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著者	安達　文幸
journal or publication title	IEEE Transactions on Communications
volume	44
number	4
page range	416-418
year	1996
URL	http://hdl.handle.net/10097/46446

doi: 10.1109/26.489084

Decision Feedback Differential Detection of Differentially Encoded 16APSK Signals

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***Abstract*— Multiple-symbol differential detection (DD) with reference signal estimation based on the feedback of past detected symbols is presented for differentially encoded 16-level amplitude/phase shift keying (16DAPSK). A suboptimal decision is derived. The approximate analysis of the bit-error rate (BER) performance taking into account the decision error propagation effect is presented in an additive white Gaussian noise (AWGN) channel and the BER performance is compared with those of 16DPSK and 16 quadrature amplitude modulation (16QAM).**

I. Introduction

RECENTLY, differentially encoded 16-level amplitude/phase shift keying (16DAPSK) with differential detection (DD) has been attracting much attention, particularly in the field of mobile radio where fast tracking, yet accurate carrier recovery, is very difficult to realize [1], [2]. Although DD eliminates the need for a carrier recovery circuit in the receiver, it suffers from a performance penalty when compared to ideal coherent detection because the received noisy signal is used as the reference. To narrow the performance gap between differential and coherent detection (CD), decision feedback multiple-symbol differential detection (DFDD) was proposed for M-ary DPSK [3]. The DFDD scheme extracts the reference signal from a number of past symbols; the additive white Gaussian noise (AWGN)-produced phase noise is smoothed to increase the signal-to-noise power ratio (SNR) of the resultant reference signal. A similar idea can be applied to 16DAPSK. In this paper, we will present the DFDD scheme for 16DAPSK and analyze its bit-error rate (BER) performance in an AWGN channel. Computer simulation results are also presented.

II. DFDD Scheme

The signal constellation of 16DAPSK is shown in Fig. 1. 16DAPSK is a combination of independent 8DPSK and 2DASK. The transmitted signal can be expressed in the complex form as $s(t) = \sqrt{2S}\sum_{n=-\infty}^{\infty} s_n p(t - nT)$, where with $s_n = r_n \exp j\phi_n$ with $r_n = \{r_L, r_H\}$ and $\phi_n = \{m\pi/4;\ m = 0, 1, \cdots, 7\}$, S is the average signal power, T is the symbol duration, and $p(t) = 1$ for $-T < t \leq 0$ and zero elsewhere. Let $a = r_H/r_L\,(>1)$ be the amplitude ratio. Employing the normalization of $0.5\,(r_L^2 + r_L^2) = 1$, we use $r_L = \sqrt{2/(a^2+1)}$ and $r_H = a\,r_L$. The four-bit symbol $(a_n\, b_n\, c_n\, d_n)$ to be transmitted is represented by $\Delta r_n \exp j\Delta\phi_n$, where $\Delta\phi_n = \phi_n - \phi_{n-1}$ and $\Delta r_n = r_n/r_{n-1}$. For 8DPSK, Gray code bit mapping of $(a_n\, b_n\, c_n)$ to $\Delta\phi_n$ is applied. For 2DASK, the signal amplitude changes from r_L to r_H or vice versa if $d_n = 1$, while no change happens otherwise; thus a three-level amplitude ratio is produced, i.e., $\Delta r_n = a$ or a^{-1} if $d_n = 1$ and $\Delta r_n = 1$ otherwise. The modulated signal is transmitted and received via an AWGN channel. We assume perfect automatic carrier frequency control (AFC) so that no frequency offset is produced between the transmitter and receiver. The received signal is passed through a matched filter with impulse response $h(t) = (1/T)p(-t)$ and sampled at $t = nT$ giving $z_n = \sqrt{2S}s_n \exp j\theta + w_n$, where θ is the unknown, constant phase and w_n is the sample of the filtered AWGN with power N. Since $s_n = (s_n/s_{n-1})s_{n-1} = (\Delta r_n \exp j\Delta\phi_n)s_{n-1}$, the probability density function (pdf) of z_n can be expressed as

$$p(z_n|\Delta r_n, \Delta\phi_n, s_{n-1}, S, \theta) = \frac{1}{2\pi N}\exp\left[-\frac{|z_n - (\Delta r_n e^{j\Delta\phi_n})(\sqrt{2S}s_{n-1}e^{j\theta})|^2}{2N}\right]. \quad (1)$$

The optimal decision on Δr_n and $\Delta\phi_n$ is to choose the pair of Δr_n and $\Delta\phi_n$ that maximizes the *a posteriori* probability calculated from (1). Unfortunately, the reference signal $\sqrt{2S}s_{n-1} \exp j\theta$ is unknown; thus, we use its estimate $z_{r,\,n-1}$ to find an implementable solution. The proposed suboptimal receiver structure is illustrated in Fig. 2.

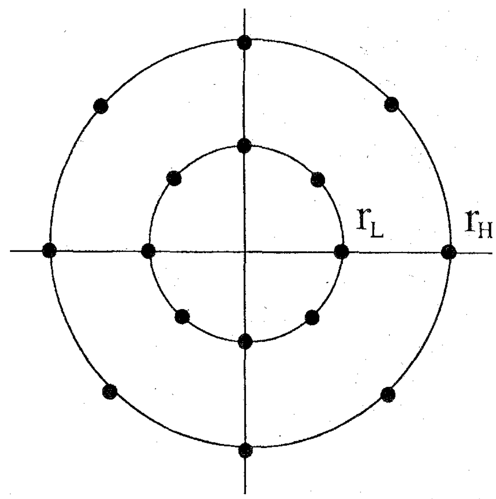

Fig. 1. Signal constellation of 16DAPSK.

Paper approved by A. Duel-Hallen, the Editor for Communications Theory of the IEEE Communications Society. Manuscript received April 5, 1993; revised May 17, 1994. This paper was presented in part at the 1993 Joint Technical Conference on Circuits/Systems, Computers and Communications, Nara, Japan, July 26–28, 1993.

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Publisher Item Identifier S 0090-6778(96)03149-2.

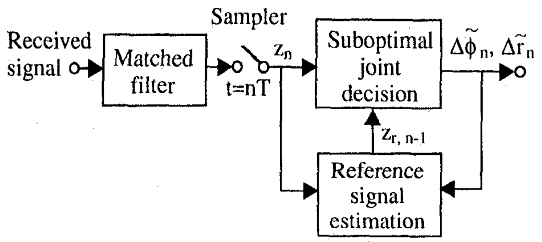

Fig. 2. Suboptimal receiver structure.

The reference signal estimation is based on feedback of the past $L-1$ pairs of decisions, $\Delta\tilde{\phi}_{n-l}$ and $\Delta\tilde{r}_{n-l}$, $l = 1, 2, \cdots, L-1$. Since $s_{n-l} = s_{n-1}\prod_{i=1}^{l-1} \Delta r_{n-i}^{-1} \cdot \exp -j\Delta\phi_{n-i}$, the joint pdf of $\overline{\mathbf{z}} = (z_{n-1}, z_{n-2}, \cdots, z_{n-L})$ can be expressed as

$$p(\overline{\mathbf{z}}) = \frac{1}{(2\pi N)^L} \exp \cdot \left[-\frac{1}{2N} \sum_{l=1}^{L} \left| z_{n-l} - \sqrt{2S}s_{n-1}e^{j\theta} \prod_{i=1}^{l-1} \Delta r_{n-i}^{-1} e^{-j\Delta\phi_{n-i}} \right|^2 \right]. \tag{2}$$

The exact sequences of $\Delta\phi_{n-l}$ and Δr_{n-l} are unknown to the receiver; thus, we substitute their decisions $\Delta\tilde{\phi}_{n-l}$ and $\Delta\tilde{r}_{n-l}$. The best estimate of $\sqrt{2S}s_{n-1} \exp j\theta$ is the maximum likelihood estimate that maximizes the joint pdf of (2), and can be found as

$$z_{r,\,n-1} = \frac{\sum_{l=1}^{L} z_{n-l} \prod_{i=1}^{l-1} \Delta\tilde{r}_{n-i}^{-1} e^{j\Delta\tilde{\phi}_{n-i}}}{\sum_{l=1}^{L} \prod_{i=1}^{l-1} \Delta\tilde{r}_{n-i}^{-2}} \tag{3}$$

for $L \geq 2$. Note that if $L = 1$, $z_{r,\,n-1} = z_{n-1}$. Using the estimated reference, the suboptimal joint decision is to choose the pair of $\Delta\tilde{\phi}_n$ and $\Delta\tilde{r}_n$ if $|z_n - (\Delta\tilde{r}_n \exp j\Delta\tilde{\phi}_n) z_{r,\,n-1}|^2$ is maximum. This decision can be decomposed into two steps:

1) *First Step:* Choose $\Delta\tilde{\phi}_n$ if $\mathrm{Re}\,[(z_n z_{r,\,n-1}^*) \exp -j\Delta\tilde{\phi}_n]$ is maximum [differential phase detection (DPD)].
2) *Second Step:* Choose $\Delta\tilde{r}_n$ if

$$|\Delta\tilde{r}_n - \mathrm{Re}\,[z_n z_{r,\,n-1}^* \exp -j\Delta\tilde{\phi}_n]/|z_{r,\,n-1}|^2|$$

is maximum [differential amplitude detection (DAD)].
3) The transmitted four-bit symbol is then recovered based on $\Delta\tilde{\phi}_n$ and $\Delta\tilde{r}_n$.

III. Approximate BER Analysis

Decision feedback produces error propagation. When a decision error is caused by AWGN, the reference signal for the next decision suffers from distortion since an incorrectly detected symbol is fed back. This results in a succeeding decision error with high probability irrespective of the value of L (except for $L = 1$). Assuming large L, we examined how this succeeding error affects the reference signal estimation, and found that no further decision error is likely to follow because the reference signal is almost always pulled back to the correct position. This implies that once a single decision error is caused by AWGN, a double symbol error is most likely to be produced[1] (the same can be found for M-ary DPSK [4]). Therefore, the BER is approximately given as twice the BER for the case that correct symbols are fed back. Neglecting simultaneous 8DPSK and 2DASK decision error,[2] the average BER of 16DAPSK can be approximated as

$$P_b = \tfrac{1}{2}\left(3P_{\mathrm{8DPSK}} + P_{\mathrm{2DASK}}\right) \tag{4}$$

for $L \geq 2$, where P_{8DPSK} and P_{2DASK} are the BER's of 8DPSK and 2DASK with correct symbol feedback, respectively. On the other hand, when $L = 1$, a factor of two increase in the BER is not necessary since no decision feedback is involved.

Assuming that correct symbols are fed back as in [3], i.e., $\Delta\tilde{\phi}_{n-l} = \Delta\phi_{n-l}$, and $\Delta\tilde{r}_{n-l} = \Delta r_{n-l}$, $l = 1, 2, \cdots, L-1$, the estimated reference signal $z_{r,\,n-1}$ becomes Gaussian. It can be shown from (3) that the mean and variance of $z_{r,\,n-1}$ are given by $E(z_{r,\,n-1}) = \sqrt{2S}s_{n-1} \exp j\theta$ and $\mathrm{var}\,(z_{r,\,n-1}) = N/\sum_{l=1}^{L}\prod_{i=1}^{l-1} \Delta r_{n-i}^{-2}$, respectively. The mean is exactly what we want to estimate; thus, $z_{r,\,n-1}$ is an unbiased estimator. The reference signal SNR ρ defined by $0.5[E(z_{r,\,n-1})]^2/\mathrm{var}\,(z_{r,\,n-1})$ becomes $\rho = \rho_0 \sum_{l=1}^{L} r_{n-l}^2$, where $\rho_0 = S/N$ is the SNR of z_n. The BER of 8DPSK can be evaluated from the distribution of the phase noise $\Delta\tilde{\eta} = \arg\,(z_n z_{r,\,n-1}^*) - \Delta\tilde{\phi}_n$. For obtaining the phase noise distribution, we apply [5, eq. (11)]. PDPSK is expressed as [6]

$$P_{\mathrm{8DPSK}} = -\frac{2}{3}\left[F\left(\frac{\pi}{8}\middle|\alpha, \beta\right) + F\left(\frac{3\pi}{8}\middle|\alpha, \beta\right)\right] \tag{5}$$

where $F(\Delta\eta|\alpha, \beta)$ is obtained as

$$\begin{aligned} F(\Delta\eta|\alpha, \beta) = \frac{-\sqrt{\alpha\beta}}{2\pi} \sin \Delta\eta \int_{-\pi/2}^{\pi/2} & \cdot \{\exp\{-0.5\rho_0[\alpha+\beta-(\alpha-\beta)\sin t \\ & - 2\sqrt{\alpha\beta}\cos\Delta\eta\cos t]\}/ \\ & \{\alpha+\beta-(\alpha-\beta)\sin t \\ & - 2\sqrt{\alpha\beta}\cos\Delta\eta\cos t\}\,dt \end{aligned} \tag{6}$$

with $\alpha = r_n^2$ and $\beta = \sum_{l=1}^{L} r_{n-l}^2$.

The DAD can be performed using two thresholds, Δr_L and Δr_H, where $a^{-1} < \Delta r_L < 1$ and $1 < \Delta r_H < a$. Define $\Delta\tilde{r} = \mathrm{Re}\,[z_n z_{r,\,n-1}^* \exp -j\Delta\tilde{\phi}_n]/|z_{r,\,n-1}|^2$. The BER can be evaluated from Prob $[\Delta\tilde{r} < \Delta r]$ which is the probability of $\Delta\tilde{r}$ being less than some positive value Δr. Denoting $z_1 = z_{r,\,n-1} \exp j\Delta\phi_n$ and $z_2 = z_n - \Delta r\, z_{r,\,n-1} \exp j\Delta\phi_n$, we find that Prob $[\Delta\tilde{r} < \Delta r] =$ Prob $[\mathrm{Re}\,(z_1 * z_2) < 0]$. Applying [7, pp. 320–323 and p. 586] and after some manipulation, we

[1] Statistics of burst symbol errors at average signal energy per bit-to-AWGN power spectrum density ratio $E_b/N_0 = 13$ dB were obtained by computer simulation. It was found that the probability of any symbol error falling into double symbol errors is 68% even for $L = 2$; it is around 80% when $L > 2$.

[2] It was found by computer simulation that the probability of 2DASK decision error conditioned on 8DPSK decision error at $E_b/N_0 = 13$ dB is only 1.1% when $L = 2$; thus, simultaneous decision errors can be neglected.

can show that

$$P_{2\text{DASK}} = \begin{cases} G(\Delta r_L|\alpha, \beta) - G(\Delta r_H|\alpha, \beta), & if r_{n-1} = r_n \\ G(\Delta r_H|\alpha, \beta) - G(\Delta r_L|\alpha, \beta), & \text{if } r_{n-1} \neq r_n \end{cases} \tag{7}$$

where

$$G(\Delta r|\alpha, \beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{U}{1 + U^2 - |1 - U^2| \cos t} - \frac{A}{2} \right] \cdot \exp\left[-0.5\rho_0 \beta (1 + U^2 - |1 - U^2| \cos t)\right] dt$$

$$U = \left(1 - \frac{\Delta r}{\Delta r_n}\right) \sqrt{\frac{\alpha}{\alpha\left(\frac{\Delta r}{\Delta r_n}\right)^2 + \beta}}$$

$$A = -\frac{\Delta r}{\Delta r_n} \sqrt{\frac{\alpha}{\alpha\left(\frac{\Delta r}{\Delta r_n}\right)^2 + \beta}}. \tag{8}$$

IV. Numerical Calculation and Computer Simulation

The BER depends on the sequence of $L + 1$ amplitudes r_{n-l}, $l = 0, 1, \cdots, L$, and on the amplitude ratio a and the choice of the decision thresholds Δr_L and Δr_H. Letting k be the number of r_L taken by r_{n-l}'s, $l = 2, 3, \cdots, L$, we have $\beta = r_{n-1}^2 + [2(L-1)a^2 - 2k(a^2 - 1)]/(a^2 + 1)$, where the pdf of k is characterized by the binomial distribution $p(k) = \binom{L-1}{k} 2^{-(L-1)}$. The overall average BER is thus obtained by first calculating the conditional BER from (4)–(8) using the statistical property of β for the given r_n and r_{n-1}, and then averaging over r_n and r_{n-1}. The optimum values of the amplitude ratio a and the thresholds $(\Delta r_L, \Delta r_H)$ at $E_b/N_0 = 15$ dB are found to be $a = 2$ and $(\Delta r_L, \Delta r_H) = (0.68, 1.5)$. Both loosely depend on variations in E_b/N_0 (note that $E_b/N_0 = 0.25\rho_0$). The calculated BER performance is shown in Fig. 3. The required E_b/N_0 value to attain BER $= 10^{-3}$ is 15.1 dB when $L = 1$ (no decision feedback is used). As L increases, the performance improves and approaches that achieved by the use of a pure reference signal, i.e., coherent detection (CD) with DD. The BER performance with $L = 8$ improves by about 1.6 dB and becomes only about 0.2 dB inferior to that of CD. For comparison, we plotted the BER performances of 16DPSK with DD and CD and also those of 16 QAM with and without differential encoding (DE).[3] The BER performance of 16DAPSK with DFDD using $L = 8$ is only about 2.6 dB inferior to that of 16 QAM with DE. The computer simulation results of 16DAPSK and 16DPSK are also plotted in Fig. 3. We see fairly good agreements between calculated and simulated BER performances.

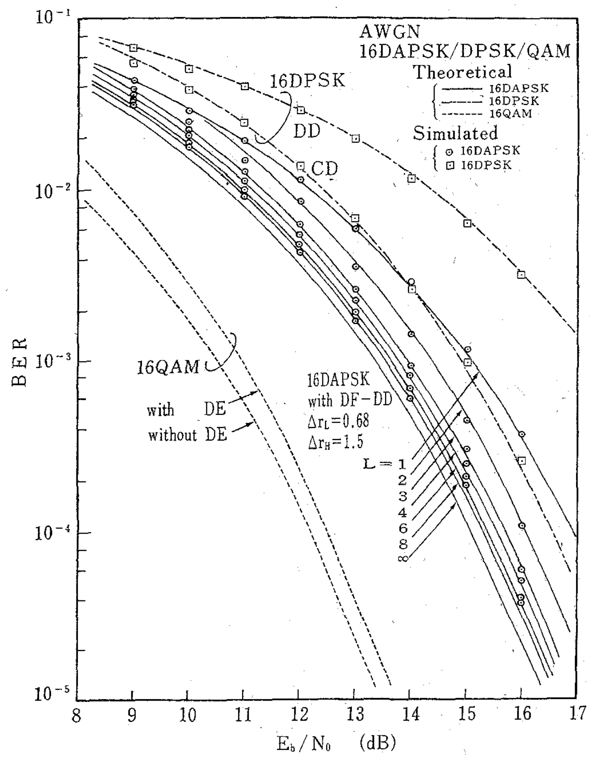

Fig. 3. BER performance.

V. Conclusion

Multiple-symbol DD with reference signal estimation based on the feedback of past detected symbols has been presented for 16DAPSK. An approximate BER analysis has shown that when eight detected symbols are fed back ($L = 8$), the required E_b/N_0 at BER $= 10^{-3}$ can be reduced by about 1.6 dB and the performance approaches that of 16 QAM with DE within about 2.6 dB. This was confirmed by computer simulation.

[3]The BER of 16 QAM is exactly given by $P_b = 3/8\,\text{erfc}\sqrt{0.4\rho_0} + 1/4\,\text{erfc}\sqrt{3.6\rho_0} - 1/8\,\text{erfc}\sqrt{10\rho_0}$ [2] and differential encoding/decoding increases the BER by 1.67 times [8].

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