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著者	安達　文幸
journal or publication title	IEEE Transactions on Vehicular Technology
volume	41
number	2
page range	199-210
year	1992
URL	http://hdl.handle.net/10097/46485

doi: 10.1109/25.142778

Postdetection Selection Diversity Reception with Correlated, Unequal Average Power Rayleigh Fading Signals for $\pi/4$-Shift QDPSK Mobile Radio

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Abstract—**The diversity gain degradations due to fading correlation and unequal average power are investigated for practical, two-branch postdetection selection diversity reception. The average bit error rate (BER) of $\pi/4$-shift QDPSK is theoretically analyzed taking into account additive white Gaussian noise (AWGN), cochannel interference, and multipath channel delay spread; exact diversity gain degradations are calculated. This paper also presents very simple and useful approximate expressions for the gain degradations. When the fading correlation and the power ratio between the two antennas are given by $h_{12}(|h_{12}| \leqq 1)$ and $q(\leqq 1)$, respectively, the total gain degradation is $D = 5\log\{q(1-|h_{12}|^2)\}$ dB if the errors are caused by AWGN, while it is $5\log\{2q(1-|h_{12}|^2)/(1+q^2)\}$ dB if cochannel interference and delay spread are the cause of errors.**

I. Introduction

LINEAR modulation for mobile radio systems [1]–[4] has recently been attracting much attention. The North American and Japanese digital cellular systems currently under development employ $\pi/4$-shift QDPSK with 24.3 k symbol/s, and 21 k symbol/s respectively [5], [6]. Mobile channels are usually composed of many propagation paths with different time delays and produce fast multipath Rayleigh fading, which severely degrades the transmission performance [7]. In cellular mobile radio, causes of errors are additive white Gaussian noise (AWGN), the cochannel interference produced by the same frequency reused in the different cells, random FM noise produced by random phase fluctuations of the fading signals, and multipath channel time delay spread [7]. Diversity reception is one of the most powerful techniques to combat fading [4], [7]–[11]. There are many possible implementations of diversity reception systems, but for mobile radio, postdetection diversity is attractive because the cophasing function necessary in predetection combiners is not required and because switching noise due to abrupt phase change is not produced. Previous analyses [8]–[11] of postdetection diversity reception assumed that the received signal on each antenna is subject to uncorrelated, equal average power Rayleigh fading. However, in practical situations, this is seldom true. The space available for diversity antennas is limited so that the fading signals are correlated [15]. Furthermore, it was reported [12] that a built-in diversity antenna scheme suitable for hand-held portable transceiver produces an antenna gain difference of about 1 dB and a fading correlation of close to 0.5. Both fading correlation and unequal average signal power between antennas degrade the benefits of diversity reception. Therefore, it is very important to investigate these effects before applying diversity reception to practical mobile radio systems.

This paper considers the most practical, two-branch postdetection selection diversity scheme for $\pi/4$-shift QDPSK signal transmission. We describe the transmission system model in Section II. Section III analyzes the influence of fading correlation between two antennas on average bit error rate (BER) performance taking into account AWGN, cochannel interference, and multipath channel delay spread.[1] The influence of unequal average signal power is analyzed in Section IV. Section V derives a very simple and useful approximate expression for diversity gain degradation and compares it with exact results obtained in Sections III and IV. The gain degradation for predetection diversity reception is also discussed assuming that AWGN is the only cause of errors. Finally, experimental laboratory results are presented in Section VI.

II. Transmission System Model

The transmission system with diversity reception is shown in Fig. 1. In $\pi/4$-shift QDPSK signal transmission, the input binary data sequence is grouped into the symbol sequence $(a_n, b_n), n$ is the integer, and mapped to the nth differential phase $\Delta\phi_n = \phi_n - \phi_{n-1}$ of the carrier, where ϕ_n is called the nth phase symbol. $\Delta\phi_n$ considered in this paper is represented as

$$\Delta\phi_n = \phi_n - \phi_{n-1} = \begin{cases} 3\pi/4, & \text{for } (a_n, b_n) = (-1,1) \\ \pi/4, & (1,1) \\ -\pi/4, & (1,-1) \\ -3\pi/4, & (-1,-1). \end{cases} \quad (1)$$

[1] For a vehicle speed of 100 km/h and a carrier frequency of 900 MHz, the average BER due to random FM noise without diversity is below 10^{-4} for a 25 k symbol/s transmission [8]. Therefore, the effect of random FM noise can be neglected.

Manuscript received July 9, 1991; revised Sept. 17, 1991.

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IEEE Log Number 9106293.

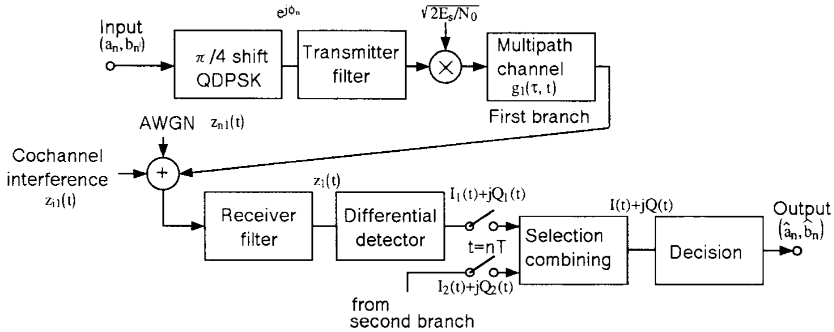

Fig. 1. Transmission system model employing postdetection two-branch selection diversity.

The modulated signal is spectrum-shaped by the transmitter filter. The transmitted signal is received over a multipath Rayleigh fading channel. AWGN is added to the received signal and they are bandlimited by the receiver filter for differential detection. We assume square-root raised cosine (RC) filters with roll-off factor $a(0 \leqq a \leqq 1)$ as the transmitter and receiver filters. The differential detector input $z_m(t)$ of the mth branch $(m = 1, 2)$ can be represented in the complex form as

$$z_m(t) = \sqrt{\frac{2E_{sm}}{T}} \int_{-\infty}^{\infty} d_R(t-\tau) g_m(\tau, t) d\tau + z_{im}(t) + z_{nm}(t) \tag{2}$$

where

$$d_R(t) = \sum_{n=-\infty}^{\infty} e^{j\phi_n} \frac{\sin(\pi(t - nT)/T)}{n(t - nT)/T} \frac{\cos(\alpha\pi(t - nT)/T)}{1 - (2\alpha(t - nT)/T)^2} \tag{3}$$

is the overall (transmitter and receiver) filter response, E_{sm} is the average received signal energy per symbol, $g_m(\tau, t)$ is the multipath channel complex impulse response for the mth antenna measured from the instant of application of a unit impulse at the transmitter at $t, z_{im}(t)$ is the cochannel interference which can be represented in a form similar to that of the desired signal, and $z_{nm}(t)$ is the bandlimited AWGN. We assume that the multipath channel impulse response at τ is due to the sum of many independent impulses with the same time delay τ, so $g_m(\tau, t)$ becomes a zero-mean complex Gaussian process of time t, and

$$\langle g_m(\tau, t) g_m^*(\tau - \lambda, t)\rangle = \xi(\tau) \cdot \delta(\lambda) \tag{4}$$

where $\xi(\tau) = \langle |g_m(\tau, t)|^2 \rangle$ is the delay power profile and $\int_{-\infty}^{\infty} \xi(\tau)\, d\tau = 1$. Two branches have identical delay power profiles.

In the differential detector (DD), the input signal is amplitude-limited, and is multiplied by its one-symbol delayed version. The mth branch DD output at the sampling instant nT can be represented as $I_m(nT) + jQ_m(nT) = z_m(nT) z_m{}^*((n-1)T)$ in the complex form, where $I_m(nT)$ and $Q_m(nT)$ are the in-phase and quadrature outputs, respectively [8]. The postdetection selection diversity combiner selects the detector output of the branch with the larger input signal envelope. The combiner output is denoted by $I(nT) + jQ(nT) = I_1(nT) + jQ_1(nT)$ if $|z_1(nT)| \geqq |z_2(nT)|$, and $I_2(nT) + jQ_2(nT)$ otherwise. A decision on a_n and b_n of the nth symbol can be based on the polarity of $I(nT)$ and $Q(nT)$, respectively.

III. Influence of Fading Correlation

A. Average BER

We assume very slow, correlated, equal average power Rayleigh fading signals $(E_{s1} = E_{s2} = E_s)$. Since the average powers of the two fading signals are identical, the average BER P_e can be obtained by averaging the conditional BER $p_e(R)$ with the probability density function (pdf) $p_R(R)$ of $R = \max(R_1, R_2)$, where $R_m = |z_m(nT)|$:

$$P_e = \int_0^{\infty} p_e(R) p_R(R)\, dR. \tag{5}$$

$$\rho = \left(\Gamma \int \xi(\tau) d_R(-\tau) d_R^*(-T-\tau)\, d\tau + \Gamma/\Lambda \int \xi(\tau) d_i(\Delta T - \tau) d_i^*(\Delta T - T - \tau)\, d\tau\right) \Bigg/ \left(\sqrt{\Gamma \int \xi(\tau) |d_R(-\tau)|^2 d\tau + \Gamma/\Lambda \int \xi(\tau) |d_i(\Delta T - \tau)|^2 d\tau + 1} \cdot \sqrt{\Gamma \int \xi(\tau) |d_R(-T-\tau)|^2 d\tau + \Gamma/\Lambda \int \xi(\tau) |d_i(\Delta T - T - \tau)|^2 d\tau + 1}\right) \tag{7}$$

$$P_e = \frac{1}{2}\left[1 - 2\frac{a_0 p_c}{\sqrt{1-\rho_s^2}} + \frac{a_0\rho_c}{\sqrt{1-|\rho|^2}}\sqrt{\frac{(1-|\eta_{12}|^2)(1-|\rho|^2)}{(1-|\eta_{12}|^2)\rho_c^2 + 2\cdot(1-|\rho|^2)}} \cdot \sum_{m=0}^{\infty} |\eta_{12}|^{2m} \sum_{r=0}^{m} \varepsilon_r \frac{(4m-2r-1)!!}{(2m-2r)!!(2m)!!}\left\{\frac{(1-|\rho|^2)}{(1-|\eta_{12}|^2)\rho_c^2 + 2(1-|\rho|^2)}\right\}^{2m-r}\right]. \quad (12)$$

Let h_{12} be the complex fading correlation between $g_1(\tau, t)$ and $g_2(\tau, t)$.[2] If $|h_{12}|$ is not too close to unity, we can use the conditional BER $p_e(R)$ derived for the uncorrelated fading case (see Appendix I). We assume that the zeroth symbol (a_0, b_0) is detected. Since the receiver filter frequency response is symmetrical, the error rates for a_0 and b_0 are identical. The conditional BER is given by [8], [13]:

$$\begin{aligned} p_e(R) &= \frac{1}{2}\text{erfc}\left(\frac{a_0\rho_c}{\sqrt{1-|\rho|^2}}\frac{R}{\sqrt{2}\sigma}\right) \\ &= \frac{a_0\rho_c}{\sqrt{1-|\rho|^2}}\frac{1}{\sqrt{2\pi}\sigma}\int_R^\infty e^{-\frac{\rho_c^2}{1-|\rho|^2}\frac{x^2}{2\sigma^2}}\,dx \end{aligned} \quad (6)$$

where σ^2 is the average power of $z_m(nT), \rho = \rho_c + j\rho_s = 1/2 < z_m(0)\cdot z_m{}^*(-T)\rangle/\sigma^2$, and ρ is (see (7) on the previous page) where Γ is the average energy per symbol-to-noise power spectrum density ratio (E_s/N_0) of the desired signal, and Λ is the average desired signal-to-interference power ratio (SIR). The average energy per bit-to-noise power spectrum density ratio (E_b/N_0) is given by $0.5 \times$ average E_s/N_0. The overall filter response to the cochannel interference is $d_i(t)$, which is also given by (3). We assume that decision timing is jitter-free and synchronous to the desired modulation timing, and that cochannel interference modulation timing is independent from that of the desired signal. ΔT is the timing offset for the interference modulation relative to the desired signal, and is assumed to be distributed uniformly within $|\Delta T| \leqq T/2$.

Substituting (6) into (5), the average BER can be calculated from

$$P_e = \int_0^\infty \frac{a_0\rho_c}{\sqrt{1-|\rho|^2}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\rho_c^2}{1-|\rho|^2}\frac{R^2}{2\sigma^2}}\cdot P_R(R)dR \quad (8)$$

where $P_R(R)$ is the cumulative distribution function (cdf) of R which can be given by (see Appendix II)

$$P_R(R) = 1 - e^{-\frac{R^2}{2\sigma^2}}\left[2 - e^{-\frac{1+|\eta_{12}|^2}{1-|\eta_{12}|^2}\frac{R^2}{2\sigma^2}} \sum_{r=0}^{\infty}\varepsilon_r|\eta_{12}(0)|^r I_r\left(\frac{|\eta_{12}|R^2}{\sigma^2(1-|\eta_{12}|^2)}\right)\right] \quad (9)$$

[2]We assume that $g_m(\tau, t)$ at any time delay τ is the sum of many independent impulses arriving from all direction uniformly with equal amplitude, thus, each impulse equally contributes to form both $g_1(\tau, t)$ and $g_2(\tau, t)$. Hence, the correlation between $g_1(\tau, t)$ and $g_2(\tau, t)$ at any time delay τ can be assumed identical.

where $\varepsilon_r = 1$ for $r = 0$, and 2 elsewhere. η_{12} is the complex correlation between $z_1(0)$ and $z_2(0)$ and

$$\eta_{12} = \frac{\Gamma + \Gamma/\Lambda}{\Gamma + \Gamma/\Lambda + 1}h_{12} \quad (10)$$

where h_{12} is already defined the complex fading correlation between two antennas. Using

$$\begin{aligned} I_r(x) &= \left(\frac{x}{2}\right)^r \sum_{n=0}^{\infty}\frac{(x/2)^{2n}}{n!(n+r)!} \\ \int_0^\infty x^{2n}e^{-ax^2}dx &= \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{2n+1}}} \end{aligned} \quad (11)$$

the average BER becomes as shown in (12). For the uncorrelated fading case, $\eta_{12} = 0$, and (12) becomes

$$P_e = \frac{1}{2}\left[1 - 2\frac{a_0\rho_c}{\sqrt{1-\rho_s^2}} + \frac{a_0\rho_c}{\sqrt{\rho_c^2 + 2(1-|\rho|^2)}}\right] \quad (13)$$

which is identical to the result obtained in [8]. The average BER without diversity is given by

$$P_e = \frac{1}{2}\left[1 - \frac{a_0\rho_c}{\sqrt{1-\rho_s^2}}\right]. \quad (14)$$

B. Numerical Results

The average BER can be approximately represented as the sum of BER's due to AWGN, cochannel interference, and delay spread. This section calculates the individual average BER's. When the delay spread is very small compared to the symbol duration, all the multipath waves can be assumed to arrive at the receiver at the same time; hence, the delay power profile $\xi(\tau) = \delta(\tau)$. In this case, the fading is called multiplicative and the AWGN and cochannel interference are the major cause of errors. For the square-root RC filter transmission scheme, there is no ISI at the sampling instant on the desired signal and

$$d_R(0)d_R^*(-T) = \frac{a_0 + jb_0}{\sqrt{2}}. \quad (15)$$

ρ becomes

$$\begin{aligned} \rho &= \frac{\Gamma(a_0+jb_0)/\sqrt{2} + \Gamma/\Lambda\{d_i(\Delta T)d_i^*(\Delta T - T)\}}{\sqrt{\Gamma + \Gamma/\Lambda|d_i(\Delta T)|^2 + 1}\sqrt{\Gamma + \Gamma/\Lambda|d_i(\Delta T - T)|^2 + 1}} \\ &= \begin{cases} \dfrac{\Gamma}{\Gamma+1}\dfrac{a_0+jb_0}{\sqrt{2}}, & \Lambda \to \infty \\ \dfrac{\Lambda\frac{a_0+jb_0}{\sqrt{2}} + d_i(\Delta T)d_i^*(\Delta T - T)}{\sqrt{\Lambda + |d_i(\Delta T)|^2}\sqrt{\Lambda + |d_i(\Delta T - T)|^2}}, & \Gamma \to \infty. \end{cases} \end{aligned} \quad (16)$$

The calculation of average BER due to AWGN is very simple because there is no ISI. To calculate the average BER due to cochannel interference, however, the values of $d_i(\Delta T)$ and $d_i(\Delta T - T)$ have to be determined from (3) taking into account ISI from several adjacent phase symbols $\phi_{i,n}$, which are represented as ϕ_n in (1). In this paper, we consider ISI from two phase symbols on each side, i.e., $(\phi_{i,-2}, \phi_{i,-1}, \phi_{i,0}, \phi_{i,1}, \phi_{i,2})$ for $d_i(\Delta T)$ and $(\phi_{i,-3}, \phi_{i,-2}, \phi_{i,-1}, \phi_{i,0}, \phi_{i,1})$ for $d_i(\Delta T - T)$. Hence, the six-phase symbol pattern $(\phi_{i,-3}, \phi_{i,-2}, \phi_{i,-1}, \phi_{i,0}, \phi_{i,1}, \phi_{i,2})$ is used for BER calculation. The values of ρ are then derived from (16) and substituted into (12) for evaluating the average BER with the given six-phase symbol pattern and timing offset ΔT. The overall average BER can be obtained by averaging the conditional average BER over all six-phase symbol patterns and timing offsets. The transmitted symbol (a_0, b_0) of the desired channel is assumed to be $(a_0, b_0) = (1, 1)$ without loss of generality.

When delay spread cannot be neglected, the fading is called frequency selective and errors are produced by ISI due to delay spread. The average BER due to delay spread can be calculated using

$$\rho = \frac{\int_{-\infty}^{\infty} \xi(\tau) d_R(-\tau) d_R^*(-T-\tau) d\tau}{\sqrt{\int_{-\infty}^{\infty} \xi(\tau)|d_R(-\tau)|^2 d\tau \cdot \int_{-\infty}^{\infty} \xi(\tau)|d_R(-T-\tau)|^2 d\tau}}. \tag{17}$$

Two adjacent phase symbols on each side are considered to calculate $d_R(-\tau)$ and $d_R(-T-\tau)$; the six-phase symbol pattern $(\phi_{-3}, \phi_{-2}, \phi_{-1}, \phi_0, \phi_1, \phi_2)$ is used for BER calculation as in the case of BER calculation for cochannel interference (note that $\Delta\phi_n = \phi_n - \phi_{n-1}$ corresponds to the nth symbol (a_n, b_n)). The average BER with a given six-phase symbol pattern is evaluated using (12) and is averaged over all patterns to obtain the overall average BER. Since the shape of delay profile does not have a profound impact on the average BER [8], a double-spike delay profile is assumed in this paper.

Fig. 2 shows the calculated average BER performance with the squared fading correlation $|h_{12}|^2$ as a parameter.[3] (Dotted lines in Fig. 2 (a) show the results for predetection selection diversity reception. Comparison between predetection and postdetection diversity reception is presented in Section V.) Diversity gain is defined as the reduced quantity of average E_b/N_0 or average SIR in decibels necessary to achieve a certain average BER. With frequency selective fading, it is defined as the reduced quantity of 20 log $(\tau_{\rm rms})$ that achieves a certain average BER, where $\tau_{\rm rms}$ is the rms delay spread. It can be seen that if $|h_{12}|^2$ is less than 0.4, the fading correlation does not degrade the BER performance much; gain degradation at average BER = 10^{-4} is only 1 dB when $|h_{12}|^2 = 0.4$. Even when $|h_{12}|^2 = 0.8$, the degradation is only 3.5 dB.

When multipath waves arrive at the mobile station from all directions uniformly with equal amplitude and the two antennas have an omnidirectional radiation pattern, the squared fading corr elation is $|h_{12}|^2 = J_0^2(2\pi d/\lambda)$ where d is the antenna separation and λ is the carrier wavelength. When $d = 0.2\lambda$, the value of $|h_{12}|^2$ is 0.4. When the radiation patterns are different for the two antennas, then the fading correlation becomes smaller than this value. This suggests that an antenna separation as small as 0.2 λ can be used without significantly increasing the degradation. Note that, however, a much larger antenna separation is required at the base station.

[3] In Rayleigh fading environments, $|h_{12}|^2$ is the power correlation and also an approximate value of the envelope correlation [14].

IV. Influence of Unequal Average Power

A. Average BER

We have, so far, assumed that the received signal powers of the two branches are identical. In the following, we assume that the two fading signals are uncorrelated, and the average signal power of the second branch is $q(< 1)$ times that of the first branch. Furthermore, we assume that the average SIR is the same as both branches (since we are assuming that all multipath waves arrive from all directions with equal amplitude, the average SIR is not affected by the difference in antenna radiation patterns). Let σ_m^2 and ρ_m be the average power of the mth branch detector input and the correlation between $z_m(0)$ and $z_m(-T)$, respectively. ρ_m is the value of ρ given by (7), where Γ of the second branch should be replaced by $q\Gamma$. The conditional BER is $P_e = \frac{1}{2}$ erfc $[(a_0\rho_{c1}/\sqrt{1-|\rho_1|^2})R_1/(\sqrt{2}\sigma_1)]$ when $R_1 \geq R_2$ and $p_e = \frac{1}{2}$ erfc $[(a_0\rho_{c2}/\sqrt{1-|\rho_2|^2})R_2/(\sqrt{2}\sigma_2)]$ otherwise. Therefore, the average BER can be given by

$$\begin{aligned} P_e = &\frac{1}{2}\left[1 - \frac{a_0\rho_{c1}}{\sqrt{1-\rho_{s1}^2}} - \frac{a_0\rho_{c2}}{\sqrt{1-\rho_{s2}^2}}\right. \\ &+ \frac{1}{1+Q^{-1}} \frac{a_0\rho_{c1}}{\sqrt{\rho_{c1}^2 + (1+Q^{-1})(1-|\rho_1|^2)}} \\ &\left. + \frac{1}{1+Q} \frac{a_0\rho_{c2}}{\sqrt{\rho_{c2}^2 + (1+Q)(1-|\rho_2|^2)}}\right] \end{aligned} \tag{18}$$

where

$$Q = \frac{\sigma_2^2}{\sigma_1^2} = \frac{q(\Gamma + \Gamma/\Lambda) + 1}{\Gamma + \Gamma/\Lambda + 1}. \tag{19}$$

B. Numerical Results

Calculated BER performance is shown in Fig. 3 with q as a parameter. (In Fig. 3(a), the results for predetection selection diversity are shown as dotted lines. Section V compares the effects with predetection and postdetection diversity.) When $q = -6(-10)$ dB, gain degradation at an average BER of 10^{-4} is 3(5) dB if AWGN is the cause of errors and 1.6(3.5) dB if cochannel interference or delay spread is the cause of errors. The value of gain degradation due to unequal average power obviously depends on the cause of errors. The result is in contrast to that of Section III, which showed that the diversity gain degradations due to fading correlation are the same irrespective of the cause of errors. The reason is that the average E_b/N_0 at the second branch reduces as the power ratio q decreases (we assume that the average power at the first branch is kept constant), while the average SIR's and rms delay spreads at both antennas remain the same; therefore, diversity gain degrades more rapidly when the cause of errors is AWGN.

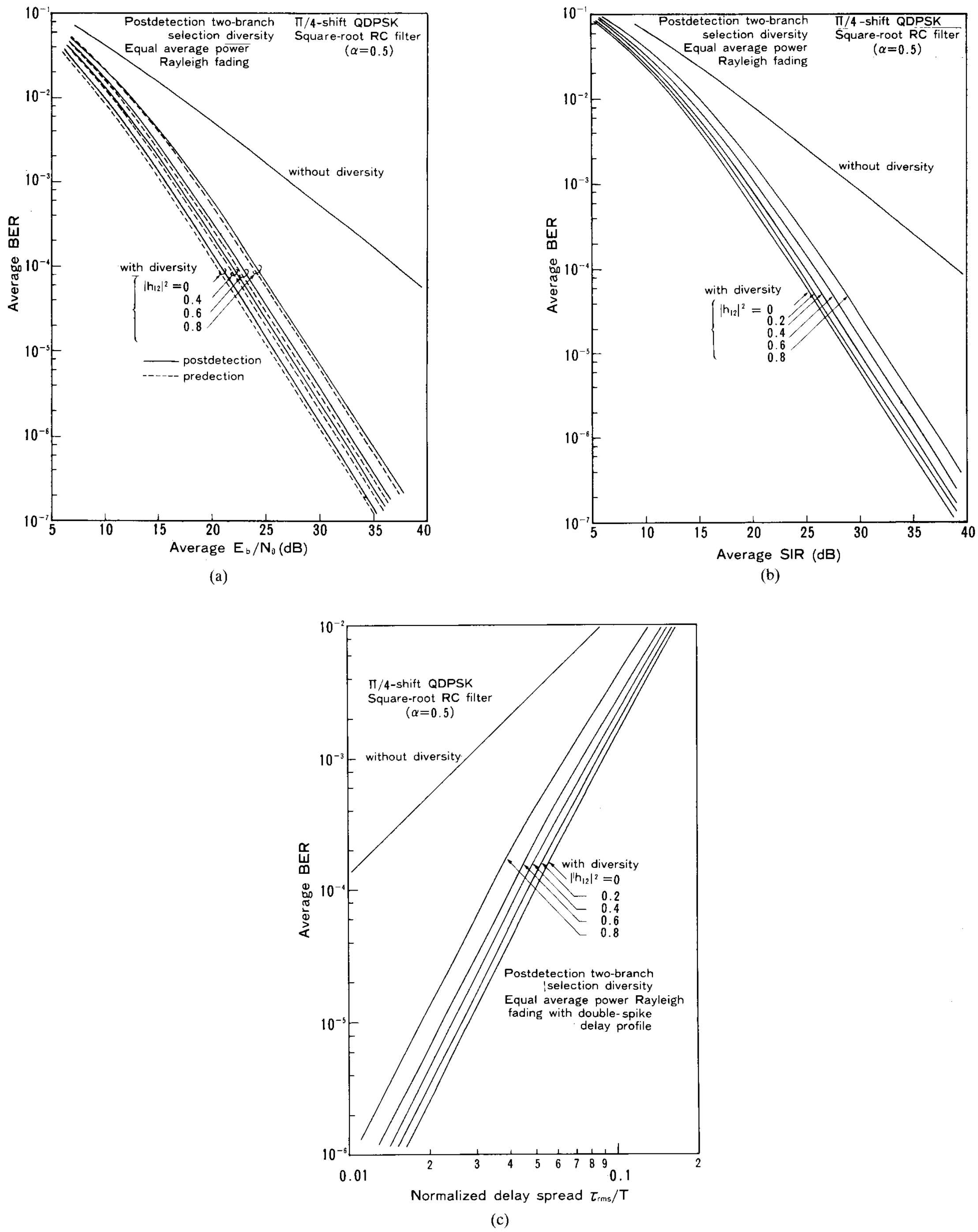

Fig. 2. Influence of fading correlation. Equal average power Rayleigh fading case. Average BER's due to (a) AWGN, (b) cochannel interference, and (c) delay spread.

V. Discussion

In this section, we derive approximate expressions for the diversity gain degradations to see more clearly how fading correlation and unequal average power degrade diversity gain. Also discussed is the gain degradation of predetection diversity reception assuming that AWGN is the only cause of errors.

A. Approximate Expression for Diversity Gain Degradations

Since the conditional BER of (6) decreases rapidly as R increases, the average BER can be calculated using the approximate joint pdf of R_1 and R_2 for small values of R_1 and R_2. Since the joint pdf can be approximated as

$$p(R_1, R_2) \approx \frac{R_1 R_2}{\sigma_1^2 \sigma_2^2 (1 - |\eta_{12}|^2)} \tag{20}$$

the average BER becomes

$$\begin{aligned} P_e &= \int_0^\infty \int_0^{R_1} \frac{1}{2} \text{erfc}\left(\frac{a_0 \rho_{c1}}{\sqrt{1 - |\rho_1|^2}} \frac{R_1}{\sqrt{2}\sigma_1} \right) \\ &\quad \cdot p(R_1, R_2) dR_1 \, R_2 \\ &\quad + \int_0^\infty \int_0^{R_2} \frac{1}{2} \text{erfc}\left(\frac{a_0 \rho_{c2}}{\sqrt{1 - |\rho_2|^2}} \frac{R_2}{\sqrt{2}\sigma_2} \right) \\ &\quad \cdot p(R_1, R_2) dR_2 \, dR_1 \\ &\approx \frac{3}{16(1 - |\eta_{12}|^2)} \left[Q^{-1} \cdot \left(\frac{1 - |\rho_1|^2}{\rho_{c1}^2} \right)^2 \right. \\ &\quad \left. + Q \cdot \left(\frac{1 - |\rho_2|^2}{\rho_{c2}^2} \right)^2 \right]. \end{aligned} \tag{21}$$

When AWGN is the only cause of errors, (21) becomes $P_e \approx 6\{\Gamma\sqrt{q(1 - |h_{12}|^2)}\}^{-2}$ since $Q \approx q$, and $\eta_{12} \approx h_{12}$ for large Γ. Therefore, the approximate degradation in diversity gain is given by $\Delta = 5 \log\{q(1 - |h_{12}|^2)\}$ dB. When the cause of errors is cochannel interference (or delay spread), ρ_2 is not a function of q since average SIR (or delay power profile) is identical for both branches. Therefore, the average BER is proportional to $(1 + q^2)/q$. It has been shown [8] that for two-branch selection diversity with uncorrelated, equal average power fading, the average BER's due to cochannel interference and delay spread are proportional to Λ^{-2} and $(\tau_{\text{rms}}/T)^2$, respectively. Thus, the gain degradation becomes $\Delta = 5 \log\{2q(1 - |h_{12}|^2)/(1 + q^2)\}$ dB.

From the above results, diversity gain degradation in decibels is the sum of those due to fading correlation and unequal average power. The fading correlation reduces the diversity gain by $5 \log(1 - |h_{12}|^2)$ dB irrespective of the cause of errors. However, the diversity gain degradation due to unequal average power depends on the cause of errors as was seen in Section IV. If AWGN is the cause of errors, then the degradation is $5 \log q$ dB, or $5 \log\{2q/(1 + q^2)\}$ dB if the cause of errors is cochannel interference or delay spread. As a consequence, we have the following result:

$$\begin{aligned} &\Delta\text{due to fading correlation } = 5 \log(1 - |h_{12}|^2) \text{dB} \\ &\Delta\text{due to unequal power } = 5 \log q \, \text{dB (AWGN)} \\ &\qquad 5 \log \left\{ \frac{2q}{1 + q^2} \right\} \text{dB} \\ &\text{(cochannel interference and delay spread).} \end{aligned} \tag{22}$$

We obtained the exact gain degradations from Figs. 2 and 3 and compared them with approximate values calculated from (22). It was found that at average BER's $< 10^{-4}$, the exact degradations agree quite well with the approximate ones (for larger average BER's, the exact gain degradations are slightly smaller than the approximate values).

B. Comparison with Predetection Diversity Reception

The cdf of the differential phase noise $\Delta\Theta$ due to AWGN is given by [16]

$$\begin{aligned} &P[-\pi < \Delta\Theta \leqq \Psi] \\ &\quad = \frac{-\sin \Psi}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\exp[-\gamma(1 - \cos \Psi \cos t)]}{1 - \cos \Psi \cos t} dt \end{aligned} \tag{23}$$

where γ is the instantaneous E_s/N_0 and $-\pi < \Psi \leqq 0$. One-bit error is produced if $\Delta\Theta$ falls between $\pi/4 < |\Delta\Theta| \leqq 3\pi/4$ and two-bit error if $3\pi/4 < |\Delta\theta| \leqq \pi$. Since the pdf of $\Delta\Theta$ is symmetrical with respect to $\Delta\Theta = 0$, the conditional BER can be given by

$$p_e(\gamma) = P[-\pi < \Delta\Theta \leqq -\pi/4] + P[-\pi < \Delta\Theta \leqq -3\pi/4]. \tag{24}$$

For the evaluation of the second term of the right-hand side in (24), we substitute $\Psi = -3\pi/4$ and replace the integral variable t with $t + \pi$ in (23). We have

$$\begin{aligned} p_e(\gamma) &= \frac{1}{4\pi\sqrt{2}} \int_0^{2\pi} \frac{\exp[-\gamma(1 - \frac{\cos t}{\sqrt{2}})]}{1 - \frac{\cos t}{\sqrt{2}}} dt \\ &= \frac{1}{4\pi\sqrt{2}} \int_0^{2\pi} \int_\gamma^\infty \exp\left[-x \left(1 - \frac{\cos t}{\sqrt{2}} \right) \right] dt \, dx. \end{aligned} \tag{25}$$

Integration with respect to t gives the following expression:

$$\begin{aligned} p_e(\gamma) &= \frac{1}{2\sqrt{2}} \int_\gamma^\infty e^{-x} I_0\left(\frac{x}{\sqrt{2}} \right) dx \\ &= \frac{1}{2} \left[1 - \frac{1}{\sqrt{2}} I_e\left(\frac{1}{\sqrt{2}}, \gamma \right) \right] \end{aligned} \tag{26}$$

where $I_e(a, b)$ is Rice's I_e-function [17]. For small values of γ, the pdf of γ can be approximated as [see Appendix III]

$$p_\gamma(\gamma) \approx \frac{2}{1 + |h_{12}|^2} \frac{1}{\Gamma} \left[e^{\frac{-\gamma}{\Gamma}} - e^{-\frac{2}{1 - |h_{12}|^2} \frac{\gamma}{\Gamma}} \right]. \tag{27}$$

Averaging (26) with (27), we obtain

$$\begin{aligned} P_e &\approx \frac{1}{2} \left[1 - \frac{1}{1 + |h_{12}|^2} \frac{2\Gamma}{\sqrt{2(\Gamma + 1)^2 - \Gamma^2}} \right. \\ &\quad \left. + \frac{1 - |h_{12}|^2}{1 + |h_{12}|^2} \frac{\Gamma(1 - |h_{12}|^2)}{\sqrt{2[\Gamma(1 - |h_{12}|^2) + 2]^2 - [\Gamma(1 - |h_{12}|^2)]^2}} \right] \end{aligned} \tag{28}$$

$$\approx \frac{5}{(\Gamma\sqrt{1 - |h_{12}|^2})^2}, \quad \text{for large } \Gamma. \tag{29}$$

On the other hand, the exact expression can be derived for the uncorrelated, but unequal average power fading case. The result is

$$\begin{aligned} P_e &= \frac{1}{2} \left[1 - \frac{\Gamma}{\sqrt{2(\Gamma + 1)^2 - \Gamma^2}} - \frac{q\Gamma}{\sqrt{2(q\Gamma + 1)^2 - (q\Gamma)^2}} \right. \\ &\quad \left. + \frac{\Gamma}{\sqrt{2(\Gamma + 1 + q^{-1})^2 - \Gamma^2}} \right] \end{aligned} \tag{30}$$

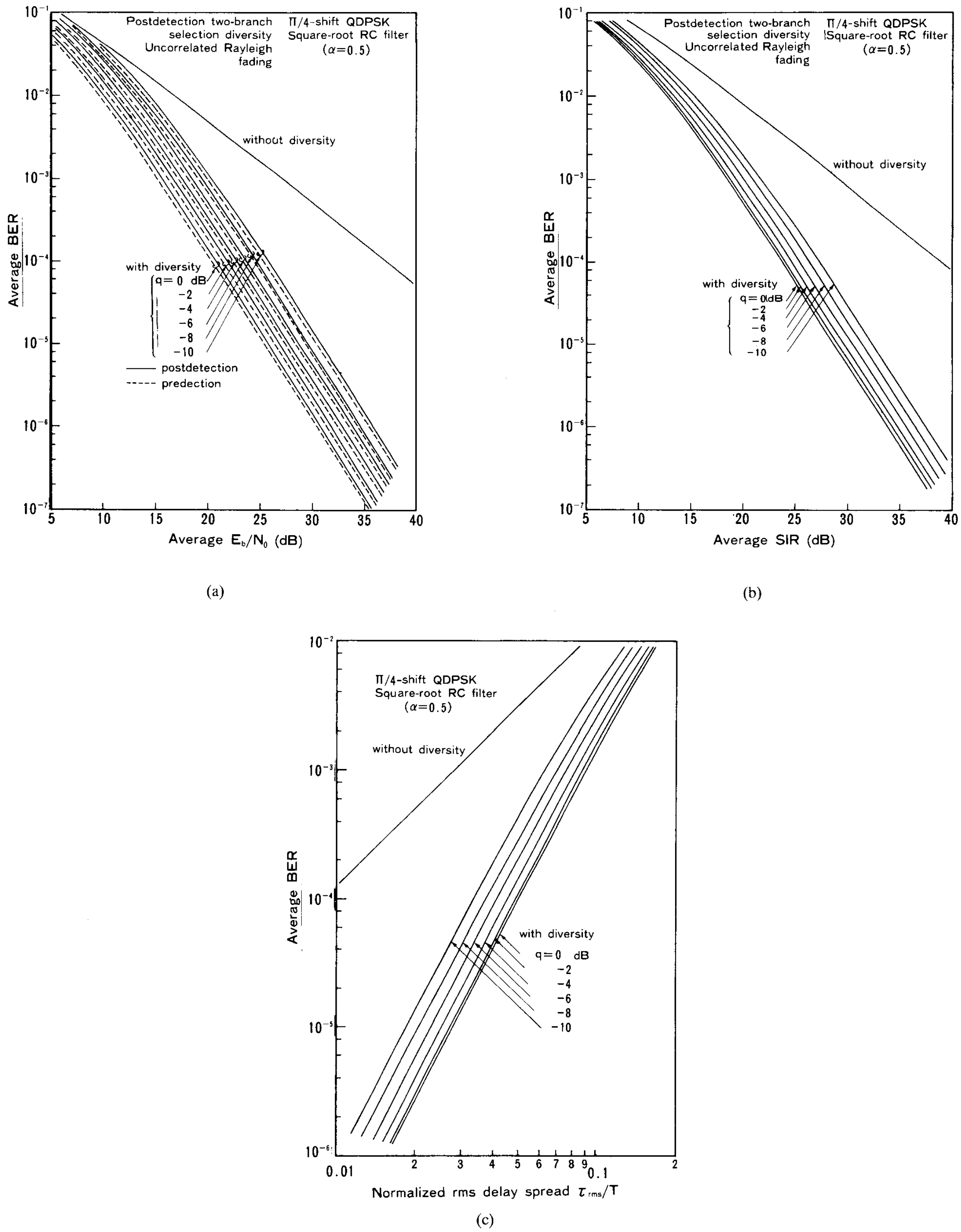

Fig. 3. Influence of unequal average power. Uncorrelated Rayleigh fading case. Average BER's due to (a) AWGN, (b) cochannel interference, and (c) delay spread.

$$\approx \frac{5}{(\sqrt{q}\Gamma)^2}, \quad \text{for large } \Gamma. \tag{31}$$

The average BER performances calculated from (28) and (30) are shown as dotted lines in Figs. 2(a) and 3(a). The performance difference between postdetection and predetection selection diversity is very small. From (21), the average BER due to AWGN for postdetection diversity is given by

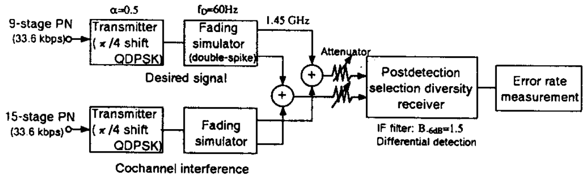

Fig. 4. Laboratory experiment block diagram.

$P_e \approx 6\{\Gamma\sqrt{q\cdot(1-|h_{12}|^2)}\}^{-2}$ for large Γ. Therefore, BER performance achieved by postdetection selection diversity is only 0.4 dB inferior to that achieved by predetection one. Furthermore, diversity gain degradations are identical for both diversity schemes. Predetection selection diversity switches the branch before differential detection and produces abrupt phase changes in the resultant signal, thereby causing decision errors.[4]

Therefore, postdetection selection diversity is preferred.

VI. Experiments

The laboratory experiment block diagram is shown in Fig. 4. A 33.6 kb/s, nine-stage PN sequence was the transmitted data. A $\pi/4$-shift QDPSK signal with roll off factor $\alpha = 0.5$ was generated at 1.45 GHz. The RC transfer function of the transmission channel was realized by the transmitter filter alone instead of sharing equally at both transmitter and receiver filters as assumed in the analysis (this filter combination may cause a slight degradation in BER performance [8]). A two-branch multipath fading simulator produced two correlated Rayleigh fading signals for diversity reception. To measure the average BER due to delay spread, the fading simulator adopted a double-spike delay power profile (each wave was subjected to independent Rayleigh fading with a maximum Doppler frequency f_D of 60 Hz). To measure cochannel interference performance, the same $\pi/4$-shift QDPSK signal modulated with 15-stage PN data was generated. Interference modulation timing was independent of the desired channel.

The measured average BER performances are shown for various squared fading correlations $|h_{12}|^2$ and for various power ratios q in Figs. 5 and 6, respectively. When measuring the average BER's due to AWGN and cochannel interference, the rms delay spread $\tau_{\rm rms}$ was set to $\tau_{\rm rms} = 0$ so that multiplicative fading signals were produced. When measuring the average BER's due to delay spread, the average E_b/N_0 was set to 47.6 dB so that the predominant cause of errors was delay spread. Compared with the calculated BER performance shown in Figs. 2 and 3, the measured BER performances are somewhat degraded, however, the measured diversity gains are close to the calculated values. The measured gain degradations at average BER $= 10^{-3}$ are plotted in Fig. 7. As was expected, gain degradations due to unequal average power are smaller when errors are caused by cochannel interference and delay spread than when errors are caused by AWGN. For comparison, the calculated results of approximate degradations are shown in the figure as the solid line. The measured results agree well with the calculated values.

[4] An antenna selection diversity proposed for TDMA portable mobile radio [18] selects antenna just before reception of data burst of the time slot allocated, and thus switching noise for antenna selection causes no errors in the received data.

VII. Conclusion

Diversity gain degradations due to fading correlation and unequal average signal power were investigated for two-branch postdetection selection diversity reception. A $\pi/4$-shift QDPSK system was assumed. The gain degradation in decibels is the sum of those due to fading correlation and unequal average power. The former is independent of the cause of errors and is approximately given by $5\log(1-|h_{12}|^2)$ dB; gain degradation is 1 dB when $|h_{12}|^2 = 0.4$. This suggests that an antenna separation of as small as 0.2 carrier wavelengths can be used at the mobile station. Diversity gain degradation due to unequal average power is approximately given by $5\log q$ dB if AWGN is the cause of errors, while it is $5\log\{2q/(1+q^2)\}$ dB if cochannel interference and delay spread are the cause of errors. When $q = -3$ dB, the degradation is 1.5 dB under AWGN, but only 0.5 dB if cochannel interference or delay spread predominates.

In this paper, we have assumed that many multipath waves arrive from all directions uniformly with equal amplitude. However, in the real environments, sometimes only a few waves arrive and furthermore, strong line-of-sight waves may appear. Diversity gain degradation in these situations is left for future study.

The results obtained in this paper will be useful in the design of the diversity reception for system TDMA digital cellular systems currently under development in North America and Japan [5], [6].

Appendix I

Since $z_1(nT), z_2(nT), z_1((n-1)T)$, and $z_2((n-1)T)$ are complex Gaussian variables, $z_1((n-1)T)$ and $z_2((n-1)T)$ are also complex Gaussian when $z_1(nT)$ and $z_2(nT)$ are given. Statistical characteristics of $z_1((n-1)T)$ and $z_2((n-1)T)$ can be exactly determined by their means and variances. Let $\boldsymbol{z}_T$ and $\boldsymbol{z}$ be the column vectors of $z_m((n-1)T)$ and $z_m(nT)$, respectively, and Ω be the partitioned column matrix of $\boldsymbol{z}_T$ and $\boldsymbol{z}$. The covariance matrix of Ω can be represented as

$$\frac{1}{2}\langle\Omega^*\Omega^T\rangle = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{g} \end{pmatrix}. \tag{32}$$

$\boldsymbol{z}_T$ is the complex Gaussian vector with a mean $\langle \boldsymbol{z}_T\rangle = \boldsymbol{k}\cdot\boldsymbol{z} = (\boldsymbol{c}\boldsymbol{g}^{-1})^*\boldsymbol{z}$ and covariance matrix $\boldsymbol{R} = \boldsymbol{a} - \boldsymbol{c}\boldsymbol{g}^{-1}\,\boldsymbol{c}^{T^*}$ [14]. For simplicity, we assume no cochannel interference and no delay spread. Therefore, $g_m(\tau, t) = g_m(t)\,\delta(\tau)$ and thus,

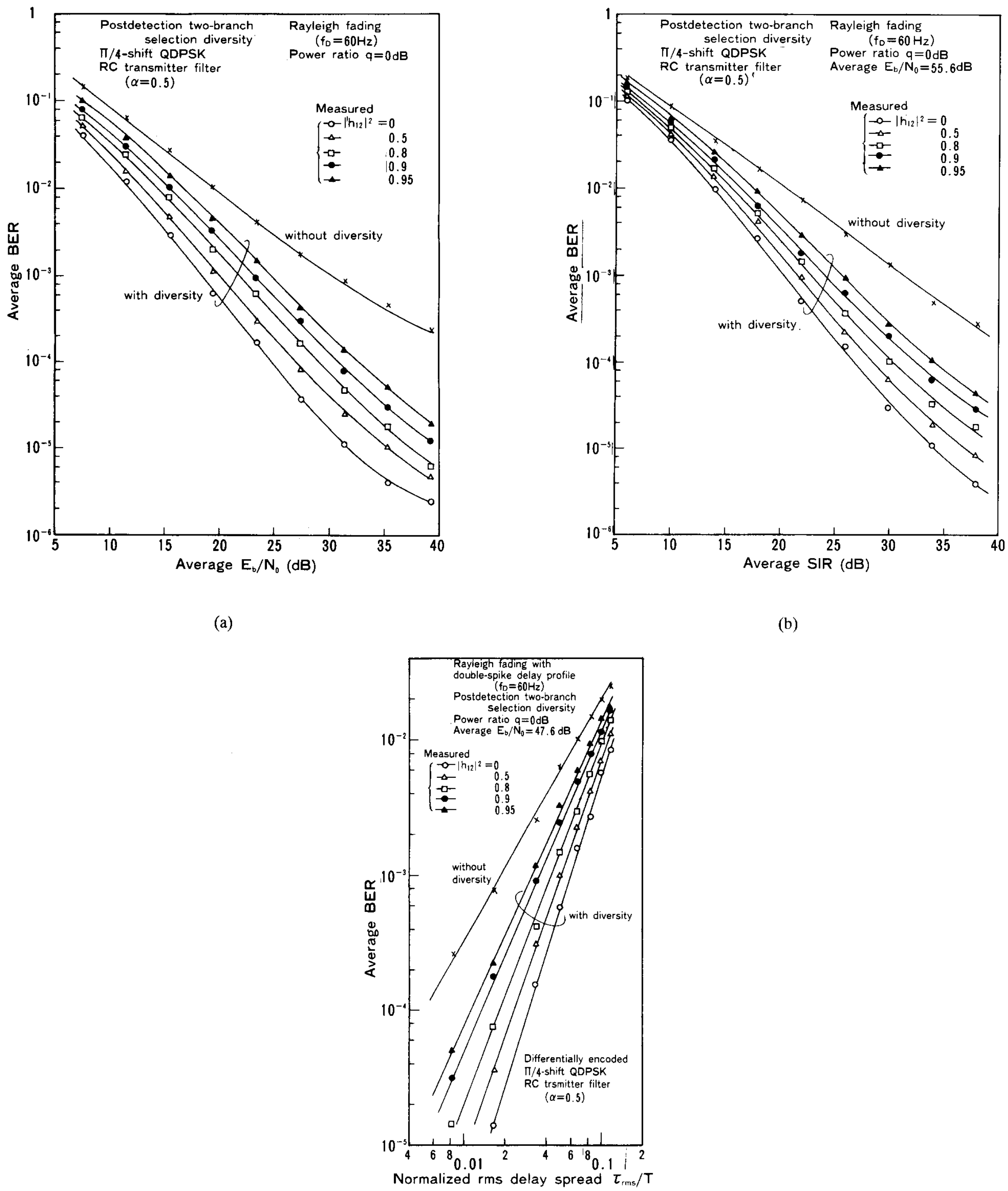

Fig. 5. Measured results of influence of fading correlation. Average BER's due to (a) AWGN, (b) cochannel interference, and (c) delay spread.

$z_m(nT) = \sqrt{2E_s/T}e^{j\phi_n} g_m(nT) + z_{nm}(nT)$. $\boldsymbol{a}$, $\boldsymbol{g}$, and $\boldsymbol{c}$ can be given as

$$\boldsymbol{a} = \boldsymbol{g} = \frac{E_s}{T}\begin{pmatrix} 1+\frac{1}{\Gamma} & h_{21} \\ h_{12} & 1+\frac{1}{\Gamma} \end{pmatrix}$$
$$\boldsymbol{c} = \frac{E_s}{T}e^{j\Delta\phi_n}\begin{pmatrix} 1 & h_{21} \\ h_{12} & 1 \end{pmatrix} \tag{33}$$

where Γ is the average symbol energy-to-noise power spectrum density ratio (E_s/N_0) and $h_{12}(= h_{21}^*)$ is the correlation between two fading signals, i.e., $h_{12} = \langle g_1(nT)g_2^*(nT)\rangle$.

Exact calculation of the average BER involves a double integral and the evaluation is difficult [13]. However, a fairly simple derivation is possible if $|h_{12}|$ is not too close to unity. When $|h_{12}|$ is small, the diagonal components of $(\boldsymbol{c}\boldsymbol{g}^{-1})^*$ are

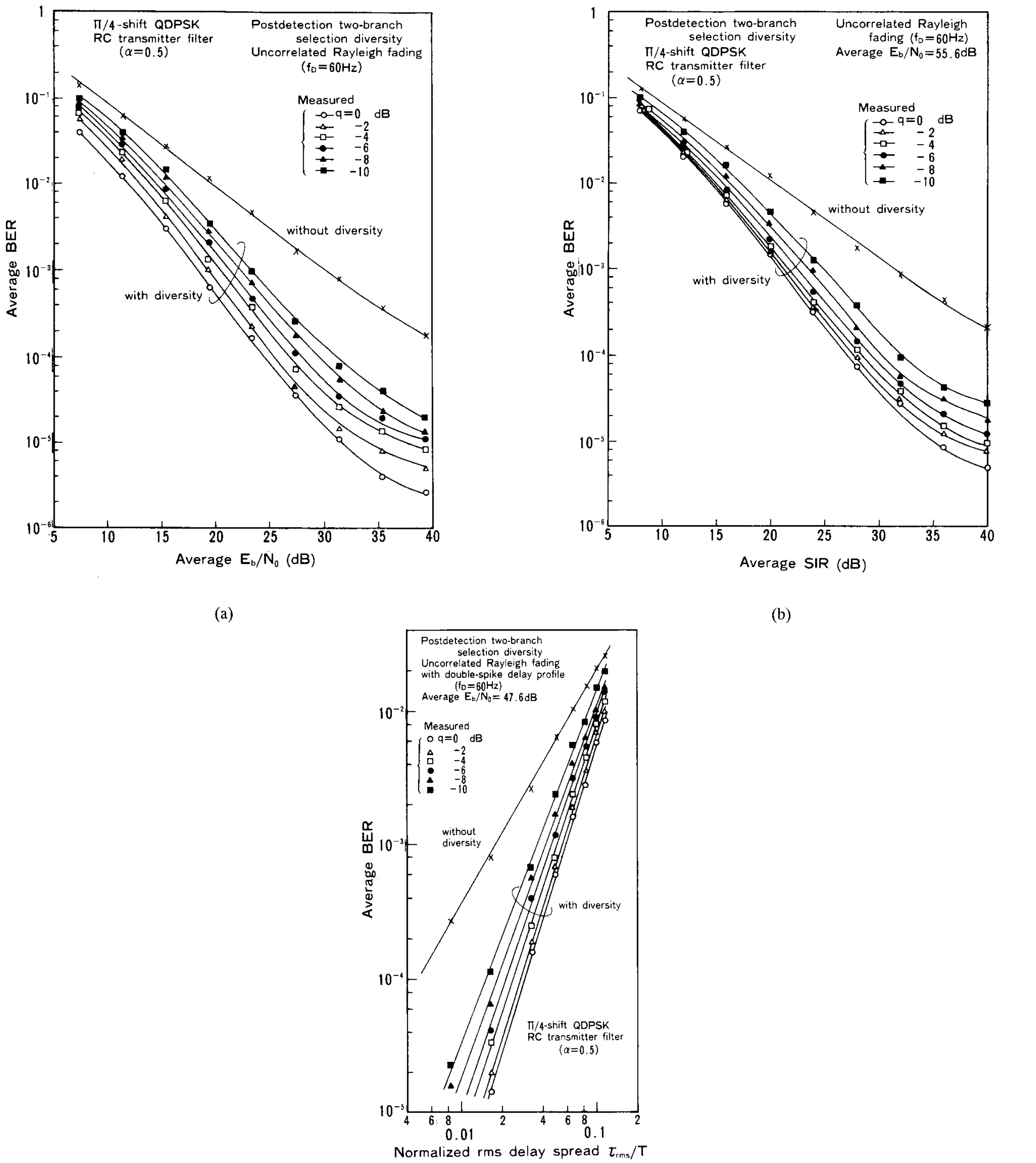

Fig. 6. Measured results of influence of unequal average power. Average BER's due to (a) AWGN, (b) cochannel interference, and (c) delay spread.

predominant

$$cg^{-1} \approx \frac{\Gamma}{\Gamma+1} e^{j\Delta\phi_n} \boldsymbol{I} \tag{34}$$

where $\boldsymbol{I}$ is the identity matrix. Therefore,

$$\langle z_T \rangle \approx e^{-j\Delta\phi_n} z \qquad \boldsymbol{R} \approx \frac{N_0}{T} \begin{pmatrix} 2 & h_{21} \\ h_{12} & 2 \end{pmatrix} \tag{35}$$

for large values of Γ. The diagonal components of $\boldsymbol{R}$ are not a function of h_{12} (or h_{21}) and are equal to those for uncorrelated fading case. Equation (35) suggests that if h_{12} is not too close to unity, the conditional mean and variance of $z_m((n-1)T)$ are approximately the same as those for the uncorrelated fading case and consequently, the conditional BER derived for the uncorrelated fading case [8] can be applied.

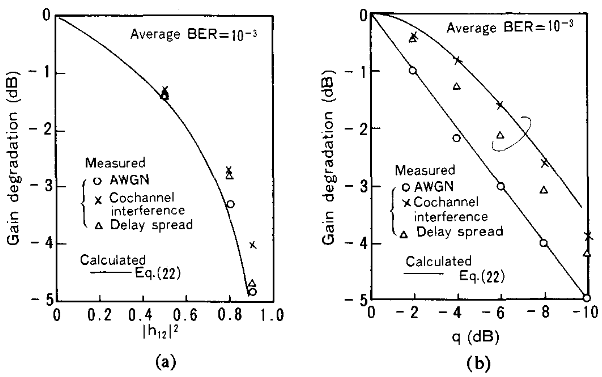

Fig. 7. Measured gain degradations. Influence of (a) fading correlation and (b) unequal average power.

APPENDIX II

The cdf of R is given by [7]

$$P_R(R) = 1 - e^{-\frac{R^2}{2\sigma^2}}[1 - Q(a,b) + Q(b,a)] \tag{36}$$

where $Q(a,b)$ is Marcum's Q-function, $a = b|\eta_{12}|$, and $b = R/(\sigma\sqrt{1-|\eta_{12}|^2})$ and η_{12} is the complex correlation between $z_1(0)$ and $z_2(0)$. $Q(a,b)$ can be expressed using the modified Bessel function $I_r(\cdot)$ [14]:

$$Q(a,b) = e^{-\frac{a^2+b^2}{2}} \sum_{r=0}^{\infty} \left(\frac{a}{b}\right)^r I_r(ab)$$
$$Q(b,a) = 1 - e^{-\frac{a^2+b^2}{2}} \sum_{r=1}^{\infty} \left(\frac{a}{b}\right)^r I_r(ab). \tag{37}$$

Substituting the above into (36), we obtain

$$P_R(R) = 1 - e^{-\frac{R^2}{2\sigma^2}} \left[2 - e^{-\frac{1+|\eta_{12}|^2}{1-|\eta_{12}|^2}\frac{R^2}{2\sigma^2}} \sum_{r=0}^{\infty} \varepsilon_r |\eta_{12}|^r \cdot I_r\left(\frac{|\eta_{12}|R^2}{\sigma^2(1-|\eta_{12}|^2)}\right)\right]. \tag{38}$$

APPENDIX III

The cdf of γ is given by [7]

$$P_\gamma(\gamma) = 1 - e^{\frac{-\gamma}{\Gamma}}[1 - Q(a,b) + Q(b,a)] \tag{39}$$

where $a = b|h_{12}|, b = \sqrt{2\gamma/\Gamma(1-|h_{12}|^2)}$. Using

$$Q(\sqrt{\beta},\sqrt{\alpha}) - Q(\sqrt{\alpha},\sqrt{\beta}) = \frac{\beta-\alpha}{\beta+\alpha} I_e\left(\frac{2\sqrt{\alpha\beta}}{\alpha+\beta}, \frac{\alpha+\beta}{2}\right). \tag{40}$$

Equation (39) can be rewritten as

$$P_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\Gamma}} \left[1 + \frac{1-|h_{12}|^2}{1+|h_{12}|^2} \cdot I_e\left(\frac{2|h_{12}|}{1+|h_{12}|^2}, \frac{1+|h_{12}|^2}{1-|h_{12}|^2}\frac{\gamma}{\Gamma}\right)\right] \tag{41}$$

where $I_e(a,b)$ is Rice's I_e function. Hence, the pdf of γ can be given by

$$p_\gamma(\gamma) = \frac{dP_\gamma(\gamma)}{d\gamma}$$
$$= \frac{1}{\Gamma}e^{-\frac{\gamma}{\Gamma}}\left[1 + \frac{1-|h_{12}|^2}{1+|h_{12}|^2} I_e\left(\frac{2|h_{12}|}{1+|h_{12}|^2}, \frac{1+|h_{12}|^2}{1-|h_{12}|^2}\frac{\gamma}{\Gamma}\right)\right]$$
$$- \frac{1}{\Gamma}e^{-\frac{2}{1-|h_{12}|^2}\frac{\gamma}{\Gamma}} I_0\left(\frac{2|h_{12}|}{1-|h_{12}|^2}\frac{\gamma}{\Gamma}\right). \tag{42}$$

Calculation of the average BER involves the infinite integration of I_e-function $\times$ I_e-function and therefore, is difficult to evaluate. However, an appropriate expression is possible. For small values of x,

$$I_e(a,x) \approx 1 - e^{-x} \qquad I_0(x) \approx 1 \tag{43}$$

and then,

$$p_\gamma(\gamma) \approx \frac{2}{1+|h_{12}|^2}\frac{1}{\Gamma}\left[e^{-\frac{\gamma}{\Gamma}} - e^{-\frac{2}{1-|h_{12}|^2}\frac{\gamma}{\Gamma}}\right]. \tag{44}$$

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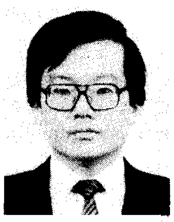

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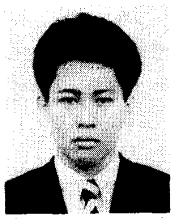
Fumiyuki Adachi (M'79–SM'90), for a photograph and biography please see page 176 of this issue.



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