### FFT VELOCITY MEASUREMENT OF MULTIPLE LEAKY WAVES BY LINE-FOCUS-BEAM ACOUSTIC MICROSCOPE

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#### Abstract

This paper describes a novel velocity measurement method of line-focus-beam acoustic microscope using an FFT analysis for V(z) curves associated with multiple leaky waves at the water/solid-sample boundary. The multiple leaky waves existing on the boundary essentially give rise to deformation in dip shape and intervals in V(z) curves. Deformed V(z)curves are represented as a simple superpositional model of elemental V(z) curves with different dip intervals corresponding to each leaky mode. In order to determine the velocities for each mode separately, an FFT method is introduced for the V(z) curve analysis. Experiments are demonstrated for a (111)-Ge sample using a sapphire line-focus-beam lens at 200 MHz and two mode spectra are separately measured corresponding to two velocities for a leaky SAW and a leaky pseudo-SAW.

### 1. Introduction

Since the line-focus-beam acoustic microscope [1], [2] has been developed as a useful measurement system for characterizing solid materials, it has been successfully applied to a variety of solid materials, both isotropic and anisotropic materials [3]-[5], and further studies on the fundamental measurement mechanisms have been successively made [6]-[9]. The system gives us a means capable of determining quantitatively acoustic properties of anisotropic materials depending on the propagation directions of leaky waves at the water/solid-sample boundary with high accuracy. The material characterization is carried out through the V(z) curve measurements [10] because dip intervals are closely related to the phase velocity of leaky waves on the boundary. So far, a model of one leaky wave on the boundary has been usually considered in the interference mechanism in the characterization region of V(z) curves [11], [12]. Recently, the multimode interference mechanism has been successfully clarified, taking up a (111)-Si sample obtained by the acoustic linefocus beam[7]. It has been clearly shown that multiple leaky waves existing on the boundary essentially give rise to deformation in the shape of V(z) curves and disorder the periodicity of dips. A simple superpositional model of elemental V(z) curves corresponding to each leaky wave has been developed on the basis of the general interference mechanism for deformed V(z) curves.

In this paper, using an FFT analysis, a general method is newly proposed and developed for velocity measurement in order to determine the velocities for each mode separately. Experiments are demonstrated for a (111)-Ge sample using a sapphire line-focusbeam lens of 1.0 mm radius at 226.3 MHz.

### 2. Multimode Interference and FFT Analysis

To describe an FFT analysis method of velocity measurement from deformed V(z) curves, let us first consider the multimode interference mechanism in V(z) curves. Figure 1 shows the cross-section of an acoustic line-focus beam used to explain the interference mechanism for two leaky modes, taking a (111)-Ge as an example. On the water/(111)-Ge boundary, there may also exist a leaky pseudo-SAW mode in addition to a fundamental leaky SAW mode. Both modes may be excited efficiently by the linefocus beam with the critical angles of  $\theta_{\mbox{\it lpsaw}}$  and θ<sub>lsaw</sub>, respectively. In such a case, three wave components, #0(a directly reflected component along the z-axis), #1(a leaky pseudo-SAW component) and #2(a leaky SAW component), contribute to the interference in the characterization region of V(z)curves. According to the analysis of the multimode interference mechanism developed previously[6], the transducer output V(z) as a function of distance z for this case is approximately represented by



Fig. 1. Cross-section of acoustic line-focus beam to explain the interference mechanism for two leaky modes in V(z) curves for water/(111)-Ge boundary.

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$$V(z) = V_{I}(z) + V_{I}(z)$$
, (1)

where  $V_L(z)$  is a characteristic lens response which is defined as the hypothetical transducer output in the case of no leaky waves on the boundary, depending uniquely on the dimensions of the acoustic linefocus-beam lens and the operating frequency. The function  $V_I(z)$  is a function that describes the interference of three wave components, and it is given as a superposition of two interference outputs: one is the interference output of  $V_{LPSCAD}(z)$  for the leaky pseudo-SAW mode between components #0 and #1, and the other is the interference output of  $V_{LSCAD}(z)$ for the leaky SAW mode between components #0 and #2, in the characterization region.

$$V_{I}(z) = V_{lpsaw}(z) + V_{lsaw}(z) .$$
 (2)

The interference outputs are given as follows:

$$V_{lpsaw}(z) = C_{1} \exp\left[-2\alpha_{w}t_{1}(z)\right]$$

$$\cdot \exp\left[-(2\pi f \alpha_{lpsaw}/v_{lpsaw})2|z|\tan\theta_{lpsaw}\right]$$

$$\cdot \sin\left(\xi_{1}|z|+\phi_{1}\right), \qquad (3)$$

$$V_{lsaw}(z) = C_{2} \exp\left[-2\alpha_{w}t_{2}(z)\right]$$

$$\cdot \exp\left[-(2\pi f \alpha_{lsaw}/v_{lsaw})2|z|\tan\theta_{lsaw}\right]$$

$$\cdot \sin\left(\xi_{2}|z|+\phi_{2}\right), \qquad (4)$$

where  $t_1(z)=\overline{A_1B_1}$  and  $t_2(z)=\overline{A_2B_2}$ , and  $\xi_1$  and  $\xi_2$  are the relative phase changes per unit translating distance z between components #0 and #1, and between components #0 and #2, respectively. The quantities  $\phi_1$  and  $\phi_2$  are the initial phase differences between components #0 and #1, and between components #0 and #2, respectively, when the (111)-Ge sample is placed at the focal plane. The quantities  $C_1$  and  $C_2$  are the arbitrary constants,  $\alpha_{ij}$  is the attenuation coefficient in water, and f is the acoustic frequency. The quantities  $v_{12800}$  and  $v_{1800}$  and the phase velocities for leaky perudo-SAW and leaky SAW, respectively, and  $\alpha_{12800}$  are the normalized attenuation factors for each mode.

In this way, V(z) curves are expressed as waveforms composed of finite periodical elemental components  $V_{\rm I}(z)$  and nonperiodical component  $V_{\rm I}(z)$ .

Now, on the basis of the construction principle of V(z) curves for multiple leaky wave modes, we apply a well known FFT waveform analysis to V(z)curves to properly obtain the elemental periodicity of dips for each mode from deformed V(z) curves. The procedure is as follows. We record V(z) curves into a wave memorizer by using the line-focus-beam acoustic microscope system. First, we synthesize the characteristic lens response  $V_{\rm L}(z)$  as given in Eq. (1) from the measured V(z) curve using digitalfiltering techniques, and then extract a  $V_{I}(z)$  by calculation of  $V(z) - V_L(z)$ . Next, we sample the measured  $V_I(z)$  curves with the distance interval  $\Delta z_g$ . Sampling points n are distributed for the waveform. In making the analysis, we provide further additional sampling points N' as dummy points both in front of and behind the  $V_{I}(z)$  curves in order to obtain a sufficiently high resolution of frequency  $\Delta f$ , which is directly related to the velocity resolution  $\Delta v$  in the present method. Applying the FFT analysis to

the waveform, we can obtain frequency spectra  $f_{\mathcal{D}}$  (mode spectra) having maxima in the frequency domain corresponding to each characteristic dip interval. The relationship between the dip interval  $\Delta z$ , corresponding to  $\xi_1$  or  $\xi_2$ , and  $f_{\mathcal{D}}$ , required for the calculation of the velocity of leaky waves, is given as

$$\Delta z = (\Delta z_s / \Delta t) (1/f_p)$$
<sup>(5)</sup>

where  $\Delta z_g/\Delta t$  is the conversion coefficient between the distance z and the time t. Substituting the relation of  $\Delta t=1/(N\Delta f)$  into Eq. (5), where  $\Delta f$  is the frequency interval in frequency domain and N is the number of sampling points, we can obtain the following equation:

$$\Delta z = N \Delta z_s \left( \Delta f / f_p \right) \quad . \tag{6}$$

With the dip interval  $\Delta z$  obtained by an FFT analysis, using the following conventional equation[2] to determine the phase velocity through the V(z) curve measurement

$$v = v_{\lambda \lambda} / (1 - (1 - v_{\lambda})/2f\Delta z)^2)^{1/2}, \tag{7}$$

we can determine the velocity v for each mode separately from the deformed V(z) curves. In Eq.(7),  $v_{ij}$  is the longitudinal velocity in water.

### 3. Experiments

Experiments are carried out for demonstration taking a (111) germanium as an example using a sapphire line-focus-beam lens of 1.0 mm radius at a frequency of 226.3 MHz. Because of the crystalline symmetry of germanium, belonging to class m3m, propagation characteristics of leaky waves on the water/ (111)-Ge boundary are completely determined from the analysis for the wave propagation directions in the range between  $\theta=0^\circ$  and  $30^\circ$ . There exist generally two leaky waves, i.e., a leaky SAW and a leaky pseudo-SAW, in all directions except  $\theta=30^\circ$  for pure Rayleigh mode. There two leaky modes would be efficiently excited on the boundary and take part in the interference in the characterization region of V(z) curves.

Figure 2 (a) and (b) show the V(z) curves for two typical directions of  $\theta=0^{\circ}$  and  $30^{\circ}$ , respectively. In the case of the V(z) curve for  $\theta=30^{\circ}$  as shown in Fig. 2 (b), an interference is observed with a constant interval of dips and simple variation in shape in the negative z region of the V(z) curve. This can be explained by a simple interference mechanism of two wave components: a directly reflected component and a leaky SAW component. The dip interval  $\Delta z$  of 21.6 µm gives a velocity of  $v_{1saw}$ =2807 m/s using Eq. (7). On the other hand, in the case of the V(z) curve for  $\theta$ =0° shown in Fig. 2 (a), the curve has apparently complex and deformed variation in both dip intervals and shape in the characterization region as compared with the curve for  $\theta$ =30°, so that it is impossible to properly determine the velocities of leaky waves from the curve. As shown in the multimode interference mechanism, the curve should be decomposed into two elemental V(z) curves with different dip intervals corresponding to the respective velocities of two leaky waves, a leaky SAW and a leaky pseudo-SAW, as presented in Table 1.

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Fig. 2. V(z) curves on water/(111)-Ge boundary measured with acoustic line-focus beam at 226.3 MHz.

Now, we apply an FFT waveform analysis to V(z)curves. Figure 3 (a) and (b) shown the results analyzed by FFT for the two V(z) curves in Fig. 2 (a) and (b), respectively. Here, we take sampling points of n=271 distributed for V(z) curves for analysis. In the present analysis, the total sampling points of N=8192 are used for a waveform, including additional sampling points as dummy to obtain a high frequency resolution in frequency domain. In Fig. 3 (a), two mode spectra for a leaky SAW (LSAW) and a leaky pseudo-SAW (LPSAW) appear clearly at frequencies of  $f_{\mathcal{D}}(LPSAW)=60.19$  Hz and  $f_p(LSAW) = 103.24$  Hz, respectively, with a frequency interval  $\Delta f$  of 0.1221 Hz. Using Eqs. (6) and (7), the velocities can be calculated as  $v_{lpsaw}$ =3431 m/s for the leaky pseudo-SAW and  $v_{lsaw}$ =2670 m/s for the leaky SAW, respectively. These values are very close to the calculated values within a difference of about 0.8%, as shown in Table 1. In the calculations, the physical constants in Ref. 13 were used for germanium, and the longitudinal velocity of  $v_{\omega}$ =1483 m/s and the density of  $\rho$ =998.2 kg/m<sup>3</sup> at 20°C



Fig. 3. FFT-analyzed mode spectra for V(z) curves shown in Fig. 2.

Table	1	Comparison of experimental results
		with calculated results for (111) Ge

Propagation direction	Leaky SAW velocity measured calculated		Leaky pseudo-SAW velocity measured calculated	
deg.	m/s		m/s	
0	2670	2691	3431	3450
30	2808	2828		

for water. In Fig. 3 (b), only one significant mode spectrum is observed, with  $f_p(LSAW) = 92.47$  Hz giving a velocity of  $v_{LSAW} = 2808$  m/s for the leaky pure Rayleigh wave on the boundary. This agrees with the values of 2807 m/s obtained by the conventional method for one leaky mode within the measurement error.

Figure 4 shows the results of velocities of leaky waves measured for the other propagation directions in the same way where experimental results are plotted as open circles, with the calcu-

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propagation direction,  $\Theta$  (deg.)

Fig. 4. Experimental and theoretical results of propagation characteristics of leaky SAWs and leaky pseudo-SAWs on water/(111)-Ge boundary. o ; measured \_\_\_\_; calculated

lated results shown by the solid line. It is easily seen that the proper determination of phase velocity is made for each mode existing on the boundary. The differences between the measured and calculated results are within 0.8% for leaky SAWs and 1.4% for leaky pseudo-SAWs.

### 4. Conclusion

An FFT waveform analysis has been newly introduced into the velocity measurement of leaky waves from the V(z) curve measurements obtained by the line-focus-beam acoustic microscope system, on the basis of a superpositional model of elemental V(z)curves derived from the multimode interference mechanism. The utility has been demonstrated for a (111)-Ge sample using a sapphire line-focus-beam lens of 1.0-mm radius at 226.3 MHz. On the water/ (111)-Ge boundary, two leaky wave modes of a leaky SAW and a leaky pseudo-SAW have been efficiently excited. The FFT method of velocity measurement has successfully revealed two mode spectra separately from the largely deformed V(z) curves, giving the proper phase velocities with high velocity resolution. With the introduction of this FFT method, the line-focus-beam acoustic microscope system has been satisfactorily expanded as a general means for material characterization, applicable to all sorts of materials supporting multiple leaky wave modes on the boundary.

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