

# Comment on the calculation of the pdf of the output of a two-branch switch and stay diversity system

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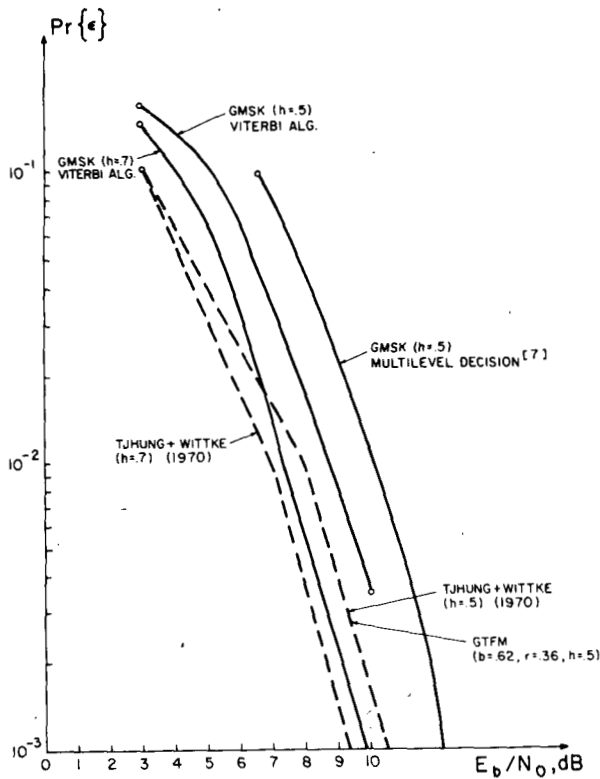


Fig. 4. GMSK detectability results ( $h = 0.5$  and  $h = 0.7$  with Viterbi algorithm) compared to binary FSK and GTFM, and multilevel decision GMSK.

We chose to detect GMSK at the discriminator output using the MLSE algorithm of [2]. By varying the premodulation Gaussian filter (3 dB) bandwidth  $B_T$ , the index of modulation  $h$ , and the receiver IF and LPF bandwidths, it is possible to generate a whole class of modulation techniques.

We chose to look at  $B_T/Rb = 0.25$  and  $h = 0.5$  (the value used in [7]), and also  $h$  equal to 0.7. As seen in Fig. 4, the results for  $h = 0.5$  represent about a 1 dB improvement over the multilevel decision method. The results for  $h = 0.7$  are almost similar to those of classical binary FSK of Tjhung and Wittke [10] but, of course, with a narrower bandwidth.

V. CONCLUSIONS

It has been shown that by properly designing digital CPM signals, it is possible to improve BER performance for discriminator detected modulations in comparison to previously reported results. In the "modified GTFM" case it is possible to achieve better BER performance than that of similar GTFM schemes, with only a small sacrifice in bandwidth. For GMSK ( $h = 0.5$ ) the discriminator-MLSE detector results in detectability performance at least 1 dB better than those previously described in the literature (for a discriminator-multilevel decision detector). The GMSK detectability results can be improved (using  $h = 0.7$ ) with only a small increase in bandwidth, due to the larger index of modulation. These results are almost equal to those of classical binary FSK with optimum modulation index ( $h = 0.7$ ).

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Comment on the Calculation of the pdf of the Output of a Two-Branch Switch and Stay Diversity System

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Expressions for the pdf of the output of a two-branch switch and stay diversity system have been derived in both [1] and [2]. The basic difference is that the analysis in [1] is based on a continuous time approach, whereas that in [2] is based on a discrete time approach.

The principal purpose of this letter is to draw attention to the fact that the pdf's obtained by the two methods are not the same; compare [1, eq. (26)] to [2, eq. (2.17)]. Moreover, while Blanco and Zdunek stated [2, sect. II-B] that they would compare their results to those of [1], they did not do so.

Having noted that the two pdf's are not the same, it is interesting to consider what different assumptions are made in the two approaches, and in what ways the problems so analyzed are different. A major assumption made in [1] is that, having switched from  $r_1$  to  $r_2$  ( $r_1$  and  $r_2$  being the two input signals), "... when we switch back from  $r_2$  to  $r_1$ , the statistics of  $r_1$  are already independent of that portion of  $r_1$  which we switched out initially." A similar assumption is made in [2]. However, an additional assumption is also made, effectively between (2.8) and (2.9), concerning the statistical independence of successive samples of the same signal. While  $x_{i-1}$  and  $x_i$  (where  $x_i$  and  $y_i$  are the  $i$ th samples of the two input signals  $x$  and  $y$ ) may be statistically independent if the time between samples is very long, it is questionable whether this is still a

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realistic assumption at a high sampling rate, certainly at a rate which is sufficiently high so that the  $[x_i]$  adequately represent  $x$ . It is also surprising that the final result is completely independent of the sampling rate.

Another difference between the two analyses is the nature of the output signal. The output of the system in [1] is continuous in time. It is not clear whether the output of the system in [2] is a series of discrete samples, or whether the output is continuous but the testing is done at discrete intervals. If the latter is the case, it is possible for the switching from one input signal to the other to be delayed by up to one sampling period from the time when the input signal actually falls below the threshold  $d$ , in the case when  $x_{i-1} > d$  and  $x_i < d$ .

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**A Numerical Method for Evaluating the Distortion of Angle-Modulated Signals in a Time Domain**

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**Abstract**—As yet there exists no analytical method of obtaining the exact output distortion of BPF's for an angle-modulated signal input. In this paper, we have introduced a numerical method in which a filtering problem is described by the state space method and then is converted to a time domain difference equation. As a result, evaluation can be carried out quickly and easily.

I. INTRODUCTION

The problem of evaluating the response of bandpass filters (BPF's) by angle-modulated signals has been investigated by many authors. J. R. Carson tackled it in the 1930's. Due to the nonlinearity involved in this problem, however, there exists no exact analytical method of expressing the response when an arbitrary modulating signal is used. Consequently, evaluation is made by either approximate analytical methods [1]-[3] or numerical ones [4]-[8]. Numerical methods consist of Monte Carlo simulation methods [4]-[6], which are mainly used for the evaluation of the interchannel interference distortion in the FM trunk radio system, and numerical techniques to evaluate the distortion of periodically modulated signals [7], [8].

Numerical methods can be roughly divided into two categories—the frequency domain [4], [7] and the time domain [5], [6], [8]. The former uses the fast Fourier transform (FFT) and the latter uses convolutional integrals in the time domain.

The method which uses the FFT is popular because it is easy to understand and the error is small. On the other hand, the

method which uses the convolutional integral is not so common because of the long computation time required for the numerical integration. However, the method which uses the FFT is ordinarily applied in cases where the angle modulation is periodic. It is difficult to apply when the modulation has transients such as frequency steps and/or frequency ramps.

In this correspondence a very simple iteration method, using the first-order difference equation derived by combining the state variable method [9] with the numerical integration formulas [10], [11], is introduced, as an improved version of the time domain convolutional method. In this method, the types of BPF and angle-modulating signal shape are arbitrary and the BPF type is specified by the state transition matrix.

In comparing this method to the FFT method, the response of a BPF for the transiently modulated FM can be evaluated quickly and easily, even using a microcomputer with a small amount of memory. A real-time application of this algorithm may lead to the discovery of new digital FM demodulation techniques.

II. A MODEL OF EVALUATION

A. A Description of the System by State Variables

The model for the analysis is shown in Fig. 1. The input angle-modulated signal  $v_{in}(t)$ , the transfer function of the BPF  $H(j\omega)$ , and the output signal of the BPF  $v_{out}(t)$  are expressed as

$$\begin{aligned}
 v_{in}(t) &= A \cos [\omega_0 t + \phi_{in}(t)] \\
 H(j\omega) &= \int_{-\infty}^{\infty} h(t) e^{j\omega t} dt \\
 v_{out}(t) &= B(t) \cos [\omega_0 t + \phi_{out}(t)] \tag{1}
 \end{aligned}$$

where  $A$ ,  $\omega_0$ , and  $\phi_{in}(t)$  represent the amplitude, the angular frequency, and the phase of the input angle-modulated signal, and  $h(t)$  is the unit impulse response of the BPF.  $B(t)$  and  $\phi_{out}(t)$  denote the amplitude and the phase of the output signal, respectively.

Given  $v_{in}(t)$  and  $H(j\omega)$  as shown in Fig. 1, the problem is to obtain  $v_{out}(t)$ . But  $\omega_0$  is arbitrary and the idea of an equivalent low-pass model [12] is used as shown in Fig. 2.

In Fig. 2,  $u(t)$  and  $v(t)$  denote the input complex low-pass signal and the output complex low-pass signal, respectively, and  $C(t) = B(t)/A$ , and  $H_L(j\omega)$  is the equivalent low-pass expression of the transfer function  $H(j\omega)$  which is written as

$$H(j\omega) = H_L[j(\omega - \omega_0)] + H_L^*[-j(\omega + \omega_0)] \tag{2}$$

where  $*$  denotes the complex conjugate.

Ordinarily, the equivalent low-pass function  $H_L(j\omega)$  has the form

$$H_L(s) = \frac{c_n s^{n-1} + c_{n-1} s^{n-2} + \dots + c_2 s + c_1}{s^n + a_n s^{n-1} + a_{n-1} s^{n-2} + \dots + a_2 s + a_1} \tag{3}$$

which is the rational function of  $s$ , where  $s = j\omega$ .

Thus, from Fig. 2 and (3), it follows that

$$\begin{aligned}
 (p^n + a_n p^{n-1} + \dots + a_2 p + a_1)v(t) \\
 = (c_n p^{n-1} + c_{n-1} p^{n-2} + \dots + c_2 p + c_1)u(t) \tag{4}
 \end{aligned}$$

where  $p$  denotes the operator  $d/dt$ . Applying the state variable representations to (4), the following expressions are obtained.

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t) \\ v(t) = CX(t) \end{cases} \tag{5}$$

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