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Error Rate Performance of Digital FM Mobile Radio with Postdetection Diversity

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Abstract—**An analysis of bit error rate (BER) in a binary digital FM system with postdetection diversity is presented. Expressions for the average BER due to additive white Gaussian noise (AWGN), random FM noise and delay-spread in the multipath channel are derived for reception using differential demodulation (DD), and frequency demodulation (FD) assuming independent fading signals. Calculated results for MSK show that the BER performance is strongly dependent on the rms-delay to bit duration ratio and that the delay-spectrum shape is of no importance when the receiver predetection filter BT product is optimized for the effect of AWGN.**

The effect of fading correlation on the diversity improvement is also analyzed for a two-branch case with multiplicative Rayleigh fading signals. Expressions for the average BER due to AWGN and random FM noise are derived. Calculated results are shown for the average BER due to random FM noise assuming a horizontally spaced antenna system at a mobile station. It is shown that the use of small antenna spacings leads to a diversity improvement greater than that obtainable for the case of independent AWGN.

I. Introduction

DIGITAL FM transmissions are a growing interest in the field of mobile radio [1], [2]. Since the radio channel is characterized by many different propagation paths with different time delays [3], [4], the frequency response of the channel over a bandwidth of the order of 100 kHz may not be constant and may vary according to the vehicle movement. Hence, for high-speed digital signal transmissions (higher than say, 64 kbits/s), the received signal suffers from frequency-selective fading; errors are caused by time-varying intersymbol interference (ISI) from delay-spread in the multipath channel [5], [6]. On the other hand, if a relatively low bit rate signal is transmitted, the received signal is subject to multiplicative fading, the major causes of errors then being additive white Gaussian noise (AWGN) and random FM noise produced by the variation in the received signal phase.

There are many possible implementations of diversity reception systems [7, ch. 6] but for mobile radio, postdetection diversity is attractive because the demodulated baseband signals can be used and the cophasing function, necessary in predetection combiners, is not required. Many previous investigations of postdetection diversity in digital FM signal transmissions have assumed multiplicative Rayleigh fading [8], [9]. For frequency-selective fading, however, the analysis available is limited to digital FM with differential demodulation (DD) using postdetection selection combining (SC) [10]. Only a double-spike delay-spectrum was treated and the effect of delay-spectrum shape was therefore not presented [10].

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Furthermore, none of the previous references, [8]–[10], took into account ISI from the receiver predetection filter.

This paper contains an analysis of the bit error rate (BER) performance of a digital FM receiver with DD and frequency demodulation (FD) using either M branch postdetection maximal-ratio combining (MRC), equal-gain combining (EGC), or SC assuming independent fading signals. The analysis takes into account ISI effects produced by both the receiver predetection filter and delay-spread in the multipath channel. General expressions for average BER are derived in Section III. In practical diversity systems, the fading signals received at the different antennas may be partially correlated and, therefore, Section IV investigates how the diversity improvement is affected by the correlation, assuming multiplicative Rayleigh fading signals. Finally, Section V illustrates the calculated results for MSK transmissions.

II. Postdetection Diversity

A. Description of Received Signal

The transmitted binary digital FM signal at an angular frequency ω_c can be represented as

$$u(t) = \mathrm{Re}\ \{s(t)e^{j\omega_c t}\} \tag{1}$$

where

$$s(t) = \sqrt{\frac{2E_b}{T}}\ e^{j\Phi_s(t)} \tag{2}$$

E_b is the signal energy per bit, T the bit duration, and $\Phi_s(t)$ the modulating phase, the time derivative of which is expressed as $\dot{\Phi}(t) = 2\pi\Delta f a_l$ for $lT < t \leqq (l+1)T$. a_l is the lth binary data symbol ($+1$ for mark, -1 for space) and Δf the frequency deviation.

Signal transmission between mobile and base stations takes place over multipath channels. The input to the kth branch demodulator of an M branch postdetection diversity receiver can be written as

$$e_k(t) = \mathrm{Re}\ \{z_k(t)e^{j\omega_c t}\} \tag{3}$$

where

$$\begin{aligned} z_k(t) &= z_{sk}(t) + z_{nk}(t) \\ &= \int_{-\infty}^{+\infty} S(f)T_k(f, t)H(f)e^{j2\pi ft}\ df \\ &\quad + \int_{-\infty}^{+\infty} N_k(f)H(f)e^{j2\pi ft}\ df. \end{aligned} \tag{4}$$

$S(f)$ and $N_k(f)$ are the spectra of $s(t)$ and of the kth branch band-limited AWGN. $H(f)$ (where $H(0) = 1$) presents the equivalent baseband characteristics of the receiver predetection filter and $T_k(f, t)$ the frequency response of the multipath channel for kth branch at time t. Introducing the complex impulse response $g_k(\tau, t)$ measured from the instant of

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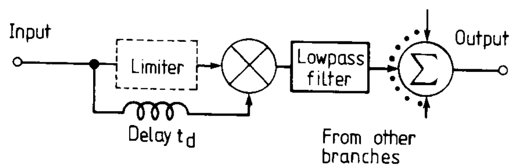

Fig. 1. Mathematical model of postdetection MRC combiner. Limiter is used for EGC.

application of a unit impulse at the transmitter at t, $T_k(f, t)$ can be represented as

$$T_k(f, t) = \int_{-\infty}^{+\infty} g_k(\tau, t) e^{-j2\pi f\tau} \, d\tau. \tag{5}$$

Assuming that the impulse response at τ is due to the sum of many independent impulses with the same time delay τ, each produced by a reflection from a different building, $g_k(\tau, t)$ and $T_k(f, t)$ become zero-mean complex Gaussian processes of t (without loss of generality, the variance of $T_k(f, t)$ at any frequency is assumed to be unity). Furthermore, we can assume that the AWGN in the different branches are independent. Hence, we have the following correlation relationships:

$$\langle g_k(\tau-\lambda, t-\mu)^* g_l(\tau, t)\rangle = \xi_{skl}(\tau, \mu)\delta(\lambda)$$

$$\frac{1}{2}\langle N_k(f)^* N_l(g)\rangle = N_0 \delta(f-g)\delta_{kl} \tag{6}$$

where N_0 is the single-sided noise power spectral density and $\xi_{skl}(\tau, \mu)$ the delay-time cross-correlation function of the multipath channel, $\delta(\cdot)$ the delta function and δ_{kl} the Dirac delta.

If the fading signals are independent, then $\xi_{skl}(\tau, \mu) = \xi_s(\tau, \mu)\delta_{kl}$. In particular, $\xi_s(\tau, 0)$ is called the delay-spectrum. The mean-delay and rms-delay are defined as

$$\tau_m = \int_{-\infty}^{+\infty} \tau \xi_s(\tau, 0)\, d\tau \Big/ \int_{-\infty}^{+\infty} \xi_s(\tau, 0)\, d\tau$$

$$\tau_0 = \sqrt{\int_{-\infty}^{+\infty} (\tau-\tau_m)^2 \xi_s(\tau, 0)\, d\tau \Big/ \int_{-\infty}^{+\infty} \xi_s(\tau, 0)\, d\tau}\,. \tag{7}$$

In this paper, we assume that $\tau_m = 0$ without loss of generality.

B. Diversity Combiner Output

Fig. 1 shows a model of a postdetection diversity receiver for reception of binary digital FM signals, including both FD and DD functions. Each branch input signal is multiplied by its delayed replica for MRC (the input signal is amplitude-limited before multiplication in case of EGC) with the time delay $t_d \approx T$ for DD, $t_d \ll T$ for FD, and $\omega_c t_d = (2n - 1/2)\pi$. In each branch demodulator, weighting for both diversity combining and demodulation are performed simultaneously. Since the weighting factor of each branch is $z_k^*(t - t_d)$ for MRC and $z_k^*(t - t_d)/|z_k(t)|$ for EGC, postdetection MRC and EGC are analogous to the predetection case.[1] It has been shown [8] that for MSK transmissions, two branch postdetection MRC (EGC) requires an average SNR only about 0.9 dB larger than that for predetection MRC (EGC) in order to obtain the same average BER due to AWGN, with independent multiplicative Rayleigh fading signals and no ISI from the receiver predetection filter. Postdetection SC is the simplest system and selects the demodulator output associated with the branch having the largest input signal envelope.

[1] Assuming multiplicative fading, i.e., $T_k(f, t) \approx T_k(0, t)$, and assuming that the AWGN in each branch has the same power, predetection MRC and EGC have the weighting factors $T_k^*(0, t)$ and $T_k^*(0, t)/|T_k(0, t)|$, respectively [11, chs. 10-5 and 10-6].

The decision as to which data symbol was sent is based on the polarity of the combiner output at the sampling instant t_s which is the end of the bit for DD and the center of the bit for FD. For simplicity in the BER analysis, the combiner output at t_s can be represented in a unified complex form as

$$Q = \begin{cases} \mathrm{Im}\,\{z_k z_k'^*\} \text{ if } |z_k| \text{ has the maximum value for SC} \\ \mathrm{Im}\left\{\sum_{k=1}^{M} \frac{z_k}{|z_k|} z_k'^*\right\} \quad \text{for EGC,} \\ \mathrm{Im}\left\{\sum_{k=1}^{M} z_k z_k'^*\right\} \quad \text{for MRC} \end{cases} \tag{8}$$

where

$$z_k = z_k(t_s),\; z_k' = \begin{cases} z_k(t_s - T) & \text{for DD} \\ -\dot{z}_k(t_s) & \text{for FD.} \end{cases} \tag{9}$$

III. Average BER with Independent Fading Signals

A. General BER Expression

In this section, we assume that the multipath channel statistical properties of different branches are independent and $\xi_{skl}(\tau, \mu) = \xi_s(\tau, \mu)\delta_{kl}$. Hence, z_k and z_k' of different branches are independent zero-mean complex Gaussian variables, as usually assumed in the case of multiplicative Rayleigh fading (no delay-spread). Therefore, we can apply the derivation technique [8] appropriate to average BER in multiplicative Rayleigh fading, assuming no ISI produced by the receiver predetection filter, which uses the fact that if all z_k are given, Q becomes a Gaussian variable, hence making the analysis easy.

With given z_k, z_k' becomes a complex Gaussian variable. If we let $\rho = \rho_c + j\rho_s = 1/2\,\langle z_k^* z_k'\rangle/\sigma\sigma'$, $\sigma^2 = 1/2\langle |z_k|^2\rangle$ and $\sigma'^2 = 1/2\langle |z_k'|^2\rangle$, the conditional mean and the conditional variance of z_k' are given by $(\sigma/\sigma')\rho^* z_k$ and $\sigma'^2(1 - |\rho|^2)$, respectively [8]. From this, the conditional BER with all z_k given, is found to be

$$p_e(R) = \frac{1}{2}\,\mathrm{erfc}\left\{a_0 \frac{\rho_s}{\sqrt{1-|\rho|^2}} \frac{R}{\sqrt{2}\sigma}\right\} \tag{10}$$

where $a_0 = \pm 1$ is the data symbol in the transmitted binary data sequence $\cdots a_{-2}, a_{-1}, a_0, a_1, a_2 \cdots$ to be detected without loss of generality and

$$R = \begin{cases} \max(R_1, R_2, \cdots R_M) & \text{for SC} \\ \frac{1}{\sqrt{M}} \sum_{k=1}^{M} R_k & \text{for EGC} \\ \sqrt{\sum_{k=1}^{M} R_k^2} & \text{for MRC,} \end{cases} \tag{11}$$

with $R_k = |z_k|$. Since all R_k are independent Rayleigh envelopes, the probability density function (pdf) of R can be expressed as

$$p(R)=\begin{cases} M\dfrac{R}{\sigma^2}\exp\left(-\dfrac{R^2}{2\sigma^2}\right)\left[1-\exp\left(-\dfrac{R^2}{2\sigma^2}\right)\right]^{M-1} & \text{for SC} \\ \dfrac{1}{(M-1)!}\dfrac{R}{\sigma^2}\left(\dfrac{R^2}{2\sigma^2}\right)^{M-1}\exp\left(-\dfrac{R^2}{2\sigma^2}\right) & \text{for MRC.} \end{cases} \tag{12}$$

For EGC, a good approximation can be obtained using the pdf of MRC, by replacing σ^2 with σ^2/ϵ_M as shown [7] for the pdf of SNR with the predetection EGC where

$$\epsilon_M = M/\{(2M-1)!!\}^{1/M}. \tag{13}$$

Therefore, the following general expression for the average BER can be obtained.

$$Pe=\int_0^\infty Pe(R)p(R)\,dR$$

$$=\begin{cases} \dfrac{1}{2}-\dfrac{1}{2}\displaystyle\sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\dfrac{a_0\rho_s}{\sqrt{\rho_s^2+k(1-|\rho|^2)}} & \text{for SC} \\ \dfrac{1}{2}-\dfrac{1}{2}\dfrac{a_0\rho_s}{\sqrt{\rho_s^2+\epsilon_M(1-|\rho|^2)}}\cdot\displaystyle\sum_{k=0}^{M-1}\dfrac{(2k-1)!!}{(2k)!!}\left\{\dfrac{(1-|\rho|^2)}{\rho_s^2+\epsilon_M(1-|\rho|^2)}\right\}^k & \text{for EGC} \\ \dfrac{1}{2}-\dfrac{1}{2}\dfrac{a_0\rho_s}{\sqrt{1-\rho_c^2}}\displaystyle\sum_{k=0}^{M-1}\dfrac{(2k-1)!!}{(2k)!!}\left\{\dfrac{1-|\rho|^2}{1-\rho_c^2}\right\}^k & \text{for MRC,} \end{cases} \tag{14}$$

$$\approx C_M\left(\frac{1-|\rho|^2}{2\rho_s^2}\right)^M \tag{15}$$

where

$$C_M=\frac{(2M-1)!!}{2}\ \text{for SC},\ \frac{1}{2}\frac{M^M}{M!}\ \text{for EGC, and}\ \frac{1}{2}\frac{(2M-1)!!}{M!}\ \text{for MRC.} \tag{16}$$

Equation (15) has been obtained using the approximate pdf of R for small R.

Taking into account the ISI effects produced by the delay-spread and by the receiver predetection filter, ρ can be obtained as

$$\rho=\begin{cases} \dfrac{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}\xi_s(\tau,T)d^*(t_s-T-\tau)d(t_s-\tau)\,d\tau+B_nT\xi_n(T)}{\sqrt{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}\xi_s(\tau,0)|d(t_s-\tau)|^2\,d\tau+B_nT}\sqrt{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}\xi_s(\tau,0)|d(t_s-T-\tau)|^2\,d\tau+B_nT}} & \text{for DD} \\ \dfrac{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}\{-\xi_s(\tau,0)d(t_s-\tau)\dot{d}^*(t_s-\tau)+\dot{\xi}_s(\tau,0)|d(t_s-\tau)|^2\}\,d\tau+B_nT\dot{\xi}_n(0)}{\sqrt{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}\xi_s(\tau,0)|d(t_s-\tau)|^2\,d\tau+B_nT}\ast\sqrt{\Gamma_0\displaystyle\int_{-\infty}^{+\infty}[\xi_s(\tau,0)|\dot{d}(t_s-\tau)|^2-2\,\mathrm{Im}\,\{\dot{\xi}_s(\tau,0)\}\,\mathrm{Im}\,\{\dot{d}^*(t_s-\tau)d(t_s-\tau)\}-\ddot{\xi}_s(\tau,0)|d(t_s-\tau)|^2]\,d\tau-B_nT\ddot{\xi}_n(0)}} & \text{for FD} \end{cases} \tag{17}$$

where

$$d(t)=\int_{-\infty}^{+\infty}S(f)H(f)e^{j2\pi ft}\,df\bigg/\sqrt{\frac{2E_b}{T}}=\int_{-\infty}^{+\infty}e^{j\Phi_s(\tau)}h(t-\tau)\,d\tau$$

$$h(t)=\int_{-\infty}^{+\infty}H(f)e^{j2\pi ft}\,df,\ \xi_n(t)=\int_{-\infty}^{+\infty}\frac{1}{B_n}|H(f)|^2e^{j2\pi ft}\,df,$$

$$B_n=\int_{-\infty}^{+\infty}|H(f)|^2\,df$$

$$\dot{\xi}_s(\tau,0)=\frac{\partial}{\partial\mu}\xi_s(\tau,\mu)|_{\mu=0},\ \ddot{\xi}_s(\tau,0)=\frac{\partial^2}{\partial\mu^2}\xi_s(\tau,\mu)|_{\mu=0}, \tag{18}$$

and $\Gamma_0 = E_b/N_0$ is the average signal energy per bit-to-noise power density ratio. In the above, $h(t)$ is the equivalent baseband impulse response of the receiver predetection filter, $\xi_n(t)$ the autocorrelation function of the band-limited AWGN and B_n the noise bandwidth.

B. Approximations

For a low bit rate transmission having a bandwidth narrower than the coherence bandwidth of the multipath channel, the received signal is subject to multiplicative Rayleigh fading. Errors are produced by the AWGN and by the random FM noise. As the transmission rate increases and the signal bandwidth approaches the coherence bandwidth, the received signal suffers from frequency-selective fading. The effect of random FM noise is negligible and most errors are produced by the AWGN and by the delay-spread. In the following, we derive simple approximate expressions, using (15), for the individual average BER's due to AWGN, random FM noise and delay-spread, separately. Perfect timing recovery at the receiver is assumed. Thus, the sampling instant is $t_s = T$ for DD and $T/2$ for FD.

1) Average BER's Due to AWGN and Random FM Noise: For multiplicative fading, the frequency response of the channel can be assumed to be constant over the bandwidth of interest, i.e., $T_k(f, t) \approx T_k(0, t)$, and therefore, $\xi_s(\tau, \mu) = \xi_s(\mu)\delta(\tau)$ where $\xi_s(\mu) = \langle T_k(0, t - \mu)^* T_k(0, t)\rangle$ is the complex fading autocorrelation function. The average BER, P_{e1}, due to AWGN can be obtained by letting $\xi_s(\tau, \mu) \approx \delta(\tau)$ in ρ (very slow fading assumption; $\xi_s(\mu) \approx 1$) and substituting into (15). For FD, both $\dot{\xi}_s(\tau, 0)$ and $\ddot{\xi}_s(\tau, 0) \approx 0$. We have

$$P_{e1} \approx \begin{cases} C_M \left[\dfrac{B_n T}{2\Gamma_0} \cdot \dfrac{|d(0)|^2 + |d(T)|^2 - 2\xi_n(T)\,\mathrm{Re}\,\{d^*(0)d(T)\}}{\mathrm{Im}^2\,\{d^*(0)d(T)\}}\right]^M & \text{for DD} \\ C_M \left[\dfrac{B_n T}{2\Gamma_0} \cdot \dfrac{|\dot{d}(T/2)|^2 - \ddot{\xi}_n(0)|d(T/2)|^2\}}{\mathrm{Im}^2\,\{\dot{d}^*(T/2)d(T/2)\}}\right]^M & \text{for FD.} \end{cases} \tag{19}$$

In the above, we assumed that $H(f)$ is symmetrical with respect to $f = 0$, hence $\xi_n(T)$ is real and $\dot{\xi}_n(0) = 0$. On the other hand, by letting $\Gamma_0 \to \infty$ in ρ, the average BER, P_{e2}, due to random FM noise can be obtained as

$$P_{e2} \approx \begin{cases} C_M \left[\dfrac{1 - |\xi_s(T)|^2}{2} \cdot \dfrac{|d^*(0)d(T)|^2}{\mathrm{Im}^2\,\{d^*(0)d(T)\}}\right]^M & \text{for DD} \\ C_M \left[-\dfrac{\{\ddot{\xi}_s(0) + |\dot{\xi}_s(0)|^2\}|d(T/2)|^4 + 2\,\mathrm{Im}\,\{\dot{\xi}_s(0)\}\,\mathrm{Im}\,\{\dot{d}^*(T/2)d(T/2)\}|d(T/2)|^2}{2\,\mathrm{Im}^2\,\{\dot{d}^*(T/2)d(T/2)\}}\right]^M & \text{for FD.} \end{cases} \tag{20}$$

2) Average BER Due to Delay-Spread: For frequency-selective fading, the time variation in multipath channel impulse response can be assumed to be negligible over T seconds and thus $\xi_s(\tau, \mu) \approx \xi_s(\tau, 0)$. Letting $\Gamma_0 \to \infty$ in ρ, the average BER, P_{e3}, due to delay-spread can be obtained as

$$P_{e3} \approx \begin{cases} C_M \left[\dfrac{\int_{-\infty}^{+\infty} \xi_s(\tau, 0)|d(-\tau)|^2\,d\tau \int_{-\infty}^{+\infty} \xi_s(\tau, 0)|d(T-\tau)|^2\,d\tau - \left|\int_{-\infty}^{+\infty} \xi_s(\tau, 0)d^*(-\tau)d(T-\tau)\,d\tau\right|^2}{2\,\mathrm{Im}^2\left\{\int_{-\infty}^{+\infty} \xi_s(\tau, 0)d^*(-\tau)d(T-\tau)\,d\tau\right\}}\right]^M & \text{for DD} \\ C_M \left[\dfrac{\int_{-\infty}^{+\infty} \xi_s(\tau, 0)|d(T/2-\tau)|^2\,d\tau \int_{-\infty}^{+\infty} \xi_s(\tau, 0)|\dot{d}(T/2-\tau)|^2\,d\tau - \left|\int_{-\infty}^{+\infty} \xi_s(\tau, 0)d(T/2-\tau)\dot{d}^*(T/2-\tau)\,d\tau\right|^2}{2\,\mathrm{Im}^2\left\{\int_{-\infty}^{+\infty} \xi_s(\tau, 0)\dot{d}^*(T/2-\tau)d(T/2-\tau)\,d\tau\right\}}\right]^M & \\ \text{for FD.} \end{cases} \tag{21}$$

Expanding $d(\cdot)$ and $\dot{d}(\cdot)$ in a power series of τ, we obtain a further approximation for small delay-spread

$$P_{e3} \approx \begin{cases} C_M \left[\dfrac{|d(0)\dot{d}(T) - \dot{d}(0)d(T)|}{\sqrt{2}\,\mathrm{Im}\,\{d^*(0)d(T)\}}\,\tau_0\right]^{2M} & \text{for DD} \\ C_M \left[\dfrac{|\dot{d}^2(T/2) - d(T/2)\ddot{d}(T/2)|}{\sqrt{2}\,\mathrm{Im}\,\{\dot{d}^*(T/2)d(T/2)\}}\,\tau_0\right]^{2M} & \text{for FD,} \end{cases} \tag{22}$$

which shows that the shape of the delay-spectrum is not important for $\tau_0/T \ll 1$.

IV. Average BER with Correlated Multiplicative Rayleigh Fading Signals

A two-branch case ($M = 2$) is considered. In order to see how the fading correlation affects the diversity improvements, we assume the receiver predetection filter to have a sufficiently wide bandwidth so that the ISI effect is negligible. Therefore, $z_{sk}(t) \approx T_k(0, t)s(t)$ in (4) and $\xi_{skl}(\tau, \mu) = \xi_{skl}(\mu)\delta(\tau)$ for $k, l = 1, 2$ where $\xi_{skl}(\mu) = \langle T_k(0, t_s - \mu)^* T_l(0, t_s)\rangle$ is the complex fading cross-correlation function (note that $\xi_{skk}(\mu)$ is expressed as $\xi_s(\mu)$ in Section III).

A. General BER Expression

It is clear from (8) that, with given z_1 and z_2, Q still remains a Gaussian variable even with correlated fading signals. Let $\langle Q\rangle$ be the conditional mean and σ_Q^2 be the conditional variance of Q with z_1 and z_2 being given. The conditional BER can then be expressed as

$$p_e = \frac{1}{2}\,\mathrm{erfc}\left[a_0 \frac{\langle Q\rangle}{\sqrt{2}\sigma_Q}\right]. \tag{23}$$

Applying the matrix theory described in [11, pp. 495–496], $\langle Q\rangle$ and σ_Q^2 are determined as follows. Let z' and z be the column matrices of z_k' and z_k, respectively, and $\mathbf{\Omega}$ be the partitioned column matrix of z' and z. Then, the covariance matrix of $\mathbf{\Omega}$ can be represented as

$$\frac{1}{2}\langle \mathbf{\Omega}^* \mathbf{\Omega}^T\rangle = \begin{pmatrix} a & c \\ c^{T*} & g \end{pmatrix}, \tag{24}$$

with the components of $\boldsymbol{a}$, $\boldsymbol{g}$, and $\boldsymbol{c}$ given by

$$a_{kl}=g_{kl}=\sigma^2\xi_{kl}(0),\ c_{kl}=\sigma^2\xi_{kl}(T), \quad \text{for DD}$$

$$\left.\begin{array}{l} a_{kl}=-\sigma^2\partial^2/\partial\mu^2\xi_{kl}(\mu)|_{\mu=0}=-\sigma^2\ddot{\xi}_{kl}(0),\\ g_{kl}=\sigma^2\xi_{kl}(0),\ c_{kl}=\sigma^2\partial/\partial\mu\xi_{kl}(\mu)|_{\mu=0}=\sigma^2\dot{\xi}_{kl}(0), \end{array}\right\} \quad \text{for FD} \tag{25}$$

where

$$\xi_{kl}(\mu)=\frac{\langle z_k(t_s-\mu)^*z_l(t_s)\rangle}{2\sigma^2} = \frac{\Gamma_0\xi_{skl}(\mu)e^{j\Delta\Phi_s(\mu)}+B_nT\xi_n(\mu)\delta_{kl}}{\Gamma_0+B_nT}, \tag{26}$$

with $\Delta\Phi_s(\mu) = \Phi_s(t_s) - \Phi_s(t_s-\mu)$. The conditional mean value $\boldsymbol{K}$ and the conditional covariance matrix $\boldsymbol{\Lambda}$ of $\boldsymbol{z}'$, with given $\boldsymbol{z}$, can be obtained from $\boldsymbol{K} = \boldsymbol{\kappa}^*\boldsymbol{z} = (\boldsymbol{cg}^{-1})^*\boldsymbol{z}$ and $\boldsymbol{\Lambda} = \boldsymbol{a} - \boldsymbol{cg}^{-1}\boldsymbol{c}^{T*}$, respectively [11]. Using the components κ_{kl} and λ_{kl} of $\boldsymbol{\kappa}$ and $\boldsymbol{\Lambda}$ and introducing the variable transformations in (8); $|z_1| = R\cos\Psi$, $|z_2| = R\sin\Psi$, and $\arg(z_1^*z_2) = \theta_{12}$ where $R \geqq 0$, $\pi/2 \geqq \Psi \geqq 0$ and $\pi \geqq \theta_{12} > -\pi$, we have

$$\frac{\langle Q\rangle}{\sqrt{2}\sigma_Q}=\begin{cases} \dfrac{R}{\sqrt{2}}\dfrac{\operatorname{Im}\left\{\kappa_{11}\cos^2\Psi+\kappa_{22}\sin^2\Psi+\frac{1}{2}\sin(2\Psi)(\kappa_{12}e^{-j\theta_{12}}+\kappa_{21}e^{j\theta_{12}})\right\}}{\sqrt{\lambda_{11}\cos^2\Psi+\lambda_{22}\sin^2\Psi+\sin(2\Psi)\operatorname{Re}\{\lambda_{12}e^{-j\theta_{12}}\}}}, & \text{for MRC}\\[2ex] \dfrac{R}{\sqrt{2}}\dfrac{\operatorname{Im}\{\kappa_{11}\cos\Psi+\kappa_{22}\sin\Psi+\kappa_{12}\sin\Psi e^{-j\theta_{12}}+\kappa_{21}\cos\Psi e^{j\theta_{12}}\}}{\sqrt{\lambda_{11}+\lambda_{22}+2\operatorname{Re}\{\lambda_{12}e^{-j\theta_{12}}\}}}, & \text{for EGC}\\[2ex] \left.\begin{array}{ll} \dfrac{R}{\sqrt{2}}\dfrac{\operatorname{Im}\{\kappa_{11}\cos\Psi+\kappa_{12}\sin\Psi e^{-j\theta_{12}}\}}{\sqrt{\lambda_{11}}}, & \frac{\pi}{4}>\Psi\geqq 0\\[2ex] \dfrac{R}{\sqrt{2}}\dfrac{\operatorname{Im}\{\kappa_{22}\sin\Psi+\kappa_{21}\cos\Psi e^{j\theta_{12}}\}}{\sqrt{\lambda_{22}}}, & \frac{\pi}{2}\geqq\Psi\geqq\frac{\pi}{4} \end{array}\right\} & \text{for SC.} \end{cases} \tag{27}$$

Since z_1 and z_2 are Gaussian variables with cross-correlation $\xi_{12}(0)$, the joint pdf $p(R, \Psi, \theta_{12})$ can be obtained as

$$p(R,\Psi,\theta_{12})=\frac{R^3\sin(2\Psi)}{4\pi\sigma^4(1-|\xi_{12}(0)|^2)} \cdot \exp\left[-R^2\frac{1-\sin(2\Psi)\operatorname{Re}(\xi_{12}(0)^*e^{j\theta_{12}})}{2\sigma^2(1-|\xi_{12}(0)|^2)}\right]. \tag{28}$$

Putting

$$\frac{\langle Q\rangle}{\sqrt{2}\sigma_Q}=\frac{R}{\sqrt{2}\sigma\sqrt{1-|\xi_{12}(0)|^2}}\alpha, \tag{29}$$

we can obtain the following general expression for BER:

$$P_e=\int_{-\pi}^{\pi}\int_0^{\pi/2}\int_0^{\infty}p_e\cdot p(R,\Psi,\theta_{12})\,dR\,d\Psi\,d\theta_{12} = \frac{1-|\xi_{12}(0)|^2}{4}\int_{-\pi}^{\pi}\int_0^{\pi/2}\frac{\sin(2\Psi)}{2\pi} \cdot \frac{2+a_0\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2}}}{(\alpha^2+\beta^2)^2\left(1+a_0\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)^2}\,d\Psi\,d\theta_{12} \tag{30}$$

where

$$\beta=\sqrt{1-\sin(2\Psi)\operatorname{Re}(\xi_{12}(0)^*e^{j\theta_{12}})}. \tag{31}$$

C. Approximations

In order to see the fading correlation effects clearly, approximate expressions for the average BER's due to AWGN and random FM noise are derived, separately. We assume that the power spectra of $T_1(0, t)$ and $T_2(0, t)$ are identical and are symmetrical with respect to the dc component. Furthermore, we assume the AWGN to have a symmetrical power spectrum.

1) Average BER Due to AWGN: Under slow fading conditions, the complex envelope of the received signal is almost constant over one bit duration, i.e., $\xi_{skl}(\mu) \approx \xi_{skl}(0)$. It can be shown that when $\xi_{s12}(0)$ is not close to unity

$$\boldsymbol{\kappa}\approx e^{j\Delta\Phi_s}\boldsymbol{I},\ \boldsymbol{\Lambda}\approx\sigma^2\frac{2}{\Gamma_0/B_nT}\boldsymbol{I}, \quad \text{for DD}$$
$$\boldsymbol{\kappa}\approx j\dot{\Phi}_s\boldsymbol{I},\ \boldsymbol{\Lambda}\approx\sigma^2\frac{\dot{\Phi}_s^2-\ddot{\xi}_n(0)}{\Gamma_0/B_nT}\boldsymbol{I}, \quad \text{for FD} \tag{32}$$

for a large Γ_0 where $\boldsymbol{I}$ is the identity matrix, $\Delta\Phi_s\ (=\Delta\Phi_s(t_s)) = a_0 2\pi\Delta fT$ and $\dot{\Phi}_s\ (=\dot{\Phi}_s(t_s)) = a_0 2\pi\Delta f$. For DD, we have assumed that $\xi_n(T) \approx 0$.

It can be shown using (32) that $|\alpha| \gg \beta$ since β is always less than unity. Hence, the double integration in (30) can be performed. Using the fact that $\xi_{12}(0) \approx \xi_{s12}(0)$ in (26), the approximate expression for P_{e1} is

$$P_{e1}=\begin{cases} C_{M=2}\left[\dfrac{1}{\sin^2(2\pi\Delta fT)}\dfrac{B_nT}{\Gamma_0\sqrt{1-|\xi_{s12}(0)|^2}}\right]^2 & \text{for DD}\\[2ex] \dfrac{C_{M=2}}{4}\left[\dfrac{(2\pi\Delta f)^2-\ddot{\xi}_n(0)}{(2\pi\Delta f)^2}\dfrac{B_nT}{\Gamma_0\sqrt{1-|\xi_{s12}(0)|^2}}\right]^2 & \text{for FD.} \end{cases} \tag{33}$$

Since $\xi_{s12}(0)$ is the fading signal cross-correlation, the correlation effect is identical with that for predetection diversity.

2) Average BER Due to Random FM Noise: When $\Gamma_0 \to \infty$ in (26), $\xi_{kl}(\mu) \approx \xi_{skl}(\mu)\exp[j\Delta\Phi_s(\mu)]$. Since we are assuming a symmetrical fading power spectrum, $\xi_{s11}(T) = \xi_{s22}(T) =$ real and $\dot{\xi}_{s11}(0) = \dot{\xi}_{s22}(0) = 0$. The maximum Doppler frequency of Rayleigh fading can be assumed to be much smaller than the bit rate, so that $\xi_{skl}(T) \approx \xi_{skl}(0) + T\dot{\xi}_{skl}(0) + (T^2/2)\ddot{\xi}_{skl}(0)$ for DD. $\boldsymbol{\kappa}$ and $\boldsymbol{\Lambda}$ can be obtained as

$$\begin{cases} \boldsymbol{\kappa} \approx e^{j\Delta\Phi_s} \begin{bmatrix} 1 - T\dfrac{\xi_{s12}^*\dot{\xi}_{s12}}{1-|\xi_{s12}|^2} & T\dfrac{\dot{\xi}_{s12}}{1-|\xi_{s12}|^2} \\ -T\dfrac{\dot{\xi}_{s12}^*}{1-|\xi_{s12}|^2} & 1 + T\dfrac{\xi_{s12}\dot{\xi}_{s12}^*}{1-|\xi_{s12}|^2} \end{bmatrix} \\ \boldsymbol{\Lambda} \approx -\left(\dfrac{E_b}{T}\right) T^2 \begin{bmatrix} \ddot{\xi}_{s11} + \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2} & \ddot{\xi}_{s12} + \dfrac{\xi_{s12}^*(\dot{\xi}_{s12})^2}{1-|\xi_{s12}|^2} \\ \ddot{\xi}_{s12}^* + \dfrac{\xi_{s12}(\dot{\xi}_{s12}^*)^2}{1-|\xi_{s12}|^2} & \ddot{\xi}_{s11} + \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2} \end{bmatrix}, \quad \text{for DD} \end{cases} \tag{34}$$

$$\begin{cases} \boldsymbol{\kappa} = \begin{bmatrix} j\dot{\Phi}_s - \dfrac{\xi_{s12}^*\dot{\xi}_{s12}}{1-|\xi_{s12}|^2} & \dfrac{\dot{\xi}_{s12}}{1-|\xi_{s12}|^2} \\ -\dfrac{\dot{\xi}_{s12}^*}{1-|\xi_{s12}|^2} & j\dot{\Phi}_s + \dfrac{\xi_{s12}\dot{\xi}_{s12}^*}{1-|\xi_{s12}|^2} \end{bmatrix} \\ \boldsymbol{\Lambda} = -\dfrac{E_b}{T} \begin{bmatrix} \ddot{\xi}_{s11} + \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2} & \ddot{\xi}_{s12} + \dfrac{\xi_{s12}^*(\dot{\xi}_{s12})^2}{1-|\xi_{s12}|^2} \\ \ddot{\xi}_{s12}^* + \dfrac{\xi_{s12}(\dot{\xi}_{s12}^*)^2}{1-|\xi_{s12}|^2} & \ddot{\xi}_{s11} + \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2} \end{bmatrix}, \quad \text{for FD} \end{cases} \tag{35}$$

where $\xi_{s12} = \xi_{s12}(0)$, $\dot{\xi}_{s12} = \dot{\xi}_{s12}(0)$, $\ddot{\xi}_{s11} = \ddot{\xi}_{s11}(0)$ and $\ddot{\xi}_{s12} = \ddot{\xi}_{s12}(0)$. If ξ_{s12} is not close to unity, the diagonal components of $\boldsymbol{\kappa}$ are predominant. Using the fact that $|\alpha| \gg \beta$, the integration in (30) can be performed. The approximate expression for P_{e_2} is

$$Pe_2 \approx \begin{cases} \dfrac{3}{16}(vT)^4 \dfrac{\left(-\ddot{\xi}_{s11} - \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2}\right)^2 + \dfrac{1}{3}\left|\ddot{\xi}_{s12} + \dfrac{\xi_{s12}^*(\dot{\xi}_{s12})^2}{1-|\xi_{s12}|^2}\right|^2}{1-|\xi_{s12}|^2}, & \text{for MRC} \\ \dfrac{1}{4}(vT)^4 \dfrac{\left(-\ddot{\xi}_{s11} - \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2}\right)^2 + \dfrac{1}{2}\left|\ddot{\xi}_{s12} + \dfrac{\xi_{s12}^*(\dot{\xi}_{s12})^2}{1-|\xi_{s12}|^2}\right|^2}{1-|\xi_{s12}|^2}, & \text{for EGC} \\ \dfrac{3}{8}(vT)^4 \dfrac{\left(-\ddot{\xi}_{s11} - \dfrac{|\dot{\xi}_{s12}|^2}{1-|\xi_{s12}|^2}\right)^2}{1-|\xi_{s12}|^2}, & \text{for SC} \end{cases} \tag{36}$$

where $v = 1/\sin(2\pi\Delta f T)$ for DD and $1/(2\pi\Delta f T)$ for FD.

V. Numerical Results for MSK

A. Independent Fading Case

We assume the receiver predetection filter to have a Gaussian bandpass characteristic with a 3 dB bandwidth of B. $h(t)$ and $\xi_n(t)$ are given by

$$h(t) = \frac{\beta}{\sqrt{\pi}T}\exp\left[-\beta^2\left(\frac{t}{T}\right)^2\right],$$

$$\xi_n(t) = \exp\left[-\frac{\beta^2}{2}\left(\frac{t}{T}\right)^2\right] \tag{37}$$

where

$$\beta = \frac{\pi\,\text{BT}}{\sqrt{2\ln 2}} = \sqrt{2}\pi B_n T. \tag{38}$$

For a receiver BT product >0.5, it is sufficient to take into account two adjacent bits (one on each side) for the calculation of $d(t)$ [12]. However, in this paper we use four adjacent bits (if the rms-delay to bit duration ratio is small, only two adjacent bits need be used). Thus, $d(t)$ of MSK ($2\Delta f T = 0.5$) is given by

$$\begin{aligned} d(t) = & -a_{-2}a_{-1}C_0(t+2T) + C_0(t) - a_0a_1C_0(t-2T) \\ & + j\{-a_{-1}C_0(t+T) + a_0C_0(t-T) - a_0a_1a_2C_0(t-3T)\} \end{aligned} \tag{39}$$

where

$$C_0(t) = \frac{\beta}{\sqrt{\pi}}\int_{-1}^{1}\cos\left(\frac{\pi}{2}x\right)\exp\left[-\beta^2\left(\frac{t}{T}-x\right)^2\right]dx. \tag{40}$$

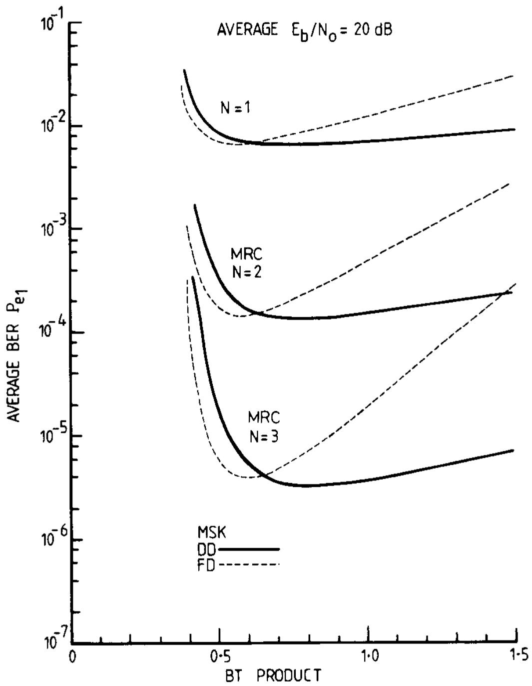

Fig. 2. The effect of BT product on the average BER due to AWGN with postdetection MRC.

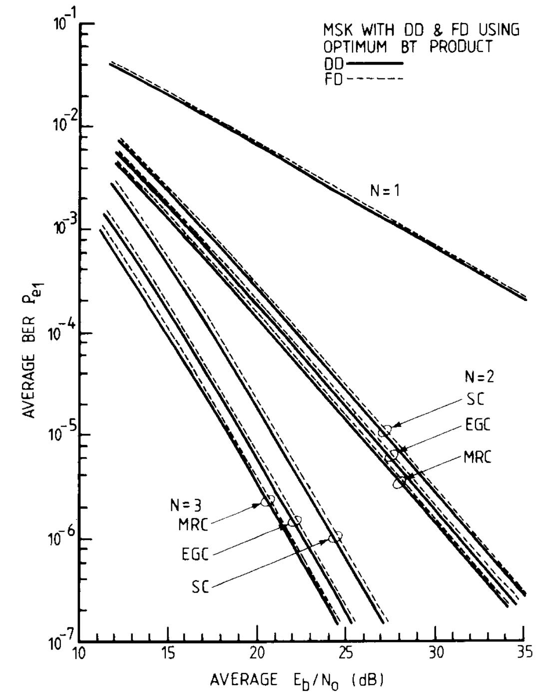

Fig. 3. The average BER due to AWGN for DD and FD reception (the optimum BT product).

TABLE I
OPTIMUM BT PRODUCT FOR DD AND FD

Diversity Order	DD	FD
N = 1	0.73	0.55
2	0.78	0.57
3	0.82	0.59
4	0.85	0.61

The overall average BER can be obtained by averaging the BER calculated using (14) or (19)–(22) over all equally likely data sequences (16 four-adjacent-bit patterns). In the following, the exact results calculated using (14) are presented for individual average BER's due to AWGN, random FM noise and delay-spread.

1) Average BER Due to AWGN: For the evaluation of average BER due to AWGN, we let $\xi_s(\tau, \mu) \approx \delta(\tau)$. The effect of the receiver BT product is shown in Fig. 2 for an average $E_b/N_0 = 20$ dB. The BER performance with DD reception is much less sensitive to BT product than with FD reception. DD reception is superior to FD reception for BT products larger than about 0.7.[2] For each demodulation scheme, an optimum value of BT product (the optimum BT product) will exist, which is a function of the diversity order and the average E_b/N_0. However, for average E_b/N_0 larger than 20 dB, it is almost constant and is listed in Table I. Fig. 3 shows the average BER performances for DD and FD reception using the optimum BT product. Comparison of the three combiners shows that the MRC achieves the greatest improvement, and the BER's with EGC and SC are 4/3 and 2 times as large as that with MRC for $M = 2$.

2) Average BER Due to Random FM Noise: For the evaluation of average BER due to random FM noise, we let $\xi_s(\tau, \mu) \approx \xi_s(\mu)\delta(\tau)$. Assuming that multipath waves having the same amplitude and independent random phases arrive at the mobile station from all directions with equal probability, $\xi_s(\mu) = J_0(2\pi f_D \mu)$ [14] where $J_0(\cdot)$ is the Bessel function and f_D is the maximum Doppler frequency (vehicle speed/carrier wavelength). The effect of BT product is shown in Fig. 4 for $f_D T = 0.01$. As the BT product decreases the average BER

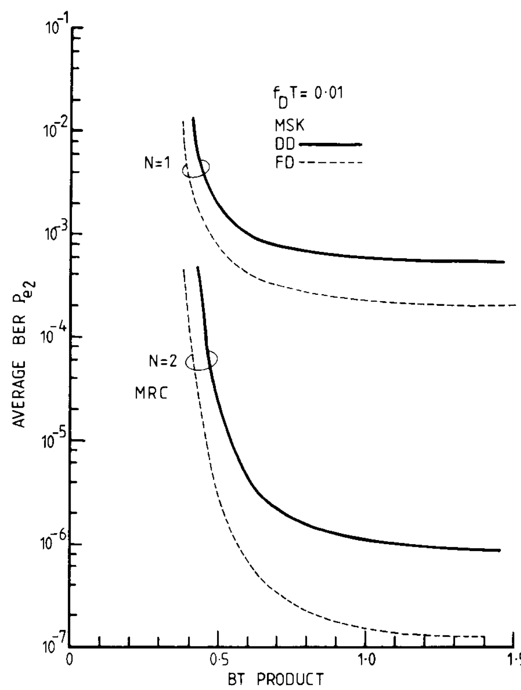

Fig. 4. The effect of BT product on the average BER due to random FM noise with postdetection MRC for $f_D T = 0.01$.

[2] Simon and Wang [13] have shown that in the no fading condition the BER performance of MSK with FD reception is identical to that of DD reception for $1 < BT < 3$. However, their results are valid only when the demodulator input SNR is larger than about 3 dB since Rice's click model is used. Note that in the fading condition, almost all errors are produced for the instantaneous SNR less than about 3 dB [11, p. 409].

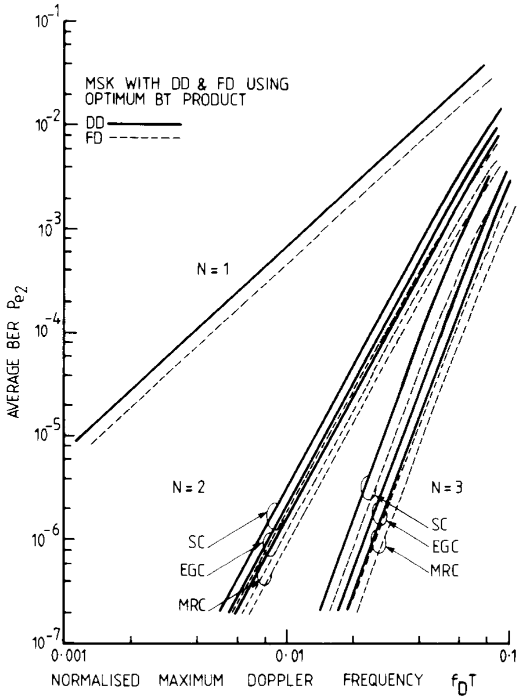

Fig. 5. The average BER due to random FM noise for DD and FD reception (the optimum BT product).

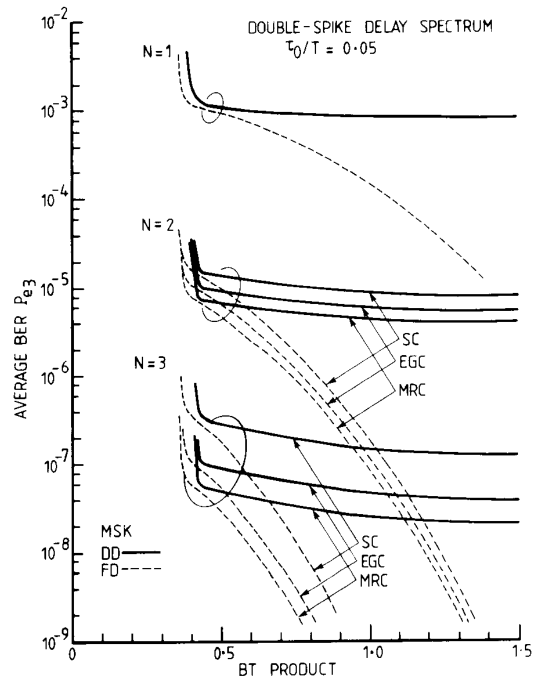

Fig. 6. The effect of BT product on the average BER due to random FM noise for a double-spike delay-spectrum with $\tau_0/T = 0.05$.

increases because of the ISI from the receiver predetection filter. When the same BT product is used for both demodulation schemes, then FD reception exhibits superior performance. However, when the optimum BT product is used for each demodulation scheme, FD reception is slightly more resistant to random FM noise than DD reception. The average BER using the optimum BT product versus $f_D T$ is shown in Fig. 5.

3) Average BER Due to Delay-Spread: For the evaluation of average BER due to delay-spread, we let $\xi_s(\tau, \mu) \approx \xi_s(\tau, 0)$. According to measurements [3], [4], the delay-spectrum can be approximated by a one-sided exponential, sometimes with several spikes. In order to examine the effects of the spectrum shape, we assume a one-sided exponential spectrum and a double-spike spectrum, and treat them, separately. We also consider a Gaussian spectrum. Therefore, the delay-spectra used for calculation are

$$\xi_s(\tau, 0) = \begin{cases} \frac{1}{\tau_0} \exp\left[-(\tau+\tau_0)/\tau_0\right], \\ \quad \tau \geqq -\tau_0 \text{ (one-sided exponential)} \\ \frac{1}{2}\delta(\tau-\tau_0) + \frac{1}{2}\delta(\tau+\tau_0) \\ \quad \text{(double-spike)}, \\ \frac{1}{\sqrt{2\pi}\tau_0} \exp\left[-\tau^2/2\tau_0^2\right] \\ \quad \text{(Gaussian)} \end{cases} \tag{41}$$

with zero mean-delay ($\tau_m = 0$).

The effect of BT product is shown in Fig. 6 for a double-spike delay-spectrum with $\tau_0/T = 0.05$. The BER performance with DD reception depends loosely on BT product. FD reception can provide much better BER performance than DD reception for a large BT product. This is because ISI caused by delay-spread is predominant at both ends of the bit and hence is smallest at the center of the bit. The average BER using the optimum BT product versus τ_0/T is shown in Fig. 7 for a double-spike delay-spectrum. The average BER's for two

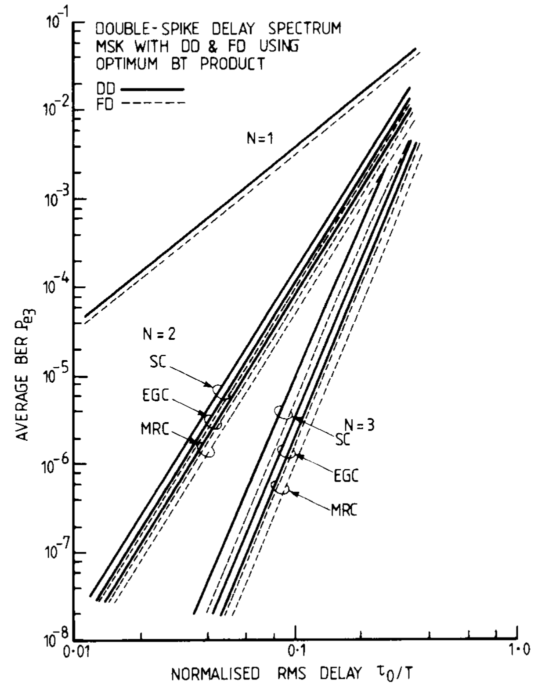

Fig. 7. The average BER due to delay-spread for a double-spike delay-spectrum (the optimum BT product).

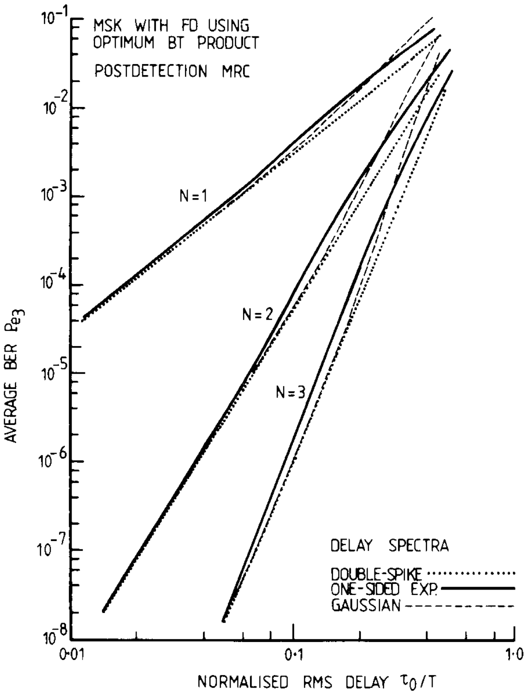

Fig. 8. The effect of delay-spectrum for FD reception with postdetection MRC (the optimum BT product).

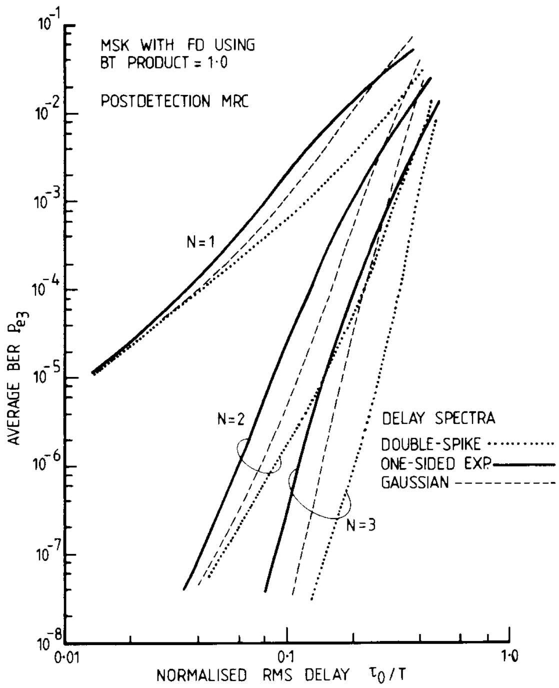

Fig. 9. The effect of delay-spectrum for FD reception with postdetection MRC (BT product = 1.0).

other types of delay-spectrum are also calculated and compared in Fig. 8 to FD reception using MRC (for DD reception, similar results are obtained). It can be seen that when the optimum BT product is used, BER performance is strongly dependent on τ_0/T and the spectrum shape has a negligible impact on BER performance. However, the delay-spectrum shape has a profound influence on FD reception (not on DD reception) for BT products larger than the optimum as shown in Fig. 9. Of the three types of delay-spectrum considered, the double-spike has the least influence because the two other delay-spectra have components at delays larger than τ_0. The approximate BER performances calculated using (22) are also shown in Fig. 10 for FD reception, along with the exact results calculated using (14) assuming the double-spike delay-spectrum. It can be seen that the approximate BER's agree quite well with the exact results for $\tau_0/T < 0.1$.

B. Correlated Fading Case

The effect of fading correlation on the average BER due to random FM noise is calculated assuming a space diversity system using two horizontally spaced antennas with omnidirectional radiation patterns at a mobile station. The antenna arrangement is shown in Fig. 11. In the figure, d is the antenna spacing, η is the angle between the antenna axis and the direction of vehicle motion. Since we are assuming that many incoming multipath waves having the same amplitude and independent random phases arrive from all directions with equal probability, $\xi_{s11}(\mu)(=\xi_s(\mu)) = J_0(2\pi f_D \mu)$ and $\xi_{s12}(\mu) = J_0(2\pi\sqrt{(f_D\mu)^2 + (d/\lambda)^2 - 2(f_D\mu)(d/\lambda)\cos\eta}$, respectively, where λ is the carrier wavelength. The exact results for the average BER of MSK ($2\Delta f T = 0.5$) due to random FM noise for $f_D = 0.01$ are obtained by using (34) and (35) for κ and Λ and performing the double integration in (30). The BER performance of DD is shown in Fig. 12 for MRC. Dashed lines show the approximate results obtained using (36). Fairly good agreements are obtained if the antenna spacings are not too small. When $d \to 0$, the two fading signal envelopes

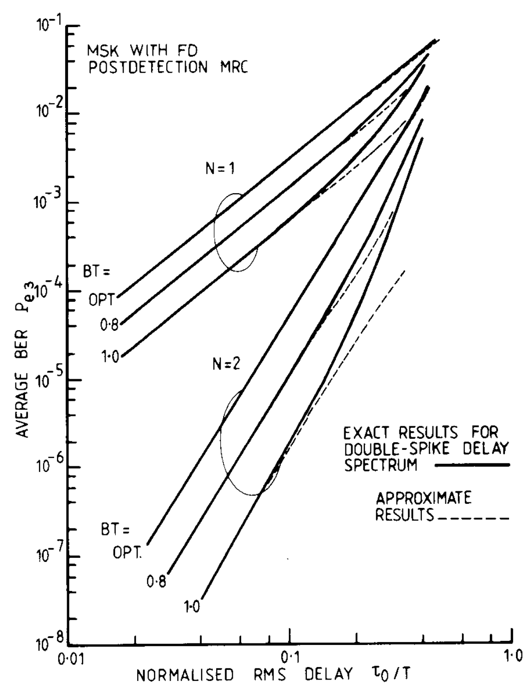

Fig. 10. Approximate average BER due to delay-spread with postdetection MRC for FD reception. Exact results for a double-spike delay-spectrum are also shown for a comparison.

become the same, and hence, from (8), the average BER value is found to approach that of no diversity reception, which is 4.9×10^{-4} for DD [15, eq. (59)]. It can be seen that the use of small antenna spacings leads to further improvements in the average BER due to random FM noise over that obtainable in the case of independent envelope fading ($d \to \infty$). The

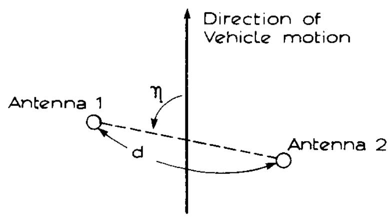

Fig. 11. Antenna arrangement at a mobile station.

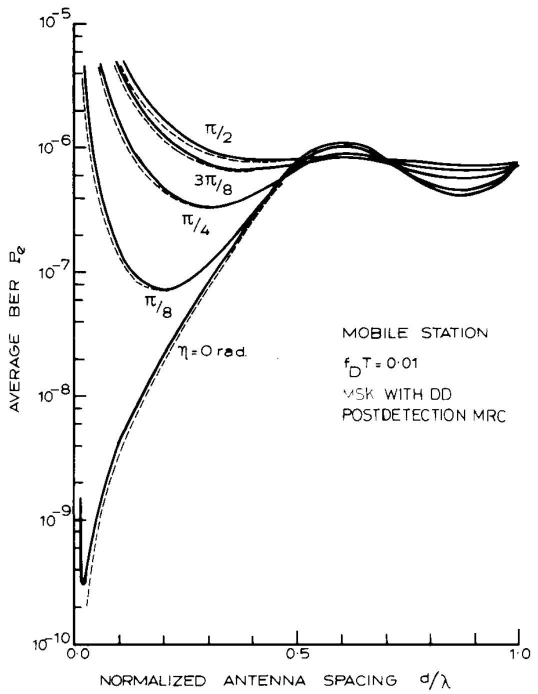

Fig. 12. The effect of antenna spacing on the average BER due to random FM noise with postdetection MRC for DD reception. Dashed lines show approximate results.

"tradeoff" of course is the corresponding increase in envelope correlation and the resultant effects on the receiving system.

VI. Conclusions

Expressions have been derived for the average BER of a binary digital FM system with DD and FD reception using postdetection SC, EGC, and MRC, taking into account ISI effects produced by the delay-spread of the multipath channel and by the receiver predetection filter. Calculated results have been presented for MSK transmission. FD reception has been found to be more resistant to both random FM noise and delay-spread than DD reception. When the optimum BT product is used for each demodulation scheme, the shape of the delay-spectrum is of no importance and BER performance is strongly dependent on the normalized rms-delay τ_0.

The effects of fading correlation on the average BER have also been analyzed for the two branch case in a multiplicative Rayleigh fading signal environment. Calculated results have shown that a substantial reduction in the average BER due to random FM noise can be obtained if space diversity with two horizontally close-spaced antennas parallel with the direction of vehicle motion is employed at a mobile station.

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