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Error-Rate Performance of Digital FM with Differential Detection in Land Mobile Radio Channels

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Abstract—Error-rate performance of a digital FM with differential detection in the presence of both thermal Gaussian noise and cochannel interference is theoretically analyzed in the fast Rayleigh fading environment encountered in the typical UHF or microwave land mobile radio channels. The temporal correlation of the fades is included in the performance analysis. The error probability is presented by a simple closed form for the important situations where both effects of Gaussian noise and cochannel interference predominate in causing errors. Finally, a comparison with the other detection schemes, e.g., discriminator and coherent detections, is given.

I. INTRODUCTION

DIGITAL FM with a low modulation index, which is also called a continuous-phase FSK, is considered to be one of the most useful modulation schemes for digital mobile radio communication systems because of its constant envelope property and small RF bandwidth requirement. There are three typical demodulation schemes for such signals: discriminator detection [1]-[3], differential detection [4], [5], and coherent detection [6]-[10].

Though the error-rate performance of differential detection is slightly inferior to that of coherent detection, the receiver structure is greatly simplified since it does not require the complicated phase-locked loop carrier recovery circuit, and only a simple delay line with time delay equal to the signaling interval is used to provide a direct estimate of the phase difference. It is not easy to construct a simple carrier recovery circuit which enables one to regenerate the reference carrier precisely and stably in the fast Rayleigh fading channels such as UHF or microwave land mobile radio channels. Furthermore, compared with discriminator detection, differential detection is affected little by severe delay distortions [4] which occur in most narrowband mobile radio transceivers. Thus, the importance and usefulness of digital FM with differential detection will increase in the wide area of UHF or microwave digital land mobile radio communications. While there are a few reports analyzing its performance [4], [5], all of them are concerned with the error-rate performance in the nonfading condition, and the effect of cochannel interference has never been considered in the nonfading and the fading conditions.

This paper presents a theoretical analysis of error-rate performance of digital FM with differential detection in the fast Rayleigh fading environment, such as is usually encountered in UHF or microwave land mobile radio channels. Not only the effect of Gaussian noise present in the channel or generated in the receiver but also that of cochannel interference caused by the geographical frequency reuse are taken into consideration. Our method is an extension of Stein's method [11] which was used for evaluating the error-rate performance of certain coherent and noncoherent binary communication systems in the nonfading condition. A fairly complete solution of the error-rate performance is given by a simple closed form for the important situation where both effects of Gaussian noise and cochannel interference predominate in causing errors. Finally, comparison with the other detection schemes, e.g., discriminator and coherent detections, is made from various aspects.

II. SYSTEM MODEL

Fig. 1 shows the mathematical model of a digital FM receiver using the differential detection scheme which is analyzed in this paper. It is assumed that not only the desired signal but also an undesired cochannel interference are received simultaneously and that both of them are digital FM signals which are modulated by different baseband pulse sequences with the identical signaling rate and are subject to mutually independent fast Rayleigh fadings. Furthermore, it is assumed that white Gaussian noise is added at the receiver front end. After being combined, those are passed through the predetection bandpass filter and are detected by the differential detector. The differential detector consists of a product demodulator followed by a baseband filter. The baseband filter is used only for filtering out the double-frequency component of the product demodulator output, and its noise suppresion effect is not taken into consideration. In addition, the intersymbol interferences caused by the bandwidth limitation of the predetection and postdetection filters are assumed to be negligible. The differential detector output is then synchronously sampled by using the recovered timing information so

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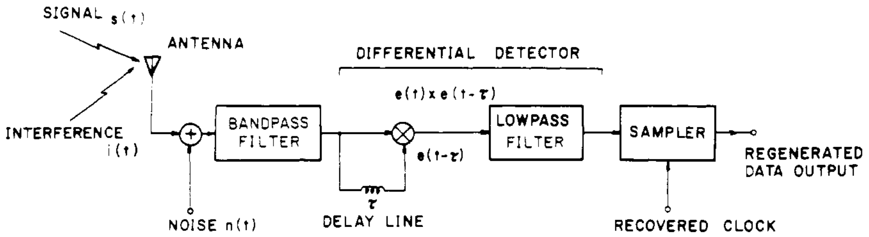

Fig. 1. Block diagram of digital FM receiver model using differential detection scheme.

that the transmitted digital baseband pulses can be sequentially regenerated.

III. MATHEMATICAL REPRESENTATION OF DETECTOR OUTPUT

According to the above-described system model, the desired signal $s(t)$, the undesired cochannel interference $i(t)$, and the additive Gaussian noise $n(t)$ at the predetection bandpass filter output can be respectively represented as the following narrowband Gaussian processes:

$$\begin{aligned} s(t) &= \mathrm{Re}\,\{z_s(t)\exp j[2\pi f_c t + \phi_s(t)]\}, \\ i(t) &= \mathrm{Re}\,\{z_i(t)\exp j[2\pi f_c t + \phi_i(t)]\}, \\ n(t) &= \mathrm{Re}\,\{z_n(t)\exp j[2\pi f_c t], \end{aligned} \tag{1}$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of $\{\cdot\}$, f_c is the nominal center frequency of the carrier, and $\phi_s(t)$ and $\phi_i(t)$ are the instantaneous modulation phase changes relative to $2\pi f_c t$ of $s(t)$ and $i(t)$, respectively. Furthermore, $z_s(t)$, $z_i(t)$, and $z_n(t)$ are the mutually independent stationary zero-mean complex Gaussian processes. Then, the differential detector input signal $e(t)$ is obtained as

$$e(t) = s(t) + i(t) + n(t) = \mathrm{Re}\,\{z(t)\exp j[2\pi f_c t]\}, \tag{2}$$

where $z(t)$ is given by

$$z(t) = z_s(t)\exp j[\phi_s(t)] + z_i(t)\exp j[\phi_i(t)] + z_n(t). \tag{3}$$

Since the differential detector for digital FM signals operates as a quadrature product demodulator of $e(t)$ and $e(t-T)$, the detector output $\upsilon(t)$ can be written as

$$\upsilon(t) = \tfrac{1}{2}\,\mathrm{Re}\,\{-jz(t)z^*(t-T)\}, \tag{4}$$

where the asterisk symbol denotes the complex conjugate and T is the signaling period. Considering that $\upsilon(t)$ is sampled at some particular instant, $t = \nu T$ ($\nu = 0, \pm 1, \pm 2, \cdots$), the sampled output is

$$\upsilon(\nu T) = \tfrac{1}{2}\,\mathrm{Re}\,\{-jz_1 z_2^*\}, \tag{5}$$

where z_1 and z_2 are respectively given by

$$\begin{aligned} z_1 &= z(\nu T) = z_{s1}\exp(j\phi_{s1}) + z_{i1}\exp(j\phi_{i1}) + z_{n1}, \\ z_2 &= z(\overline{\nu - 1}T) = z_{s2}\exp(j\phi_{s2}) + z_{i2}\exp(j\phi_{i2}) + z_{n2}, \end{aligned} \tag{6}$$

where z_{s1}, z_{s2}, z_{i1}, z_{i2}, z_{n1}, z_{n2}, ϕ_{s1}, ϕ_{s2}, ϕ_{i1}, and ϕ_{i2} are defined by $z_s(\nu T)$, $z_s(\overline{\nu - 1}T)$, $z_i(\nu T)$, $z_i(\overline{\nu - 1}T)$, $z_n(\nu T)$, $z_n(\overline{\nu - 1}T)$, $\phi_s(\nu T)$, $\phi_s(\overline{\nu - 1}T)$, $\phi_i(\nu T)$, and $\phi_i(\overline{\nu - 1}T)$, respectively. Defining the new random variables ζ_1 and ζ_2 by

$$\begin{aligned} \zeta_1 &= \tfrac{1}{2}(z_1 + jz_2) = R_1 \exp(j\theta_1), \\ \zeta_2 &= \tfrac{1}{2}(z_1 - jz_2) = R_2 \exp(j\theta_2), \end{aligned} \tag{7}$$

where R_1 and $R_2 \geqslant 0$, $-\pi \leqslant \theta_1$, and $\theta_2 \leqslant \pi$, (5) can be written in the following simplified form:

$$\upsilon(\nu T) = \tfrac{1}{2}(|\zeta_1|^2 - |\zeta_2|^2) = \tfrac{1}{2}(R_1^2 - R_2^2). \tag{8}$$

The decision as to whether a mark or a space was sent is based on the polarity of $\upsilon(\nu T)$. Therefore, in order to analyze the error-rate performance it is necessary to know the joint probability density function (pdf) of R_1 and R_2, $p(R_1, R_2)$, when a mark or a space was sent in $s(t)$.

IV. DERIVATION OF $p(R_1, R_2)$

Since $z_s(t)$, $z_i(t)$, and $z_n(t)$ are mutually independent stationary zero-mean complex Gaussian processes, z_{s1}, z_{s2}, z_{i1}, z_{i2}, z_{n1}, and z_{n2} become zero-mean complex Gaussian variables having the complex covariance matrix $\mathbf{H}_a$ given by

$$H_a = \tfrac{1}{2}\langle [a - \langle a\rangle_{av}]^* [a - \langle a\rangle_{av}]^t\rangle_{av}, \tag{9}$$

where $\langle\cdot\rangle_{av}$ is an ensemble average, $[\cdot]^t$ is a transpose-matrix symbol, and $\mathbf{a}$ is a column matrix defined by

$$a = \begin{bmatrix} z_{s1} \\ z_{s2} \\ z_{i1} \\ z_{i2} \\ z_{n1} \\ z_{n2} \end{bmatrix}. \tag{10}$$

Using H_s, H_i, and H_n defined by

$$H_s = \frac{1}{2}\langle [z_s - \langle z_s \rangle_{av}]^* [z_s - \langle z_s \rangle_{av}]^t \rangle_{av} = \sigma_s^2 \begin{bmatrix} 1 & , \rho_s^*(T) \\ \rho_s(T), & 1 \end{bmatrix},$$

$$H_i = \frac{1}{2}\langle [z_i - \langle z_i \rangle_{av}]^* [z_i - \langle z_i \rangle_{av}]^t \rangle_{av} = \sigma_i^2 \begin{bmatrix} 1 & , \rho_i^*(T) \\ \rho_i(T), & 1 \end{bmatrix}, \quad (11)$$

$$H_n = \frac{1}{2}\langle [z_n - \langle z_n \rangle_{av}]^* [z_n - \langle z_n \rangle_{av}]^t \rangle_{av} = \sigma_n^2 \begin{bmatrix} 1 & , \rho_n^*(T) \\ \rho_n(T), & 1 \end{bmatrix},$$

where z_s, z_i, and z_n are respectively defined by

$$z_s = \begin{bmatrix} z_{s1} \\ z_{s2} \end{bmatrix}, \quad z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}, \quad z_n = \begin{bmatrix} z_{n1} \\ z_{n2} \end{bmatrix}, \quad (12)$$

the complex covariance matrix H_a can be represented as

$$H_a = \begin{bmatrix} H_s, & O, & O \\ O, & H_i, & O \\ O, & O, & H_n \end{bmatrix}, \quad (13)$$

where $\mathbf{O}$ is a 2×2 zero-matrix and σ_S^2, σ_i^2, σ_n^2 and $\rho_S(T)$, $\rho_i(T)$, $\rho_n(T)$ are the average powers and the normalized autocorrelation functions of $s(t)$, $i(t)$, and $n(t)$, which are respectively given by

$$\sigma_s^2 \rho_s(T) = \int_{-\infty}^{\infty} W_s(f) \exp(j2\pi fT)\, df,$$

$$\sigma_i^2 \rho_i(T) = \int_{-\infty}^{\infty} W_i(f) \exp(j2\pi fT)\, df, \quad (14)$$

$$\sigma_n^2 \rho_n(T) = \int_{-\infty}^{\infty} W_n(f) \exp(j2\pi fT)\, df,$$

where $W_S(f)$, $W_i(f)$, and $W_n(f)$ are the equivalent baseband power spectra of $s(t)$, $i(t)$, and $n(t)$, respectively.

Define a new column matrix $\mathbf{b}$ by

$$b = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (15)$$

Since (6) is a linear transformation, z_1 and z_2 are also zero-mean complex Gaussian variables whose complex covariance matrix $\mathbf{H}_b$ is given by

$$H_b = \frac{1}{2}\langle [b - \langle b \rangle_{av}]^* [b - \langle b \rangle_{av}]^t \rangle_{av} = \sigma^2 \begin{bmatrix} 1, & \rho^* \\ \rho, & 1 \end{bmatrix}, \quad (16)$$

where σ^2 and ρ are respectively given by

$$\sigma^2 = \sigma_s^2 + \sigma_i^2 + \sigma_n^2,$$
$$\sigma^2 \rho = \sigma_s^2 \rho_s(T) \exp j(\phi_{s1} - \phi_{s2}) + \sigma_i^2 \rho_i(T) \langle \exp j(\phi_{i1} - \phi_{i2}) \rangle_{av} + \sigma_n^2 \rho_n(T). \quad (17)$$

Since the sampling instant of interest, $t = \nu T$ ($\nu = 0, \pm 1, \pm 2, \cdots$), is perfectly locked to the timing phase of $\phi_s(t)$ and the timing sources of $\phi_S(t)$ and $\phi_i(t)$ are asynchronous, ρ in (17) becomes

$$\sigma^2 \rho = \begin{cases} \sigma^2 \rho_+ = \sigma_s^2 \rho_s(T) \exp(jm_s\pi) + \sigma_i^2 \rho_i(T) R_i(T) + \sigma_n^2 \rho_n(T) & \text{for mark,} \\ \sigma^2 \rho_- = \sigma_s^2 \rho_s(T) \exp(-jm_s\pi) + \sigma_i^2 \rho_i(T) R_i(T) + \sigma_n^2 \rho_n(T) & \text{for space,} \end{cases} \quad (18)$$

where $R_i(T)$ is given by [12]

$$R_i(T) = \langle \exp j(\phi_{i1} - \phi_{i2}) \rangle_{av} = \frac{1}{2}\left(\cos m_i\pi + \frac{\sin m_i\pi}{m_i\pi} \right), \quad (19)$$

and m_s and m_i are modulation indices of $s(t)$ and $i(t)$, which are respectively defined by

$$m_s = 2\Delta f_s T, \qquad m_i = 2\Delta f_i T, \quad (20)$$

where Δf_s and Δf_i are frequency deviations of $s(t)$ and $i(t)$, respectively. By letting the complex Gaussian variables z_1 and z_2, given by (6) and (7), be written in terms of their real and imaginary components,

$$z_1 = x_1 + jy_1, \qquad z_2 = x_2 + jy_2, \quad (21)$$

their joint pdf is obtained as

$$p(x_1, y_1, x_2, y_2) = \frac{1}{(2\pi)^2 \det H_b} \exp\{-\tfrac{1}{2}[b - \langle b \rangle_{av}]^t H_b^{-1} [b - \langle b \rangle_{av}]^*\}, \quad (22)$$

where det $\mathbf{H}_b$ and $\mathbf{H}_b^{-1}$ are the determinant and the inverse matrix of $\mathbf{H}_b$, which are respectively given by

$$\det H_b = \sigma^4(1 - |\rho|^2), \tag{23}$$

$$H_b^{-1} = \frac{1}{\sigma^2(1-|\rho|^2)} \begin{bmatrix} 1, & -\rho^* \\ -\rho, & 1 \end{bmatrix}. \tag{24}$$

From (15), (23), and (24), (22) can be written as

$$p(x_1, y_1, x_2, y_2) = \frac{1}{(2\pi)^2\sigma^4(1-|\rho|^2)} \cdot \exp\left\{-\frac{1}{2\sigma^2(1-|\rho|^2)}[x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2\rho_c(x_1x_2 + y_1y_2) - 2\rho_s(x_2y_1 - x_1y_2)]\right\}, \tag{25}$$

where ρ_c and ρ_s are the real and imaginary components of ρ, respectively.

Using (7) and (21), the relationship between x_1, y_1, x_2, y_2 and $R_1, \theta_1, R_2, \theta_2$ can be written as

$$\begin{aligned} x_1 &= R_1 \cos\theta_1 + R_2 \cos\theta_2, \\ y_1 &= R_1 \sin\theta_1 + R_2 \sin\theta_2, \\ x_2 &= R_1 \sin\theta_1 - R_2 \sin\theta_2, \\ y_2 &= -R_1 \cos\theta_1 + R_2 \cos\theta_2. \end{aligned} \tag{26}$$

with Jacobian

$$\frac{\partial(x_1, y_1, x_2, y_2)}{\partial(R_1, \theta_1, R_2, \theta_2)} = 4R_1R_2. \tag{27}$$

Consequently, from (25)-(27), the joint pdf of R_1, θ_1, R_2, and θ_2 can be derived as [13]

$$p(R_1, \theta_1, R_2, \theta_2) = \frac{4R_1R_2}{(2\pi)^2\sigma^4(1-|\rho|^2)} \exp\left\{-\frac{1}{\sigma^2(1-|\rho|^2)} \cdot [(1-\rho_s)R_1^2 + (1+\rho_s)R_2^2 - 2\rho_cR_1R_2 \sin(\theta_1 - \theta_2)]\right\}. \tag{28}$$

Integrating (28) with respect to θ_1 and θ_2, the joint pdf of R_1 and R_2 is then obtained as the following joint Rayleigh distribution [14]:

$$p(R_1, R_2) = \frac{4R_1R_2}{\sigma^4(1-|\rho|^2)} \cdot \exp\left\{-\frac{(1-\rho_s)R_1^2 + (1+\rho_s)R_2^2}{\sigma^2(1-|\rho|^2)}\right\} \cdot I_0\left[\frac{2\rho_cR_1R_2}{\sigma^2(1-|\rho|^2)}\right], \tag{29}$$

where $I_0(\cdot)$ denotes the zeroth order modified Bessel function of the first kind, which is defined by

$$I_0(\xi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[\xi \cos\psi]\, d\psi. \tag{30}$$

V. ERROR PROBABILITY

Assuming that the probability of mark transmission in $s(t)$ is denoted by p and that the events of mark and space transmission in $s(t)$ are respectively represented as "M" and "S," the net error probability P_e is given by

$$P_e = p\int_0^\infty dR_2 \int_0^{R_2} p(R_1, R_2 \mid \text{“}M\text{”})\, dR_1 + (1-p)\int_0^\infty dR_2 \int_{R_1}^\infty p(R_1, R_2 \mid \text{“}S\text{”})\, dR_1, \tag{31}$$

where $p(R_1, R_2 \mid \text{“}M\text{”})$ and $p(R_1, R_2 \mid \text{“}S\text{”})$ denote the conditional joint pdf's of R_1 and R_2 for mark and space transmission in $s(t)$, respectively.

Substituting (29) and (30) into (31), letting $R_1/R_2 = u$, and calculating the two integrals in (31) with respect to R_2 firstly, ψ secondly, and u finally, the error probability P_e is given by

$$P_e = \frac{p}{2}\left\{1 - \frac{\rho_{s+}}{\sqrt{1-\rho_{c+}^2}}\right\} + \frac{1-p}{2}\left\{1 - \frac{\rho_{s-}}{\sqrt{1-\rho_{c-}^2}}\right\}, \tag{32}$$

where $\rho_{c\pm}$ and $\rho_{s\pm}$ denote the real and imaginary components of $\rho_\pm$ defined by (18), respectively.

In particular, when $W_s(f)$, $W_i(f)$, and $W_n(f)$ in (14) are assumed to be even symmetrical with respect to f, then $\rho_s(T)$, $\rho_i(T)$, and $\rho_n(T)$ are real functions. Equation (32) becomes

$$P_e = \frac{1}{2}\left[1 - \frac{\Gamma\Lambda\rho_s(T)\sin m_s\pi}{\left\{(\Gamma\Lambda + \Gamma + \Lambda)^2 - \left[\Gamma\Lambda\rho_s(T)\cos m_s\pi + \frac{\Gamma}{2}\rho_i(T)\left(\cos m_i\pi + \frac{\sin m_i\pi}{m_i\pi}\right) + \Lambda\rho_n(T)\right]^2\right\}^{1/2}}\right], \tag{33}$$

where Γ and Λ are the average signal-to-noise ratio (SNR) and the average signal-to-interference ratio (SIR), defined

$$\Gamma = \sigma_s^2/\sigma_n^2, \qquad \Lambda = \sigma_s^2/\sigma_i^2. \tag{34}$$

Especially when the cochannel interference is not present, that is, when $\Lambda \to \infty$, (33) reduces to

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma\rho_s(T)\sin m_s\pi}{\{(\Gamma+1)^2 - [\Gamma\rho_s(T)\cos m_s\pi + \rho_n(T)]^2\}^{1/2}}\right]. \tag{35}$$

Furthermore, by letting $\Gamma \to \infty$ in (35), (33) reduces to

$$P_{e_{\Gamma,\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\rho_s(T)\sin m_s\pi}{\{1-\rho_s^2(T)\cos^2 m_s\pi\}^{1/2}}\right], \tag{36}$$

which corresponds to the error probability due to random FM noise.

For the particular case of $m_s = m_i = 0.5$, which corresponds to the case of minimum-shift-keying (MSK), (33), (35), and (36) are respectively given by

$$P_e = \frac{1}{2}\left[1 - \frac{\Gamma\Lambda\rho_s(T)}{\left\{(\Gamma\Lambda+\Gamma+\Lambda)^2 - \left[\frac{\Gamma}{\pi}\rho_i(T) + \Lambda\rho_n(T)\right]^2\right\}^{1/2}}\right], \tag{37}$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma\rho_s(T)}{\{(\Gamma+1)^2 - \rho_n^2(T)\}^{1/2}}\right], \tag{38}$$

$$P_{e_{\Gamma,\Lambda\to\infty}} = \frac{1}{2}[1 - \rho_s(T)]. \tag{39}$$

Especially when $\rho_n(T) = 0$ in (38), $P_{e_{\Lambda\to\infty}}$ is given by

$$P_{e_{\Lambda\to\infty}} = \frac{\Gamma\{1-\rho_s(T)\}+1}{2(\Gamma+1)}. \tag{40}$$

This is identical to the Voelcker's result [15] which was obtained for binary phase-shift-keying (BPSK) with differential detection for the case of neglecting the cross correlation between the consecutive noise samples. Considering that differential detection for MSK signals is equivalent to that for BPSK signals when the noise correlation can be neglected, it is clear that (40) is equal to Voelcker's equation (21).

VI. NUMERICAL CALCULATION

In order to analyze the error probability numerically, let us assume that $W_s(f)$, $W_i(f)$, and $W_n(f)$ in (14) are respectively given by

$$W_s(f) = \begin{cases} \dfrac{\sigma_s^2}{\pi(f_D^2 - f^2)^{1/2}}, & |f| \leq f_D, \\ 0, & |f| > f_D, \end{cases} \tag{41}$$

$$W_i(f) = \begin{cases} \dfrac{\sigma_i^2}{\pi(f_D^2 - f^2)^{1/2}}, & |f| \leq f_D, \\ 0, & |f| > f_D, \end{cases} \tag{42}$$

$$W_n(f) = \begin{cases} \sigma_n^2/B, & |f| \leq B/2, \\ 0, & |f| > B/2, \end{cases} \tag{43}$$

where f_D is the maximum Doppler frequency and B is the bandwidth of the predetection bandpass filter. Equations (41) and (42) mean that the fading phenomena appeared on $s(t)$, and $i(t)$ are the fading phenomena which are usually encountered when the vehicle having a vertical dipole antenna is moving through the multipath propropagation field with a constant speed [16]. Furthermore, (43) means that the ideal rectangular bandpass filter having a center frequency of f_c and a bandwidth of B is used as the predetection bandpass filter. From (14), $\rho_s(T)$, $\rho_i(T)$, and $\rho_n(T)$ can then be obtained as

$$\rho_s(T) = \rho_i(T) = J_0(2\pi f_D T), \qquad \rho_n(T) = \frac{\sin(\pi BT)}{(\pi BT)}, \tag{44}$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. Since the bandwidth of the predetection bandpass filter is usually set as $BT = 1$, $\rho_n(T)$ reduces to zero. Therefore, (33), (35), and (36) are respectively given by

$$P_e = \frac{1}{2}\left[1 - \frac{\Gamma\Lambda J_0(2\pi f_D T)\sin m_s\pi}{\left\{(\Gamma\Lambda+\Gamma+\Lambda)^2 - \Gamma^2 J_0^2(2\pi f_D T)\left[\Lambda\cos m_s\pi + \frac{1}{2}\left(\cos m_i\pi + \frac{\sin m_i\pi}{m_i\pi}\right)\right]^2\right\}^{1/2}}\right], \tag{45}$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma J_0(2\pi f_D T)\sin m_s\pi}{\{(\Gamma+1)^2 - \Gamma^2 J_0^2(2\pi f_D T)\cos^2 m_s\pi\}^{1/2}}\right], \tag{46}$$

$$P_{e_{\Gamma,\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{J_0(2\pi f_D T)\sin m_s\pi}{\{1 - J_0^2(2\pi f_D T)\cos^2 m_s\pi\}^{1/2}}\right]. \tag{47}$$

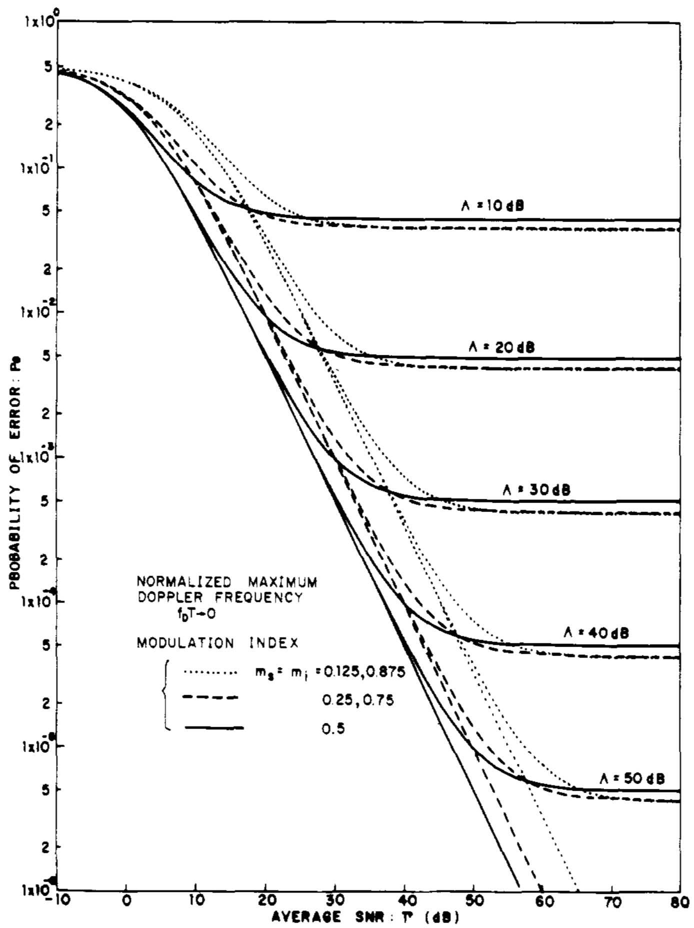

Fig. 2. Error-rate performance of digital FM with differential detection in slow Rayleigh fading environment.

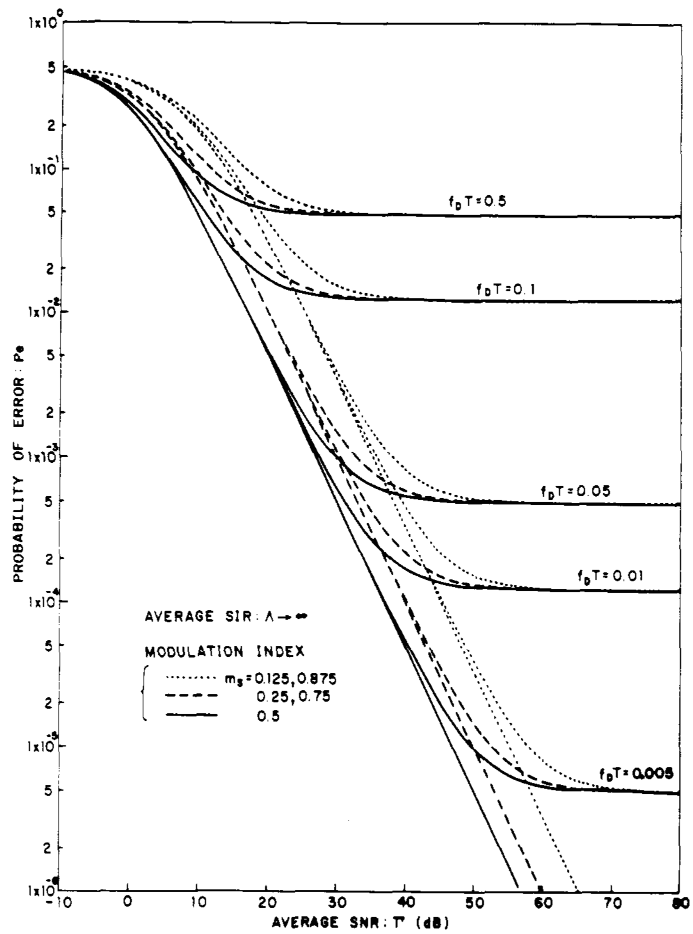

Fig. 3. Error-rate performance of digital FM with differential detection in fast Rayleigh fading environment.

In particular, by letting $m_s = m_i = 0.5$, (45)–(47) are respectively given by

$$P_e = \frac{1}{2}\left[1 - \frac{\Gamma\Lambda J_0(2\pi f_D T)}{\{(\Gamma\Lambda + \Gamma + \Lambda)^2 - (\Gamma/\pi)^2 J_0{}^2(2\pi f_D T)\}^{1/2}}\right], \tag{48}$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma J_0(2\pi f_D T)}{(\Gamma + 1)}\right], \tag{49}$$

$$P_{e_{\Gamma,\Lambda\to\infty}} = \frac{1}{2}\,[1 - J_0(2\pi f_D T)]. \tag{50}$$

A few examples of the calculated results of (45)–(47) are shown in Figs. 2–4, respectively. From Fig. 4 it is found that the error probability due to random FM noise can be minimized by setting the modulation index of $s(t)$, m_s, equal to 0.5, which corresponds to the well-known MSK. The optimum value of $m_s = 0.5$ is also correct for minimizing the error probability due to only Rayleigh envelope fading, when $f_D T = 0$ and $\Lambda \to \infty$ in (46). Considering that the signal space discrimination between mark and space can be maximized in the case of $m_s = 0.5$, the above result can be easily understood.

VII. COMPARISON

As previously mentioned, digital FM signals can be detected not only by differential detection but also by discriminator or coherent detection. It is an important problem to compare the error-rate performances of these detection schemes. While the error-rate performance of discriminator detection in the fast Rayleigh fading environment has already been obtained by the authors in the presence of cochannel interference [17], that of coherent detection has never been obtained since the tracking behavior of the carrier recovery circuit made of a phase-locked loop has not yet been perfectly analyzed in the fast Rayleigh fading environment of interest. Therefore, we concentrate on the comparison of the error-rate performances of differential and discriminator detection schemes.

When the ideal rectangular bandpass filter of $BT = 1$ is assumed to be used as the predetection filter, the error probabilities of discriminator detection corresponding to (45)–

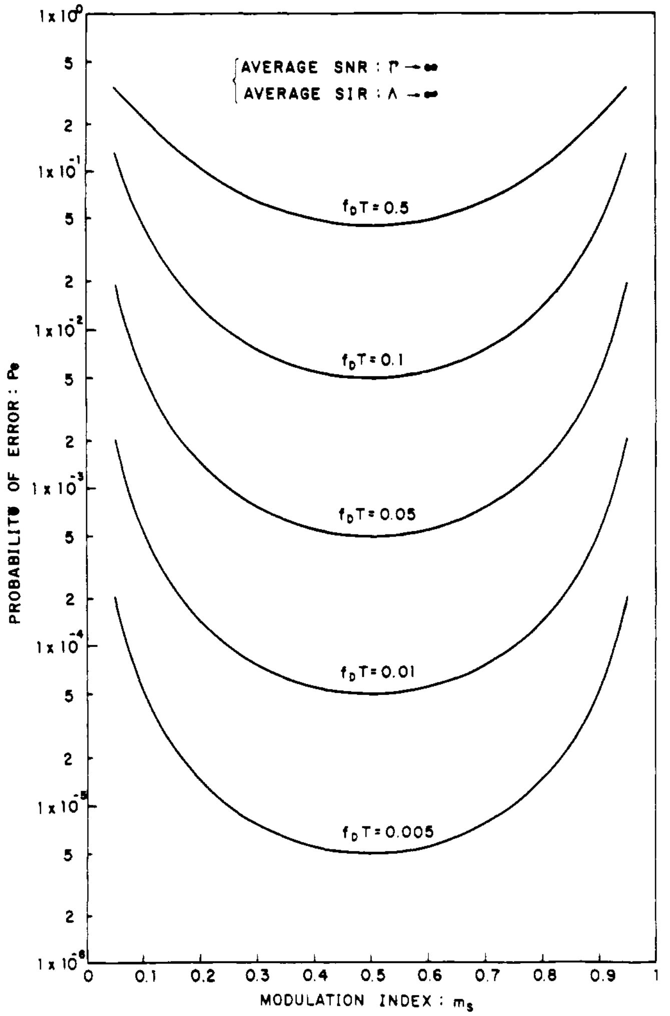

Fig. 4. Error probability of digital FM with differential detection due to random FM noise.

(47) are respectively given by [16]

$$P_e = \frac{1}{2}\left[1 - \frac{\Gamma\Lambda(m_s/\sqrt{2} f_D T)}{(\Gamma\Lambda + \Gamma + \Lambda)^{1/2}\{\Gamma\Lambda[1 + (m_s/\sqrt{2} f_D T)^2] + \Gamma[1 + (m_i/\sqrt{2} f_D T)^2] + \Lambda/(\sqrt{6} f_D T)^2\}^{1/2}}\right], \quad (51)$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma(m_s/\sqrt{2} f_D T)}{(\Gamma + 1)^{1/2}\{\Gamma[1 + (m_s/\sqrt{2} f_D T)^2] + 1/(\sqrt{6} f_D T)^2\}^{1/2}}\right], \quad (52)$$

$$P_{e_{\Gamma,\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{(m_s/\sqrt{2} f_D T)}{\{1 + (m_s/\sqrt{2} f_D T)^2\}^{1/2}}\right]. \quad (53)$$

A. Error Probability due to Rayleigh Envelope Fading

Assuming that the fading rate is much smaller than the signaling rate, i.e., $f_D T \to 0$ in (46) and (52), $P_{e_{\Lambda\to\infty}}$ of the two detection schemes are respectively given by

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma \sin m_s\pi}{\{(\Gamma + 1)^2 - \Gamma^2 \cos^2 m_s\pi\}^{1/2}}\right] \cong \frac{1}{2\Gamma \sin^2 m_s\pi} \quad \text{for differential detection,} \quad (54)$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\sqrt{3}\,\Gamma m_s}{(\Gamma + 1)^{1/2}(1 + 3\Gamma m_s^2)^{1/2}}\right] \cong \frac{1}{4\Gamma}\left(1 + \frac{1}{3m_s^2}\right) \quad \text{for discriminator detection.} \quad (55)$$

These are the respective error probabilities of the two detection schemes when the cochannel interference is not present in the quasi-stationary slow Rayleigh fading environment. Equations (54) and (55) suggest that the error-rate performance of differential detection is slightly superior to that of discriminator detection unless $m_s \gtrsim 0.558$.

Fortunately, in the sepcial case of $m_s = 0.5$, i.e., MSK, the error probability of coherent detection in the quasi-stationary slow Rayleigh fading environment, i.e., $f_D T \to 0$, can be easily obtained as

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma^{1/2}}{(\Gamma + 1)^{1/2}}\right] \cong \frac{1}{4\Gamma} \quad \text{for coherent detection,} \quad (56)$$

since the detection mechanism is equivalent to that of the well-known binary phase-shift-keying with coherent detection [9]. By letting $m_s = 0.5$ in (54) and (55), the error probabilities of the other two detection schemes in this special case are respectively given by

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2(\Gamma + 1)} \cong \frac{1}{2\Gamma} \quad \text{for differential detection,} \quad (57)$$

$$P_{e_{\Lambda\to\infty}} = \frac{1}{2}\left[1 - \frac{\Gamma}{(\Gamma + 1)^{1/2}(\Gamma + 4/3)^{1/2}}\right] \cong \frac{7}{12\Gamma} \quad \text{for discriminator detection.} \quad (58)$$

Comparing (56)-(58) it is found that coherent detection has the best performance for MSK transmission in the quasi-stationary Rayleigh fading environment. The required values of average SNR for obtaining the error probability of 1 X

10^{-3} are 24.0 dB, 27.0 dB, and 27.7 dB for coherent, differential, and discriminator detection schemes, respectively.

B. Error Probability due to Random FM Noise

From (47) and (53) the error probabilities due to random FM noise caused by the random phase change in the fast Rayleigh fading environment are respectively given by

$$P_{e_{\Gamma,\Lambda\to\infty}} \cong \frac{1}{2}\left(\frac{\pi f_D T}{\sin m_s \pi}\right)^2 \quad \text{for differential detection,} \tag{59}$$

$$P_{e_{\Gamma,\Lambda\to\infty}} \cong \frac{1}{2}\left(\frac{f_D T}{m_s}\right)^2 \quad \text{for discriminator detection,} \tag{60}$$

where the approximation formula

$$J_0(z) \cong 1 - (z/2)^2 \tag{61}$$

was used for the derivation of (59). From (59) and (60) it is found that the error-rate performance of differential detection is inferior to that of discriminator detection irrespective of the values of the modulatin index. In the special case of $m_s = 0.5$, i.e., MSK, the error probability of differential detection is nearly equal to 2.5 times that of discriminator detection. This curious result can be understood from the fact that the discriminator, whose detection mechanism is considered to be equivalent to that of a differential detector using the delay line with the infinitesmal time delay, is able to track the fast phase change more quickly than the differential detector in the high SNR and SIR conditions. In order to clarify the error-rate performance of coherent detection in such a condition, it is necessary to solve the tracking behavior of the carrier recovery circuit. While this is one of the most important problems which should be solved in the future, a qualitative comparison can be made. Since the reference carrier is usually recovered from the received digital FM signal itself by the use of a narrowband phase-locked loop with decision-directed channel measurement, its tracking behavior against the fast random phase change, i.e., random FM noise, is considered to be inferior to that of the reference carrier recovered by the delay line in a difference detector. Thus, the error-rate performance of coherent detection in such conditions may be inferior to that of differential detection. Therefore, it is concluded that the coherent detection has the worst performance compared with the other detection schemes.

C. Error Probability due to Cochannel Interference

By letting $\Gamma \to \infty$, $f_D T \to 0$, and $m_s = m_i = m$ in (45) and (51), the error probabilities, when the cochannel interference predominates in causing errors, are respectively given by

$$P_{e_{\Gamma\to\infty}} = \frac{1}{2}\left[1 - \frac{\Lambda \sin m\pi}{\left\{(\Lambda+1)^2 - \left[\Lambda \cos m\pi + \frac{1}{2}\left(\cos m\pi + \frac{\sin m\pi}{m\pi}\right)\right]^2\right\}^{1/2}}\right] \quad \text{for differential detection,} \tag{62}$$

$$P_{e_{\Gamma\to\infty}} = \frac{1}{2(\Lambda+1)} \quad \text{for discriminator detection.} \tag{63}$$

Therefore, it is concluded that the error-rate performance of differential detection is slightly superior to that of discriminator detection unless $m \gtrsim 0.5$.

VIII. CONCLUSION

The error-rate performance of digital FM with differential detection in the presence of not only thermal Gaussian noise but also cochannel interference has been theoretically analyzed in the fast Rayleigh fading environment encountered in the typical UHF or microwave land mobile radio channels. While the effect of intersymbol interference caused by band restrictions has been neglected, the fading spectrum effect has been taken into consideration. The error probability is presented by a simple closed form which is useful for the system designer of digital FM land mobile radio communication systems. Though the cochannel interference has been assumed to be a single digital FM signal, the above analysis may be easily extended to the case when the multiple cochannel interferences are presented. Furthermore, it may also be extended to the case when the cochannel interference is an analog FM signal.

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