discriminator detection [2, pp. 334–340], differential detection [3], and coherent detection [4]. In the UHF or microwave mobile radio channels characterized by fast Rayleigh fading, it is not easy to design the stable carrier recovery system for coherent detection. Noncoherent detection (differential and discriminator detections) can avoid this problem. Although the error rate performance of differential detection is slightly inferior to that of coherent detection, the receiver structure is greatly simplified since the phase reference is derived from the past signaling interval by a simple delay line. Also discriminator detection has the advantage that the conventional analog FM receivers can be used for both voice and data detections. In this paper both differential and discriminator detections are considered.

Few papers [5]-[7] have studied the error rate performance of digital FM in the fast Rayleigh fading environment which characterizes typical land mobile radio channels in an urban area. Likewise, few have pointed out that the irreducible errors due to random FM noise can not be negligible compared with the error rates due to Rayleigh envelope fading in fast Rayleigh fading and high signal-to-noise ratio (SNR).

Diversity techniques [8] may be used to combat both effects of envelope fading and random FM noise. For mobile radio use, the selection diversity is attractive since it is presumably less expensive to implement. Despite extensive study of digital mobile radio, there are still few papers [9]-[11] devoted to the study of selection diversity effects on digital FM signal transmission in the Rayleigh fading signal environment. The literature [9] does not include the differential detection scheme and random FM noise effect since slow fading is assumed. The literature [10] considers both differential and discriminator detection schemes and includes random FM noise effect, however, the analysis assumes the selection diversity with total power method whose selection operation depends on the instantaneous fading signal plus receiver thermal noise power at the detector input. It has been found in [11] that the selection diversity with total power method is somewhat inferior to the ideal selection diversity whose selection operation depends only on the instantaneous fading signal power. In addition a predetection selection diversity which does the selection operation before the demodulation causes switching noises whenever a different diversity branch is selected due to the fact that the branches are not phase coherent. However, a postdetection selection diversity which does the selection operation after the demodulation has the advantage of causing no switching noise.

This paper considers the ideal postdetection selection diversity and presents the theoretical analysis of diversity effects on the error rate performance of a binary digital FM with both differential and discriminator detections in the fast Rayleigh fading signal environment. For analysis we apply Stein's method [12] which was used for evaluating the error rate performance of certain coherent and noncoherent binary communication systems in the nonfading condition. While the effect of intersymbol interference caused by predetection bandpass filter band restriction is neglected, the fading spectrum effect is taken into consideration and then a fairly complete solution including both effects of Rayleigh envelope fading and random FM noise is given in a simple closed form useful for the design of digital land mobile radio systems.

II. MATHEMATICAL PRESENTATION OF MODEL

A. System Model

In this paper a binary digital FM with both differential and discriminator detections is considered. Fig. 1 shows the mathematical model of the digital FM receiver with differential detection and postdetection selection diversity. It is assumed that there are available N antennas, each receiving statistically independent fast Rayleigh fading signals $s_i(t)$, and to each signal there is an additive Gaussian noise $n_i(t)$. In the ith branch receiver, the signal $s_i(t)$ plus noise $n_i(t)$ is filtered by a predetection bandpass filter (BPF) with symmetrical band-

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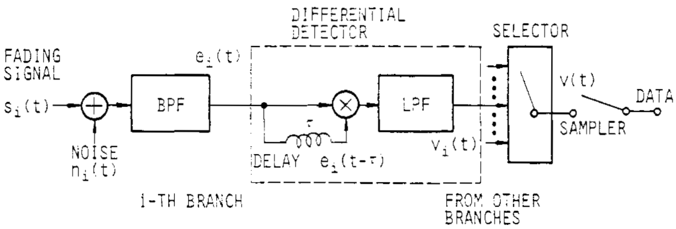

Fig. 1. Digital FM receiver model with differential detection and postdetection selection diversity.

pass characteristics to be detected by a differential detector or a discriminator. Here the intersymbol interference caused by the bandwidth limitation of the predetection BPF and postdetection lowpass filter (LPF) is assumed to be negligible.

The ideal postdetection diversity selects the detector output of the branch with the highest instantaneous received signal envelope (or power) and therefore has the advantage of causing no switching noises, while the predetection selection diversity produces switching noises whenever a different diversity branch is selected due to the fact that the branches are not phase coherent.

B. Detector Output

The predetection BPF bandwidth is assumed to be sufficiently wide so as not to distort the signal. Therefore in the Rayleigh fading signal environment, the signal and the additive Gaussian noise at the ith branch predetection BPF output can be represented, respectively, in the following complex form:

$$s_i(t) = \mathrm{Re}\,[z_{s_i}(t)\exp j\{2\pi f_c t + \phi_s(t)\}]$$
$$n_i(t) = \mathrm{Re}\,[z_{n_i}(t)\exp j\{2\pi f_c t\}]. \tag{1}$$

Re $[\cdot]$ denotes the real part of $[\cdot]$, f_c the carrier frequency, and $\phi_s(t)$ the modulating phase defined as

$$\dot{\phi}_s(t) = \begin{cases} +2\pi\Delta f_d, & \text{for mark} \\ -2\pi\Delta f_d, & \text{for space} \end{cases} \tag{2}$$

where Δf_d is the frequency deviation, and the overdot denotes the time derivative. Furthermore, $z_{s_i}(t)$ and $z_{n_i}(t)$ are the complex envelopes of the signal and noise and are independent zero-mean complex Gaussian processes.

The detector input signal $e_i(t)$ is given by

$$\begin{aligned} e_i(t) &= s_i(t) + n_i(t) \\ &= \mathrm{Re}\,[z_i(t)\exp j\{2\pi f_c t\}], \end{aligned} \tag{3}$$

where $z_i(t)$ is a complex envelope characterized by zero-mean complex Gaussian process, and is given by

$$z_i(t) = z_{s_i}(t)\exp j\{\phi_s(t)\} + z_{n_i}(t). \tag{4}$$

The differential detector for digital FM operates as a quadrature product demodulator (Fig. 1). The phase reference is derived from the past signaling interval by a delay line under the condition $\tau = T$ and $2\pi f_c T = \pi/2$ (T is a signaling period). At the detector the BPF output $e_i(t)$ is multiplied by its delayed replica $e_i(t - T)$. The postdetection LPF following the multiplier eliminates the double frequency components from the product signal and leaves the resulting signal intact. As the time delay τ of the differential detector is made small under the condition $2\pi f_c\tau = \pi/2$, the detector performance approaches that of a frequency discriminator [3]. Therefore, the detector output $v_i(t)$ for differential and discriminator detections can be represented in the following unified form:

$$v_i(t) = \tfrac{1}{2}\,\mathrm{Re}\,[z_i(t)\{jz_i(t-\tau)\}^*], \tag{5}$$

where the asterisk symbol denotes the complex conjugate, and

$$\tau \rightarrow \begin{cases} T, & \text{for differential detection} \\ 0, & \text{for discriminator detection.} \end{cases} \tag{6}$$

C. pdf of Instantaneous SNR on Selected Branch

Letting σ_s^2 and σ_n^2 represent the average signal and noise powers, respectively,

$$\begin{aligned} \sigma_s^2 &= \langle z_{s_i}^*(t) z_{s_i}(t)\rangle/2 \\ \sigma_n^2 &= \langle z_{n_i}^*(t) z_{n_i}(t)\rangle/2, \end{aligned} \tag{7}$$

where $\langle\cdot\rangle$ denotes statistical average, the instantaneous signal-to-noise ratio (SNR) γ_i and the average SNR Γ_i on the ith branch are defined as

$$\begin{aligned} \gamma_i &= z_{s_i}^*(t) z_{s_i}(t)/2\sigma_n^2 \\ \Gamma_i &= \langle\gamma_i\rangle = \sigma_s^2/\sigma_n^2. \end{aligned} \tag{8}$$

We assume that fadings on each branch are independent and all branches have equal signal power and equal noise power,i.e., all the Γ_i are equal to some value Γ,

$$\Gamma_i = \Gamma. \tag{9}$$

The ideal postdetection selection diversity is defined as selecting the detector output of the branch with the highest instantaneous SNR (or signal power). Defining γ and $v(t)$ as the instantaneous SNR and detector output of the selected branch, when $\gamma_i >$ all others,

$$\begin{aligned} \gamma &= \gamma_i \\ v(t) &= \tfrac{1}{2}\,\mathrm{Re}\,[z(t)\{jz(t-\tau)\}^*], \end{aligned} \tag{10}$$

where $z(t)$ and $z(t-\tau)$ are equal to $z_i(t)$ and $z_i(t-\tau)$ on the ith branch, respectively.

The probability density function (pdf) of γ can be easily understood to be equal to that for well-known predetection selection diversity and is given by [2, p. 435]

$$\begin{aligned} p(\gamma) &= \frac{N}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right)\left[1 - \exp\left(-\frac{\gamma}{\Gamma}\right)\right]^{N-1} \\ &= \frac{1}{\Gamma}\sum_{n=1}^{N} {}_N C_n\, n(-1)^{n+1}\exp\left(-n\frac{\gamma}{\Gamma}\right). \end{aligned} \tag{11}$$

III. AVERAGE ERROR RATE ANALYSIS

The diversity selector output $v(t)$ is sampled at some particular instant $t = \nu T$ ($\nu = 0, \pm 1, \pm 2, \cdots$). For binary digital signal transmission, the decision as to whether a mark or a space was sent is based on the polarity of $v(t)$. Let $p_{e+}(\gamma)$ and $p_{e-}(\gamma)$ represent the conditional error rates of mark and space transmissions when the instantaneous SNR $= \gamma$, respectively. Then the average error rate P_e is given by

$$P_e = p \cdot P_{e+} + (1-p) \cdot P_{e-}, \tag{12}$$

where

$$P_{e+} = \int_0^\infty p_{e+}(\gamma)p(\gamma)\, d\gamma$$
$$P_{e-} = \int_0^\infty p_{e-}(\gamma)p(\gamma)\, d\gamma \tag{13}$$

are the average error rates of mark and space transmissions, respectively, and p denotes the probability of mark transmission. The pdf of γ, $p(\gamma)$, is given by (11).

Therefore to calculate the average error rate, it is necessary to know the conditional error rates $p_{e+}(\gamma)$ and $p_{e-}(\gamma)$.

A. Conditional Error Rate

From (10) the selector output at the sampling instant can be rewritten in the form

$$v = \tfrac{1}{2}\mathrm{Re}\,\{\xi_1 \xi_2^*\}, \tag{14}$$

where

$$\xi_1 = z$$
$$\xi_2 = jz_\tau, \tag{15}$$

and v, z, and z_τ are defined by $v(\nu T)$, $z(\nu T)$, and $z(\nu T - \tau)$, respectively. Note that ξ_1 and ξ_2 are complex Gaussian variables.

Since (14) can be rewritten as

$$v = \frac{|\xi_1 + \xi_2|^2 - |\xi_1 - \xi_2|^2}{8}, \tag{16}$$

errors occur when the sign of v becomes negative for mark transmission and positive for space transmission. In order to calculate the conditional error rate, it is necessary to analyze the probability of the occurences of $|\xi_1 + \xi_2| < |\xi_1 - \xi_2|$ for mark transmission and $|\xi_1 + \xi_2| > |\xi_1 - \xi_2|$ for space transmission. We can apply Stein's method [12] to this problem. Following Stein's method, the conditional error rates for mark and space are given by

$$p_{e+}(\gamma) = \tfrac{1}{2}\,[1 - Q(\sqrt{b_+}, \sqrt{a_+}) + Q(\sqrt{a_+}, \sqrt{b_+})] - \frac{A_+}{2}\exp\left(-\frac{a_+ + b_+}{2}\right) I_0(\sqrt{a_+ b_+})$$

$$p_{e-}(\gamma) = \tfrac{1}{2}\,[1 - Q(\sqrt{b_-}, \sqrt{a_-}) + Q(\sqrt{a_-}, \sqrt{b_-})] + \frac{A_-}{2}\exp\left(-\frac{a_- + b_-}{2}\right) I_0(\sqrt{a_- b_-}), \tag{17}$$

where

$$a_\pm = \frac{1}{4\{1 - \mathrm{Im}^2(\rho)\}}\left[\frac{|\langle\xi_1\rangle|^2}{\sigma_1^2} + \frac{|\langle\xi_2\rangle|^2}{\sigma_2^2} + 2\frac{\mathrm{Im}\,(\rho)}{\sigma_1\sigma_2} \cdot \mathrm{Im}\,\{\langle\xi_1\rangle\langle\xi_2^*\rangle\} - 2\frac{\sqrt{1-\mathrm{Im}^2(\rho)}}{\sigma_1\sigma_2}\mathrm{Re}\,\{\langle\xi_1\rangle\langle\xi_2^*\rangle\}\right]$$

$$b_\pm = \frac{1}{4\{1 - \mathrm{Im}^2\,(\rho)\}}\left[\frac{|\langle\xi_1\rangle|^2}{\sigma_1^2} + \frac{|\langle\xi_2\rangle|^2}{\sigma_2^2} + 2\frac{\mathrm{Im}\,(\rho)}{\sigma_1\sigma_2} \cdot \mathrm{Im}\,\{\langle\xi_1\rangle\langle\xi_2^*\rangle\} + 2\frac{\sqrt{1-\mathrm{Im}^2\,(\rho)}}{\sigma_1\sigma_2}\mathrm{Re}\,\{\langle\xi_1\rangle\langle\xi_2^*\rangle\}\right]$$

$$A_\pm = \frac{\mathrm{Re}\,(\rho)}{\sqrt{1 - \mathrm{Im}^2\,(\rho)}}$$

$$\sigma_1^2 = 1/2 \cdot \langle(\xi_1 - \langle\xi_1\rangle)^*(\xi_1 - \langle\xi_1\rangle)\rangle$$

$$\sigma_2^2 = 1/2 \cdot \langle(\xi_2 - \langle\xi_2\rangle)^*(\xi_2 - \langle\xi_2\rangle)\rangle$$

$$\sigma_1\sigma_2\rho = 1/2 \cdot \langle(\xi_1 - \langle\xi_1\rangle)^*(\xi_2 - \langle\xi_2\rangle)\rangle \tag{18}$$

and $Q(a, b)$ is Marcum's Q-function defined as

$$Q(a, b) = \int_b^\infty x \exp\{-(x^2 + a^2)/2\} I_0(ax)\, dx, \tag{19}$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function. In (18), Im $(\cdot)$ denotes the imaginary part of $(\cdot)$. Note that ensemble average $\langle\cdot\rangle$ must be performed under the condition the instantaneous SNR $\gamma = z_s^* z_s / 2\sigma_n^2$ and that the plus and minus signs correspond to mark and space transmissions, respectively.

To derive $a_\pm$, $b_\pm$, and $A_\pm$, we rewrite ξ_1 and ξ_2 in (15), in terms of new variables ξ_{1_s}, ξ_{2_s}, ξ_{1_n}, and ξ_{2_n} as

$$\xi_1 = \xi_{1_s} + \xi_{1_n}$$
$$\xi_2 = \xi_{2_s} + \xi_{2_n}, \tag{20}$$

where supersubscripts s and n denote signal and noise components, respectively. Define that $\xi_s = (\xi_{1_s}, \xi_{2_s})^T$ and $\xi_n = (\xi_{1_n}, \xi_{2_n})^T$ are complex Gaussian vectors with the following covariance matrices:

$$H_s = 1/2 \cdot \langle(\xi_s - \langle\xi_s\rangle)^*(\xi_s - \langle\xi_s\rangle)^T\rangle = \sigma_s^2\begin{pmatrix} 1 & \rho_s^* \\ \rho_s & 1 \end{pmatrix}$$

$$H_n = 1/2 \cdot \langle(\xi_n - \langle\xi_n\rangle)^*(\xi_n - \langle\xi_n\rangle)^T\rangle = \sigma_n^2\begin{pmatrix} 1 & \rho_n^* \\ \rho_n & 1 \end{pmatrix}, \tag{21}$$

where $(\cdot)^T$ denotes transpose matrix symbol and ρ_s and ρ_n are, respectively, the covariance coefficients for signal and noise. Note that $\langle\cdot\rangle$ indicates ensemble average under the condition that $\gamma = z_s^* z_s/2\sigma_n^2$.

Realizing that both vectors ξ_s and ξ_n are independent complex Gaussian vectors, the joint pdf of ξ_s and ξ_n is given by [2, pp. 590-595]

$$p(\xi_s, \xi_n) = p(\xi_s) \cdot p(\xi_n), \tag{22}$$

with

$$p(\xi_s) = \frac{1}{(2\pi)^2 \det H_s} \exp\left[-\tfrac{1}{2}(\xi_s - \langle\xi_s\rangle)^T H_s^{-1} (\xi_s - \langle\xi_s\rangle)^*\right]$$

$$p(\xi_n) = \frac{1}{(2\pi)^2 \det H_n} \exp\left[-\tfrac{1}{2}(\xi_n - \langle\xi_n\rangle)^T H_n^{-1} (\xi_n - \langle\xi_n\rangle)^*\right] \tag{23}$$

where $(\cdot)^{-1}$ and $\det(\cdot)$ denote inverse matrix and determinant, respectively.

Since ξ_{1_s} and ξ_{2_s} are Gaussian random variables with correlation ρ_s, from (23) we can easily derive the pdf of ξ_{2_s} under the condition ξ_{1_s}. Then, considering that $\xi_{1_s} = z_s \cdot \exp j\phi_s$ and that ξ_s and ξ_n are independent vectors, the following results are obtained under the condition $\xi_{1_s} = z_s \cdot \exp j\phi_s$ which means instantaneous SNR $\gamma = z_s^* z_s/2\sigma_n^2$,

$$\begin{aligned}\langle\xi_1\rangle &= \xi_{1_s}\\ \langle\xi_2\rangle &= \rho_s^* \xi_{1_s}.\end{aligned} \tag{24}$$

Using (24), σ_1, σ_2, and ρ in (18) are given by

$$\begin{aligned}\sigma_1^2 &= \sigma_n^2\\ \sigma_2^2 &= \sigma_s^2(1 - |\rho_s|^2) + \sigma_n^2\\ \sigma_1\sigma_2\rho &= \sigma_n^2 \rho_n^*.\end{aligned} \tag{25}$$

Then

$$\begin{aligned}a_\pm &= \frac{\gamma}{2\{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)\}}\left[\Gamma(1-|\rho_s|^2) + 1 + |\rho_s|^2 - 2\rho_{n_s}\rho_{s_s} - 2\rho_{s_c}\sqrt{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}\right]\\ b_\pm &= \frac{\gamma}{2\{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)\}}\left[\Gamma(1-|\rho_s|^2) + 1 + |\rho_s|^2 - 2\rho_{n_s}\rho_{s_s} + 2\rho_{s_c}\sqrt{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}\right]\\ A_\pm &= \frac{\rho_{n_c}}{\sqrt{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}},\end{aligned} \tag{26}$$

where

$$\begin{aligned}\rho_s &= \rho_{s_c} + j\rho_{s_s}\\ \rho_n &= \rho_{n_c} + j\rho_{n_s}\end{aligned} \tag{27}$$

and supersubscripts c and s denote the real and imaginal parts, respectively.

For convenience in calculation of the average error rate in (13), we can rewrite (17) by using the following equation derived from [13]

$$Q(\sqrt{b}, \sqrt{a}) - Q(\sqrt{a}, \sqrt{b}) = \frac{b-a}{b+a} I_e\left(\frac{2\sqrt{ab}}{a+b}, \frac{a+b}{2}\right), \tag{28}$$

where $I_e(a, b)$ is Rice's I_e-function defined as

$$I_e(a, b) = \int_0^b I_0(at) \exp(-t)\, dt. \tag{29}$$

Introducing the following transformations

$$\begin{aligned}\alpha\gamma &= \frac{b+a}{2}\\ \beta\gamma &= \frac{b-a}{2},\end{aligned} \tag{30}$$

(17) becomes

$$\begin{aligned}p_{e+}(\gamma) &= \frac{1}{2}\left[1 - \left(\frac{\beta_+}{\alpha_+}\right) I_e\left(\sqrt{1 - \left(\frac{\beta_+}{\alpha_+}\right)^2}, \beta_+\gamma\right)\right] - \frac{A_+}{2}\exp(-\alpha_+\gamma) I_0\left(\alpha_+\sqrt{\left(1 - \frac{\beta_+}{\alpha_+}\right)^2}\,\gamma\right)\\ p_{e-}(\gamma) &= \frac{1}{2}\left[1 + \left(\frac{\beta_-}{\alpha_-}\right) I_e\left(\sqrt{1 - \left(\frac{\beta_-}{\alpha_-}\right)^2}, \beta_-\gamma\right)\right] + \frac{A_-}{2}\exp(-\alpha_-\gamma) I_0\left(\alpha_-\sqrt{1 - \left(\frac{\beta_-}{\alpha_-}\right)^2}\,\gamma\right),\end{aligned} \tag{31}$$

where $\alpha_\pm$, $\beta_\pm$, and $A_\pm$ are, from (26) and (30), given by

$$\begin{aligned}\alpha_\pm &= \frac{1}{2}\left[1 + \frac{|\rho_s|^2 + \rho_{n_s}{}^2 - 2\rho_{n_s}\rho_{s_s}}{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}\right]\\ \beta_\pm &= \frac{\rho_{s_c}}{\sqrt{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}}\\ A_\pm &= \frac{\rho_{n_c}}{\sqrt{\Gamma(1-|\rho_s|^2) + (1-\rho_{n_s}{}^2)}},\end{aligned} \tag{32}$$

where $\rho_s = \rho_{s_c} + j\rho_{s_s}$, $\rho_n = \rho_{n_c} + j\rho_{n_s}$, and the plus and minus signs correspond to mark transmission and space transmission, respectively.

The normalized covariance coefficient ρ_s and ρ_n are given by, from (4) and (21),

$$\begin{aligned}\rho_s &= -jk_s(\tau)\exp j(\phi_s - \phi_{s\tau})\\ \rho_n &= -jk_n(\tau),\end{aligned} \tag{33}$$

where the phases ϕ_s and $\phi_{s\tau}$ are the modulation phases at the sampling instant νT and $\nu T - \tau$, respectively, and $k_s(\tau)$ and $k_n(\tau)$ are the normalized autocorrelation functions defined by

$$k_s(\tau) = \int_{-\infty}^{\infty} G_s(f) \exp j(2\pi f \tau)\, df$$

$$k_n(\tau) = \int_{-\infty}^{\infty} G_n(f) \exp j(2\pi f \tau)\, df, \tag{34}$$

where $G_s(f)$ and $G_n(f)$ are, respectively, the power spectra of the ith branch complex envelopes $z_{s_i}(t)$ and $z_{n_i}(t)$ (for all i).

Note that in (31)-(34) while the conditional error rate of differential detection is obtained by letting time delay τ be the signaling period T, that of discriminator detection is obtained by letting τ be zero.

C. Average Error Rate

Substituting (11) and (31) into (13) and realizing that

$$\int_0^{\infty} c \exp(-cx) I_e(a, bx)\, dx = b/\sqrt{(b+c)^2 - a^2 b^2}, \tag{35}$$

the average error rates of mark and space become

$$P_{e+} = \frac{1}{2} \sum_{n=1}^{N} {}_N C_n (-1)^{n+1} \left[1 - \frac{\beta_+ \Gamma + n A_+}{\sqrt{\alpha_+ \left\{ 1 + \sqrt{1 - \left(\frac{\beta_+}{\alpha_+} \right)^2} \right\} \Gamma + n} \sqrt{\alpha_+ \left\{ 1 - \sqrt{1 - \left(\frac{\beta_+}{\alpha_+} \right)^2} \right\} \Gamma + n}} \right]$$

$$P_{e-} = \frac{1}{2} \sum_{n=1}^{N} {}_N C_n (-1)^{n+1} \left[1 + \frac{\beta_- \Gamma + n A_-}{\sqrt{\alpha_- \left\{ 1 + \sqrt{1 - \left(\frac{\beta_-}{\alpha_-} \right)^2} \right\} \Gamma + n} \sqrt{\alpha_- \left\{ 1 - \sqrt{1 - \left(\frac{\beta_-}{\alpha_-} \right)^2} \right\} \Gamma + n}} \right], \tag{36}$$

where $\alpha_\pm$, $\beta_\pm$, and $A_\pm$ are given by (32). Equation (36) shows the average error rate which includes both effects of Rayleigh envelope fading and random FM noise.

In particular, when average SNR $\Gamma \to \infty$, the irreducible error due to random FM noise becomes dominant. When $\Gamma \to \infty$, since

$$\alpha_\pm \to 1/2$$

$$\beta_\pm \to \rho_{s_c} / \sqrt{\Gamma(1 - |\rho_s|^2)}$$

$$A_\pm \to \rho_{n_c} / \sqrt{\Gamma(1 - |\rho_s|^2)} \tag{37}$$

then (36) becomes

$$P_{e+} = \frac{1}{2} \sum_{n=1}^{N} {}_N C_n (-1)^{n+1} \left[1 - \frac{\mu_+}{\sqrt{\mu_+^2 + n}} \right]$$

$$P_{e-} = \frac{1}{2} \sum_{n=1}^{N} {}_N C_n (-1)^{n+1} \left[1 + \frac{\mu_-}{\sqrt{\mu_-^2 + n}} \right], \tag{38}$$

where

$$\mu_\pm = \frac{\rho_{s_c}}{\sqrt{1 - |\rho_s|^2}}. \tag{39}$$

IV. NUMERICAL RESULTS

In Section III we have obtained the generalized results for the average error rate which include both effects of Rayleigh envelope fading and random FM noise. To analyze the average error rates numerically, let us assume that the vehicle having vertical dipole antennas for diversity antennas is moving through the multipath propagation field with a constant speed and that the each predetection BPF has an ideal rectangular characteristics with a center frequency f_c. The power spectra of the ith branch complex envelopes $z_{s_i}(t)$ and $z_{n_i}(t)$ (for all i) are given by, respectively,

$$G_s(f) = \sigma_s^2 / \pi\sqrt{f_D^2 - f^2}, \qquad |f| \leqslant f_D$$

$$G_n(f) = \sigma_n^2 / B, \qquad |f| \leqslant B/2, \tag{40}$$

where f_D and B are the maximum Doppler frequency [14] and the predetection BPF bandwidth, respectively. Thus, from (34) the normalized autocorrelation functions $k_s(\tau)$ and $k_n(\tau)$ are given by

$$\begin{cases} k_s(\tau) = J_0(2\pi f_D \tau) \\ k_n(\tau) = \sin(\pi B \tau)/(\pi B \tau), \end{cases} \tag{41}$$

where $J_0(\cdot)$ is the first kind zeroth order Bessel function.

Substituting (33) and (41) into (32), $\alpha_\pm$, $\beta_\pm$, and $A_\pm$ for differential detection are, respectively, given by

$$\alpha_+ = \alpha_- = \frac{1}{2} \left[1 + \frac{J_0^2(2\pi f_D T) + \{\sin(\pi B T)/(\pi B T)\}^2 - 2 J_0(2\pi f_D T) \cos(2\pi \Delta f_d T) \{\sin(\pi B T)/(\pi B T)\}}{\Gamma\{1 - J_0^2(2\pi f_D T)\} + (1 - \{\sin(\pi B T)/(\pi B T)\}^2)} \right]$$

$$\begin{pmatrix} \beta_+ \\ \beta_- \end{pmatrix} = \pm \frac{J_0(2\pi f_D T) \sin(2\pi \Delta f_d T)}{\sqrt{\Gamma\{1 - J_0^2(2\pi f_D T)\} + (1 - \{\sin(\pi B T)/(\pi B T)\}^2)}}$$

$$A_+ = A_- = 0. \tag{42}$$

$\alpha_\pm$, $\beta_\pm$, and $A_\pm$ for discriminator detection can be easily obtained by letting $T \to 0$ in (42) since the differential detection performance approaches that of the discriminator detection as the time delay T in the differential detector is made small. Letting $T \to 0$ in (42), $\alpha_\pm$, $\beta_\pm$, and $A_\pm$ for discriminator detection are given by

$$\alpha_+ = \alpha_- = \frac{1}{2}\left[1 + \frac{3(2\Delta f_d T)^2}{6(f_D T)^2 \Gamma + (BT)^2}\right]$$

$$\begin{pmatrix}\beta_+ \\ \beta_-\end{pmatrix} = \pm \frac{\sqrt{3}(2\Delta f_d T)}{\sqrt{6(f_D T)^2 \Gamma + (BT)^2}} \tag{43}$$

$$A_+ = A_- = 0.$$

In (42), (43), $2\Delta f_d T$ indicates the modulation index.

Substituting (42), (43) into (36), we can calculate numerically the average error rate including both effects of Rayleigh envelope fading and random FM noise. Particularly, irreducible error due to random FM noise can be calculated by (38) with

$$\begin{pmatrix}\mu_+ \\ \mu_-\end{pmatrix} = \pm \frac{J_0(2\pi f_D T)\sin(2\pi \Delta f_d T)}{1 - J_0{}^2(2\pi f_D T)}, \quad \text{for differential} \tag{44}$$

and

$$\begin{pmatrix}\mu_+ \\ \mu_-\end{pmatrix} = \pm \frac{2\Delta f_d T}{\sqrt{2} f_D T}, \quad \text{for discriminator.} \tag{45}$$

Digital FM with modulation index ($2\Delta f_d T$) 0.5, which is generally called MSK, is considered to be one of the most useful modulation schemes. In the following, we assume MSK transmission and the predetection BPF bandwidth with $BT = 1.0$.

A few examples of the calculated results for MSK are shown in Figs. 2–4. Figs. 2 and 3 show the error rate performances with diversity order N as a parameter for the maximum Doppler frequency $f_D T \to 0$ and 0.02. Fig. 4 shows the irreducible error rate due to random FM noise (average SNR $\Gamma \to \infty$). These figure show that the selection diversity can combat both effects of Rayleigh envelope fading and random FM noise.

The normalized maximum Doppler frequency $f_D T \to 0$ indicates the slow Rayleigh fading case where an error is produced dominantly due to Rayleigh envelope fading effect. In this case the diversity gain defined as the reduced quantity of the required average SNR for obtaining some specific average error rate, becomes 22.7, 30.1, 33.9, and 36.3 dB for $N = 2$, 3, 4, and 5, respectively, when $P_e = 10^{-5}$. Note that the diversity gain does not differ between two detection schemes.

The average SNR $\Gamma \to \infty$ indicates the fast Rayleigh fading case where an error is produced dominantly due to random FM noise effect (Fig. 4). When the normalized maximum Doppler frequency $f_D T = 0.0025$, which indicates typical vehicle speed of 48 km/h for 16 kbit/s and 900 MHz carrier frequency,

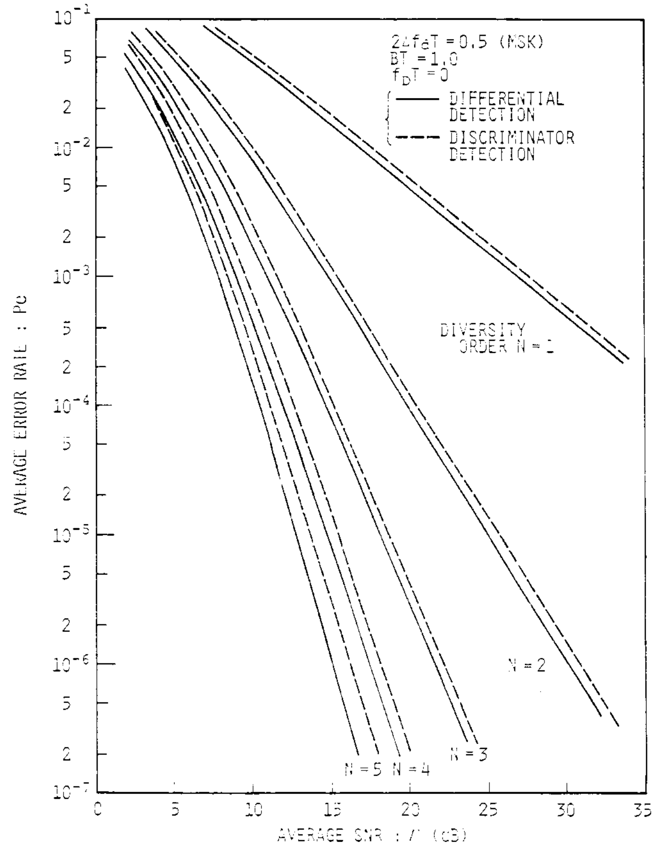

Fig. 2. Error rate performance of MSK with $f_D T \to 0$ (slow fading).

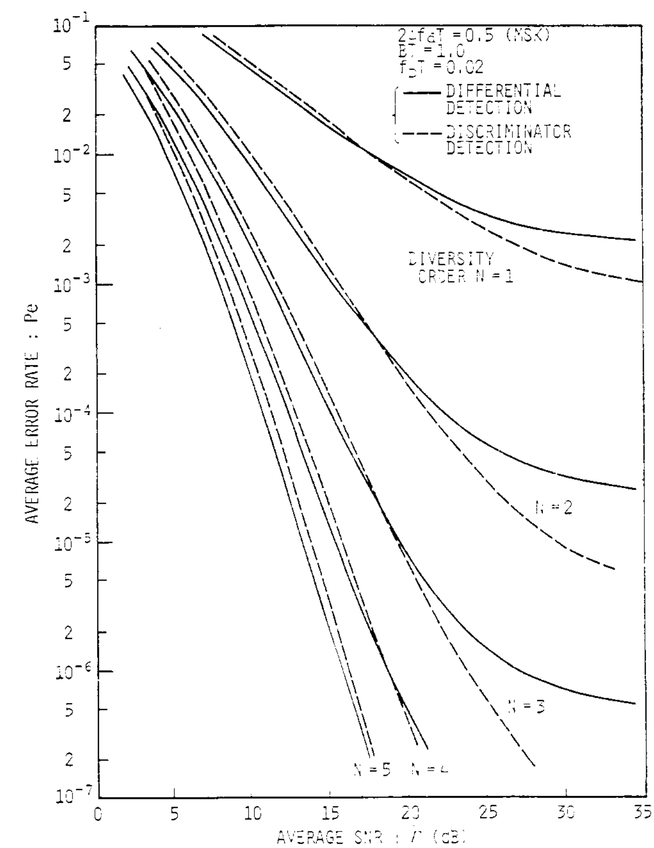

Fig. 3. Error rate performance of MSK with $f_D T = 0.02$ (fast fading).

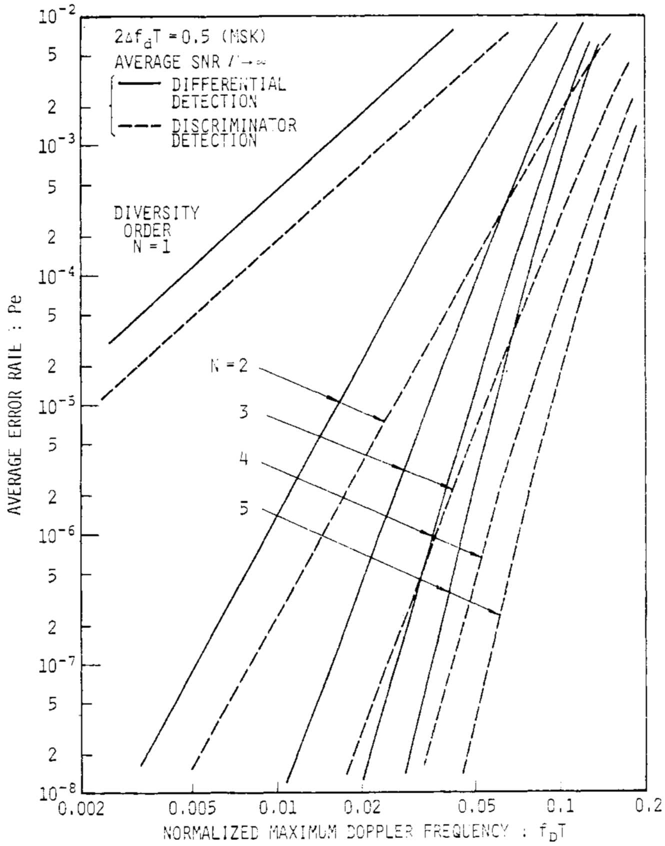

Fig. 4. Irreducible error rate of MSK due to random FM noise.

the irreducible error rate for differential detection with two branch diversity becomes 2.9×10^{-9}, while that with no diversity is 3.1×10^{-5}. However the irreducible error rate for discriminator detection with two branch diversity becomes 4.8×10^{-10}, while that with no diversity is 1.3×10^{-5} when $f_D T = 0.0025$.

It is interesting that although the differential detection performance is slightly superior to the discriminator detection performance when the Rayleigh envelope fading effect is dominant, the discriminator detection performance becomes superior to the differential detection performance as the random FM noise effect becomes dominant (average SNR $\Gamma \to \infty$). This interesting results can be qualitatively understood from the fact that the discriminator, whose detection mechanism is considered to be equivalent to that of a differential detector using the delay line with the infinitesimal time delay, is able to track the fast phase change more quickly than the differential detector in the high SNR conditions [5]. However, this is a problem which is an area for future study.

V. CONCLUSION

This paper theoretically analyzed the ideal postdetection selection diversity effects on digital FM communication system in the fast Rayleigh fading signal environment. Both differential and discriminator detections are assumed for demodulation of digital FM. While the effect of intersymbol interference caused by predetection bandpass filter band restrictions is neglected, the fading spectrum effect is taken into consideration. The average error rate with diversity including both effects of Rayleigh envelope fading and random FM noise is presented by a simple closed form which is very useful for the design of digital FM land mobile radio systems.

A few examples of the numerical results are graphically presented for MSK system and show that the postdetection selection diversity can combat both effects of Rayleigh envelope fading and random FM noise.

REFERENCES

[1] T. Brenig, "Data transmission for mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-27, pp. 77–85, Aug. 1978.

[2] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill, 1966, pp. 334–340, p. 435, pp. 590–595.

[3] R. R. Anderson, W. R. Bennett, J. R. Davey, and J. Salz, "Differential detection of binary FM," *Bell Sys. Tech. J.*, vol. 44, pp. 111–159, Jan. 1965.

[4] R. de Buda, "Coherent demodulation of frequency shift keying with low deviation ratio," *IEEE Trans. Commun.*, vol. COM-20, pp. 429–435, June 1972.

[5] K. Hirade, M. Ishizuka, F. Adachi, and K. Ohtani, "Error-rate performance of digital FM with differential detection in land mobile radio channels," *IEEE Trans. Veh. Technol.*, vol. VT-28, pp. 204–212, Aug. 1979.

[6] K. Hirade, M. Ishizuka, and F. Adachi, "Error rate performance of digital FM with discriminator detection in the presence of co-channel interference under fast Rayleigh fading environment," *Trans. IECE Japan*, vol. E61, pp. 704–709, Sept. 1978.

[7] S. Elnoub and S. C. Gupta, "Error rate performance of non-coherent detection of duobinary coded MSK and TFM in mobile radio communication systems," *IEEE Trans. Veh. Technol.*, vol. VT-30, pp. 62–76, May 1981.

[8] J. D. Parsons, M. Henze, P. A. Ratliff, and M. J. Withers, "Diversity techniques for mobile radio reception," *Radio and Electron. Eng.*, vol. 45, pp. 357–367, July 1975.

[9] P. Prapinmongkolkarn, N. Morinaga, and T. Namekawa, "Performance of digital FM system in a fading environment," *IEEE Trans. Aerosp. and Electron. Syst.*, vol. AES-10, pp. 698–709, Sept. 1974.

[10] F. Adachi, "Postdetection selection diversity effects in a digital FM land mobile radio with discriminator and differential detections," *Trans. IECE Japan*, (in Japanese), vol. J63-B, pp. 759–766, Aug. 1980.

[11] F. Adachi, "Selection and scanning diversity effects in a digital FM land mobile radio with limiter discriminator detection," *Trans. IECE Japan*, vol. E64, pp. 398–405, June 1981.

[12] S. Stein, "Unified analysis of certain coherent and noncoherent binary communication systems," *IEEE Trans. Inform. Theory*, vol. IT-10, pp. 43–51, Jan. 1964.

[13] W. R. Bennett and J. R. Davey, *Data transmision*. New York: McGraw-Hill, 1965, p. 182.

[14] M. J. Gans, "A power-spectral theory of propagation in the mobile radio environment," *IEEE Trans. Veh. Technol.*, vol. VT-21, pp. 27–38, Feb. 1972.

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