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# Magnetoelastic analysis and tensile testing of a soft ferromagnetic strip with a single-edge crack

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This article describes the results of an analytical and experimental study of the effect of magnetic fields on the stress intensity factors in a soft ferromagnetic strip under uniaxial tension. The linear magneto-elastic problem for a soft ferromagnetic strip with a single-edge crack is analyzed. Fourier transform techniques are used to formulate the mixed boundary value problem as a singular integral equation. The stress intensity factors are obtained for several values of material and geometrical parameters, and magnetic field. Static experiments are also conducted on nickel–iron soft magnetic materials with a single-edge cracked plate specimen geometry in the bore of a superconducting magnet at room temperature. A strain gauge method is employed in experiments to determine the stress intensity factor. A comparison of the stress intensity factor is made between theory and experiment, and the agreement is good for the magnetic field considered. *© 2006 American Institute of Physics*. [DOI: 10.1063/1.2222068]

## I. INTRODUCTION

In magnetomechanical devices such as fusion reactors, magneto hydro dynamics structures, and magnetically levitated vehicles, magnetic fields are automatically applied to surrounding components. Design and development of such structures require basic research on magnetic fracture mechanics. Fracture mechanics have long been concerned with the fracture and deformation of elastic materials under forces. Magnetic fields constitute an extremely powerful natural force field when electromagnetic materials are present. In the theory of brittle fracture in a strong magnetic field, usually we consider linear magneto-elastic solutions of crack problems.[1] Recently, Shindo *et al.*[2,3] confirmed theoretically and experimentally the fact that the applied magnetic field tends to intensify the stress intensity factor. Also, the excellent agreement between calculations and measurements of the stress intensity factor established the validity of the linear theory for magnetoelastic interactions in a cracked soft ferromagnetic material.

This article examines theoretically and experimentally the effects of magnetic fields on the fracture mechanics parameters of a soft ferromagnetic strip with a single-edge crack subjected to tensile load and uniform magnetic field. The theoretical analysis is based on a linear theory for magneto-elastic interactions in a soft ferromagnetic material.[4] Fourier transform method is used to formulate the problem in terms of a singular integral equation. Numerical results are obtained for the stress intensity factor with different material properties, geometries and amplitudes of the magnetic field. Tensile tests are also conducted in which strain gauge techniques[5] are used to obtain values of the stress intensity factor. Single-edge cracked plate specimens are fabricated from nickel–iron soft magnetic materials and a static magnetic field is applied perpendicular to the crack surfaces. Comparison of the predictions with experimental data is conducted.

## II. PROBLEM STATEMENT AND BASIC EQUATIONS

Consider a soft ferromagnetic isotropic linear elastic strip of width $h$ which contains a single-edge crack of length $a$ aligned with its plane normal to the free edge as shown in Fig. 1. Let $x$, $y$ and $z$ denote the Cartesian coordinates. The soft ferromagnetic strip occupies the region $(0 \leq x \leq h, |y| < \infty)$. The strip is in the plane stress or plane strain state in the $z$ direction, and is subjected to a uniform normal stress $\sigma_{yy}=\sigma_0$ and a uniform magnetic field of magnetic induction $B_{0y}=B_0$. Because of symmetry, we need only to consider the upper half space region.

We consider small perturbations characterized by the displacement vector $\mathbf{u}$ produced in the strip. All magnetic quantities are divided into two parts, those in the rigid body state and those in the perturbation state as follows:

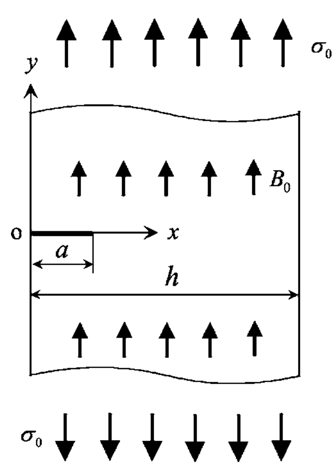


FIG. 1. A single-edge crack in a soft ferromagnetic elastic strip.

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$$\begin{aligned}\mathbf{B} &= \mathbf{B_0} + \mathbf{b}\\ \mathbf{M} &= \mathbf{M_0} + \mathbf{m}\\ \mathbf{H} &= \mathbf{H_0} + \mathbf{h}\ ,\end{aligned} \tag{1}$$

where **B**, **M** and **H** are the magnetic induction, magnetization and magnetic intensity vectors, respectively. Those quantities with the subscript 0 refer to the magnetic field in the undeformed state. The remaining ones are the corrections to account for the additional changes in magnetic field due to deformations. The magnetoelastic solution for the rigid body state can be written as

$$B_{0y} = B_0, \quad H_{0y} = \frac{B_0}{\mu_0\mu_r}, \quad M_{0y} = \frac{\chi B_0}{\mu_0\mu_r}; \quad 0 \le x \le h, 0 < y < \infty \tag{2}$$

$$B^e_{0y} = \frac{B_0}{\mu_r}, \quad H^e_{0y} = \frac{B_0}{\mu_0\mu_r}, \quad M^e_{0y} = 0; \quad x < 0, h < x < \infty, 0 < y < \infty \tag{3}$$

$$B^{ec}_{0y} = B_0, \quad H^{ec}_{0y} = \frac{B_0}{\mu_0}, \quad M^{ec}_{0y} = 0; \quad 0 \le x < a, y = 0, \tag{4}$$

where $B_{0y}$, $H_{0y}$ and $M_{0y}$ are, respectively, the $y$ components of $\mathbf{B_0}$, $\mathbf{H_0}$ and $\mathbf{M_0}$. The superscripts $e(x<0, h<x<\infty, 0<y<\infty)$ and $ec(0\le x<a, y=0)$ stand for the components of the field quantity outside the strip. Note that $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$ is the magnetic permeability of the vacuum, $\mu_r = 1+\chi$ is the specific magnetic permeability, and $\chi$ is the magnetic susceptibility.

The effect of the magnetization as induced by the deformation becomes important for a cracked soft ferromagnetic strip in a magnetic field normal to the crack surface. In this case, the body force of the type $\mu_0 \mathbf{M}\cdot\nabla\mathbf{H}$ must be considered on account of the sharp gradient of magnetic field near the crack. For the magnetic field in the strip, it is assumed that

$$h_{x,x} + h_{y,y} = 0, \tag{5}$$

$$h_{x,y} - h_{y,x} = 0, \tag{6}$$

and

$$\begin{aligned} b_x &= \mu_0\mu_r h_x\\ b_y &= \mu_0\mu_r h_y\ ,\end{aligned} \tag{7}$$

$$\begin{aligned} m_x &= \chi h_x\\ m_y &= \chi h_y\ ,\end{aligned} \tag{8}$$

where the subscript comma implies a partial derivative with respect to the coordinate, and $(h_x, h_y)$, $(b_x, b_y)$ and $(m_x, m_y)$ are the $x$ and $y$ components of **h**, **b** and **m**. Equations (5) and (6) for the perturbed state are satisfied by introducing a magnetic potential $\phi$ such that

$$h_x = \phi_{,x}, \quad h_y = \phi_{,y}, \tag{9}$$

$$\phi_{,xx} + \phi_{,yy} = 0. \tag{10}$$

By using a dipole model for the magnetization, we obtain the following equilibrium equations:

$$\begin{aligned} t_{xx,x} + t_{yx,y} + \frac{\chi B_0 \phi_{,xy}}{\mu_r} &= 0\\ t_{xy,x} + t_{yy,y} + \frac{\chi B_0 \phi_{,yy}}{\mu_r} &= 0\ .\end{aligned} \tag{11}$$

The components of magnetoelastic stresses $t_{xx}$, $t_{yy}$, $t_{xy}=t_{yx}$ and Maxwell stresses $t^M_{xx}$, $t^M_{yy}$, $t^M_{xy}=t^M_{yx}$ are

$$\begin{aligned} t_{xx} &= \sigma_{xx}\\ t_{yy} &= \sigma_{yy} + \frac{\chi B_0^2}{\mu_0\mu_r^2} + \frac{2\chi B_0\phi_{,y}}{\mu_r}\\ t_{xy} &= t_{yx} = \sigma_{xy} + \frac{\chi B_0 \phi_{,x}}{\mu_r}\ ,\end{aligned} \tag{12}$$

$$\begin{aligned} \sigma_{xx} &= 2\mu u_{x,x} + \kappa(u_{x,x} + u_{y,y})\\ \sigma_{yy} &= 2\mu u_{y,y} + \kappa(u_{x,x} + u_{y,y})\\ \sigma_{xy} &= \sigma_{yx} = \mu(u_{x,y} + u_{y,x})\ ,\end{aligned} \tag{13}$$

$$\begin{aligned} t^M_{xx} &= -\frac{B_0\phi_{,y}}{\mu_r} - \frac{B_0^2}{2\mu_0\mu_r^2}\\ t^M_{yy} &= \frac{(1+2\chi)B_0\phi_{,y}}{\mu_r} + \frac{(1+2\chi)B_0^2}{2\mu_0\mu_r^2}\\ t^M_{xy} &= t^M_{yx} = B_0\phi_{,x}\ ,\end{aligned} \tag{14}$$

where $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}=\sigma_{yx}$ are the elastic stress components, $u_x$ and $u_y$ are the displacement components, $\kappa=\lambda$ for plane strain, and $\kappa = 2\lambda\mu/(\lambda+2\mu)$ for plane stress, $\lambda = 2G\nu/(1-2\nu)$ and $\mu = G$ are the Lamé constants, $G = E/2(1+\nu)$ is the modulus of rigidity, and $E$ and $\nu$ are the Young's modulus and Poisson's ratio, respectively. By using Eqs. (12) and (13), Eqs. (11) can be written as

$$u_{x,xx} + u_{x,yy} + \left(\frac{\kappa}{\mu} + 1\right)(u_{x,x} + u_{y,y})_{,x} + \frac{2\chi B_0 \phi_{,xy}}{\mu\mu_r} = 0, \tag{15}$$

$$u_{y,xx} + u_{y,yy} + \left(\frac{\kappa}{\mu} + 1\right)(u_{x,x} + u_{y,y})_{,y} + \frac{2\chi B_0 \phi_{,yy}}{\mu\mu_r} = 0. \tag{16}$$

The boundary conditions in the perturbation state become

$$\begin{aligned} &h^{ec}_x(x,0) - h_x(x,0) = -(\chi B_0/\mu_0\mu_r) u_{y,x}(x,0) && (0 \le x < a)\\ &\phi(x,0) = 0 && (a \le x \le h)\end{aligned}, \tag{17}$$

$$\begin{aligned} &b^{ec}_y(x,0) - b_y(x,0) = 0 && (0 \le x < a)\\ &\phi^{ec}(x,0) = 0 && (0 \le x < a)\end{aligned}, \tag{18}$$

$$\sigma_{yx}(x,0) = -(\chi B_0/\mu_r) h_x(x,0) \quad (0 \le x \le h), \tag{19}$$

$$\begin{array}{ll} \sigma_{yy}(x,0)=\{\chi(\chi-2)B_0/\mu_r\}\{h_y(x,0)+B_0/2\mu_0\mu_r\} & (0\le x<a) \\ u_y(x,0)=0 & (a\le x\le h) \end{array}, \tag{20}$$

$$h_y^e(0,y)-h_y(0,y)=0 \quad (0\le y<\infty), \tag{21}$$

$$h_x^e(0,y)-\mu_r h_x(0,y)=-(\chi B_0/\mu_0\mu_r)u_{x,y}(0,y) \quad (0\le y<\infty), \tag{22}$$

$$\sigma_{xx}(0,y)=0 \quad (0\le y<\infty), \tag{23}$$

$$\sigma_{xy}(0,y)=-(\chi B_0/\mu_r)h_x(0,y) \quad (0\le y<\infty), \tag{24}$$

$$h_y^e(h,y)-h_y(h,y)=0 \quad (0\le y<\infty), \tag{25}$$

$$h_x^e(h,y)-\mu_r h_x(h,y)=-(\chi B_0/\mu_0\mu_r)u_{x,y}(h,y) \quad (0\le y<\infty), \tag{26}$$

$$\sigma_{xx}(h,y)=0 \quad (0\le y<\infty), \tag{27}$$

$$\sigma_{xy}(h,y)=-(\chi B_0/\mu_r)h_x(h,y) \quad (0\le y<\infty). \tag{28}$$

## III. SOLUTION PROCEDURE

Let the solutions of Eqs. (15) and (16) be of the form

$$\begin{aligned} u_x=&\frac{2}{\pi}\int_0^\infty\left\{A(\alpha)+\left[y-\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\frac{1}{\alpha}\right]B(\alpha)+\frac{2\chi B_0}{\mu_r(\kappa+\mu)}a(\alpha)\right\}e^{-\alpha y}\sin(\alpha x)d\alpha-\frac{2}{\pi}\\ &\times\int_0^\infty\left\{\left[x\cosh(\alpha x)-\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\frac{1}{\alpha}\sinh(\alpha x)\right]D(\alpha)\right.\\ &+\left[x\sinh(\alpha x)-\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\frac{1}{\alpha}\cosh(\alpha x)\right]F(\alpha)\\ &+\sinh(\alpha x)C(\alpha)+\cosh(\alpha x)E(\alpha)+\frac{2\chi B_0}{\mu_r(\kappa+\mu)}\\ &\left.\times\sinh(\alpha x)b(\alpha)+\frac{2\chi B_0}{\mu_r(\kappa+\mu)}\cosh(\alpha x)c(\alpha)\right\}\\ &\times\cos(\alpha y)d\alpha-a_0x, \end{aligned} \tag{29}$$

$$\begin{aligned} u_y=&\frac{2}{\pi}\int_0^\infty\{A(\alpha)+B(\alpha)y\}e^{-\alpha y}\cos(\alpha x)d\alpha\\ &+\frac{2}{\pi}\int_0^\infty\{C(\alpha)\cosh(\alpha x)+D(\alpha)x\sinh(\alpha x)\\ &+E(\alpha)\sinh(\alpha x)+F(\alpha)x\cosh(\alpha x)\}\sin(\alpha y)d\alpha+b_0y, \end{aligned} \tag{30}$$

$$\phi=\frac{2}{\pi}\int_0^\infty a(\alpha)e^{-\alpha y}\cos(\alpha x)d\alpha+\frac{2}{\pi}\int_0^\infty\{b(\alpha)\cosh(\alpha x)+c(\alpha)\sinh(\alpha x)\}\sin(\alpha y)d\alpha, \tag{31}$$

where $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D(\alpha)$, $E(\alpha)$, $F(\alpha)$, $a(\alpha)$, $b(\alpha)$ and $c(\alpha)$ are the unknown functions to be solved, and the real constants $a_0$, $b_0$ are determined from the far-field loading conditions. Application of Fourier transform to Eq. (10) yields

$$\phi^{ec}=\frac{2}{\pi}\int_0^\infty a_e(\alpha)\sinh(\alpha y)\cos(\alpha x)d\alpha \quad (0\le x<a), \tag{32}$$

$$\phi^e=\begin{cases}\dfrac{2}{\pi}\displaystyle\int_0^\infty b_e(\alpha)e^{\alpha x}\sin(\alpha y)d\alpha & (-\infty<x<0)\\ \dfrac{2}{\pi}\displaystyle\int_0^\infty c_e(\alpha)e^{-\alpha x}\sin(\alpha y)d\alpha & (h<x<\infty)\end{cases}, \tag{33}$$

where $a_e(\alpha)$, $b_e(\alpha)$ and $c_e(\alpha)$ are also unknowns. The magnetic field can be obtained by making use of Eqs. (9) and (31). The magnetic field in the void inside the crack and outside the strip can also be obtained from Eqs. (9), (32), and (33).

By applying the far-field loading conditions, the constants $a_0$ and $b_0$ are obtained as

$$a_0=\frac{\kappa}{4(\kappa+\mu)}\left(\frac{\sigma_0}{\mu}-\frac{\chi b_c^2}{\mu_r^2}\right), \quad b_0=\frac{\kappa+2\mu}{4(\kappa+\mu)}\left(\frac{\sigma_0}{\mu}-\frac{\chi b_c^2}{\mu_r^2}\right), \tag{34}$$

where

$$b_c^2=\frac{B_0^2}{\mu\mu_0}. \tag{35}$$

The boundary condition (19) gives the following equation in $A(\alpha)$, $B(\alpha)$ and $a(\alpha)$:

$$\alpha A(\alpha)=\frac{\kappa+2\mu}{\kappa+\mu}B(\alpha)-\left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+2\mu}\right)\alpha a(\alpha). \tag{36}$$

Making use of the two mixed boundary conditions (17) and (20), we have simultaneous dual integral equations:

$$\int_0^\infty\alpha\left[a(\alpha)-\left(\frac{\chi B_0}{\mu_0\mu_r}\right)A(\alpha)\right]\sin(\alpha x)d\alpha=0 \quad (0\le x<a)$$

$$\int_0^\infty a(\alpha)\cos(\alpha x)d\alpha=0 \quad (a\le x\le h), \tag{37}$$

$$\int_0^\infty \alpha\left[1-\frac{\chi^2(\kappa+2\mu)b_c^2}{2\mu_r^2(\kappa+\mu)}\left(\chi-\frac{\mu}{\kappa+2\mu}\right)\right]A(\alpha)\cos(\alpha x)d\alpha - \frac{\kappa+2\mu}{(\kappa+\mu)y_0}\int_0^\infty [G_1(\alpha)\cosh(\alpha x) + G_2(\alpha)\sinh(\alpha x)]e^{-\alpha x}d\alpha = \frac{\pi}{4}\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\left(\frac{\sigma_0}{\mu}\right) \quad (0\le x<a)$$

$$\int_0^\infty A(\alpha)\cos(\alpha x)d\alpha = 0 \quad (a\le x\le h), \tag{38}$$

where

$$G_1(\alpha)=\alpha C(\alpha)+\frac{\kappa}{\kappa+\mu}D(\alpha)+\alpha x F(\alpha) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\chi-\frac{4\kappa+2\mu}{\kappa+\mu}\right)\alpha b(\alpha)$$

$$G_2(\alpha)=\alpha x D(\alpha)+\alpha E(\alpha)+\frac{\kappa}{\kappa+\mu}F(\alpha) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\chi-\frac{4\kappa+2\mu}{\kappa+\mu}\right)\alpha c(\alpha). \tag{39}$$

The boundary conditions of Eqs. (21)–(28) lead to the following relations between unknown functions:

$$\left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left\{\alpha\sinh(\alpha h)C(\alpha)+\left[\alpha h\cosh(\alpha h) - \frac{\kappa+3\mu}{\kappa+\mu}\sinh(\alpha h)\right]D(\alpha)+\alpha\cosh(\alpha h)E(\alpha) + \left[\alpha h\sinh(\alpha h)-\frac{\kappa+3\mu}{\kappa+\mu}\cosh(\alpha h)\right]F(\alpha)\right\} + \left[\frac{2\mu\chi^2 b_c^2}{\mu_r^2(\kappa+\mu)}-\mu_r\right]\{\alpha\sinh(\alpha h)b(\alpha) + \alpha\cosh(\alpha h)c(\alpha)\} - \alpha e^{-\alpha h}c_e(\alpha) = f_1(\alpha), \tag{40}$$

$$\alpha\{\cosh(\alpha h)b(\alpha)+\sinh(\alpha h)c(\alpha)-e^{-\alpha h}c_e(\alpha)\}=f_2(\alpha), \tag{41}$$

$$\alpha\sinh(\alpha h)C(\alpha)+\left[\alpha h\cosh(\alpha h)-\frac{\mu}{\kappa+\mu}\sinh(\alpha h)\right]D(\alpha) + \alpha\cosh(\alpha h)E(\alpha)+\left[\alpha h\sinh(\alpha h) - \frac{\mu}{\kappa+\mu}\cosh(\alpha h)\right]F(\alpha) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\{\alpha\sinh(\alpha h)b(\alpha) + \alpha\cosh(\alpha h)c(\alpha)\} = f_3(\alpha), \tag{42}$$

$$\alpha\cosh(\alpha h)C(\alpha)+\left[\alpha h\sinh(\alpha h) - \frac{\kappa+2\mu}{\kappa+\mu}\cosh(\alpha h)\right]D(\alpha)+\alpha\sinh(\alpha h)E(\alpha) + \left[\alpha h\cosh(\alpha h)-\frac{\kappa+2\mu}{\kappa+\mu}\sinh(\alpha h)\right]F(\alpha)+\left(\frac{\chi B_0}{\mu\mu_r}\right) \times\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\{\alpha\cosh(\alpha h)b(\alpha)+\alpha\sinh(\alpha h)c(\alpha)\} = f_4(\alpha), \tag{43}$$

$$\alpha\{b_e(\alpha)-b(\alpha)\}=f_5(\alpha), \tag{44}$$

$$\alpha b_e(\alpha)+\left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left[\alpha E(\alpha)-\frac{\kappa+3\mu}{\kappa+\mu}F(\alpha)\right] + \left[\frac{2\mu\chi^2 b_c^2}{\mu_r^2(\kappa+\mu)}-\mu_r\right]\alpha c(\alpha)=f_6(\alpha), \tag{45}$$

$$\alpha C(\alpha)-\frac{\kappa+2\mu}{\kappa+\mu}\left[D(\alpha)+\left(\frac{\chi B_0}{\mu\mu_r}\right)\alpha b(\alpha)\right]=f_7(\alpha), \tag{46}$$

$$\alpha E(\alpha)-\frac{\mu}{\kappa+\mu}\left[F(\alpha)+\left(\frac{\chi B_0}{\mu\mu_r}\right)\alpha c(\alpha)\right]=f_8(\alpha), \tag{47}$$

where

$$f_1(\alpha)=\int_0^\infty F_1(s,\alpha)\left\{\left(\frac{\chi B_0}{\mu_0\mu_r}\right)sA(s) - \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left[\frac{2(\kappa+2\mu)}{\kappa+\mu}-sy\right]B(s) - \left[\mu_r-\frac{2\mu\chi^2 b_c^2}{\mu_r^2(\kappa+\mu)}\right]sa(s)\right\}ds$$

$$f_2(\alpha)=\int_0^\infty F_2(s,\alpha)sa(s)ds$$

$$f_3(\alpha)=\int_0^\infty F_1(s,\alpha)\left[sA(s)-\left(\frac{\kappa+2\mu}{\kappa+\mu}-sy\right)B(s) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)sa(s)\right]ds$$

$$f_4(\alpha)=\int_0^\infty F_2(s,\alpha)\left[sA(s)-\left(\frac{2\kappa+3\mu}{\kappa+\mu}-sy\right)B(s) + \left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)sa(s)\right]ds \tag{48}$$

$$f_5(\alpha)=-\int_0^\infty F_3(s,\alpha)sa(s)ds$$

$$f_6(\alpha)=0$$

$$f_7(\alpha) = \int_0^\infty F_3(s,\alpha)\left[sA(s) - \left(\frac{2\kappa+3\mu}{\kappa+\mu} - sy\right)B(s) + \left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)sa(s)\right]ds$$

$$f_8(\alpha) = 0$$

and

$$F_1(s,\alpha) = \frac{2}{\pi}\left(\frac{\alpha}{s^2+\alpha^2}\right)\sin(sh)$$

$$F_2(s,\alpha) = \frac{2}{\pi}\left(\frac{s}{s^2+\alpha^2}\right)\cos(sh) \quad (49)$$

$$F_3(s,\alpha) = \frac{2}{\pi}\left(\frac{s}{s^2+\alpha^2}\right).$$

Now define the unknown function

$$a(\alpha) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\frac{1}{\alpha}\int_0^a d(\xi)\sin(\alpha\xi)d\xi$$

$$A(\alpha) = \frac{1}{\alpha}\int_0^a d(\xi)\sin(\alpha\xi)d\xi. \quad (50)$$

The two simultaneous dual integral Eqs. (37) and (38), together with Eqs. (40)–(47), lead to the following singular integral equation:

$$\int_0^a \frac{1}{\xi-x}d(\xi)d\xi + \int_0^a\left[\frac{1}{\xi+x} + K(\xi,x)\right]d(\xi)d\xi = \frac{\pi}{2}\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\left(\frac{\sigma_0}{\mu y_0}\right), \quad (51)$$

where

$$K(\xi,x) = -\frac{2(\kappa+2\mu)}{\kappa+\mu}\int_0^\infty \{M_1(\alpha,\xi)\cosh(\alpha x) + M_2(\alpha,\xi)\sinh(\alpha x)\}d\alpha, \quad (52)$$

$$y_0 = 1 - \frac{\kappa+2\mu}{2(\kappa+\mu)}\left(\chi - \frac{\mu}{\kappa+2\mu}\right)\left(\frac{\chi b_c}{\mu_r}\right)^2 \quad (53)$$

and

$$M_1(\alpha,\xi) = \frac{\kappa}{\kappa+\mu}L_1(\alpha,\xi) + \alpha x L_2(\alpha,\xi)\alpha + L_3(\alpha,\xi) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\chi - \frac{4\kappa+2\mu}{\kappa+\mu}\right)\alpha L_5(\alpha,\xi)$$

$$M_2(\alpha,\xi) = \alpha x L_1(\alpha,\xi) + \frac{\kappa}{\kappa+\mu}L_2(\alpha,\xi)\alpha + L_4(\alpha,\xi) + \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\chi - \frac{4\kappa+2\mu}{\kappa+\mu}\right)\alpha L_6(\alpha,\xi). \quad (54)$$

The functions $L_i(\alpha,\xi)(i=1,\ldots,6)$ are

$$L_i(\alpha,\xi) = \sum_{j=1}^{8}\frac{D_j(\alpha,\xi)Q_{i,j}(\alpha)}{|\mathbf{C}|}, \quad (55)$$

where **C** and **D** are given in Appendix A, and $Q_{i,j}(\alpha)$ are the cofactors of the elements in the square matrix **C**.

Prior to the numerical solution of Eq. (52), it is normalized by introducing

$$\xi = \frac{a}{2}(t+1), \quad x = \frac{a}{2}(s+1), \quad (56)$$

$$d(\xi) = d^*(t). \quad (57)$$

If we now substitute Eqs. (56) and (57) into Eq. (52), we get

$$\int_{-1}^{1}\frac{1}{t-s}d^*(t)dt + \int_{-1}^{1}\left[\frac{1}{t+s+2} + K^*(t,s)\right]d^*(t)dt = \frac{\pi}{2}\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\left(\frac{\sigma_0}{\mu y_0}\right), \quad (58)$$

where

$$K^*(t,s) = K(\xi,x). \quad (59)$$

The solution of Eq. (58) is of the form

$$d^*(t) = \frac{G(t)}{(1-t^2)^{1/2}}. \quad (60)$$

By using the method described Ref. 6, Eq. (58) may now be reduced to the following system of equations:

$$\sum_{k=1}^{N}\frac{1}{2N}G(t_k)\left[\frac{2}{t_k-s_r} + \frac{2}{(t_k+1)+(s_r+1)} + K^*(t_k,s_r)\right] = \frac{\pi}{2}\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\left(\frac{\sigma_0}{\mu y_0}\right)(r=1,\ldots,N-1), \quad (61)$$

where in the above $t_k$ and $s_r$ are, respectively, the roots of Chebyshev polynomials of the first kind of order $N$ and of the second kind of order $N-1$. Equation (61) provides $N-1$ linear algebraic equations for the unknowns $G(t_1),\ldots,G(t_N)$. The $N$th unknown, $G(t_N)$, is assumed to be

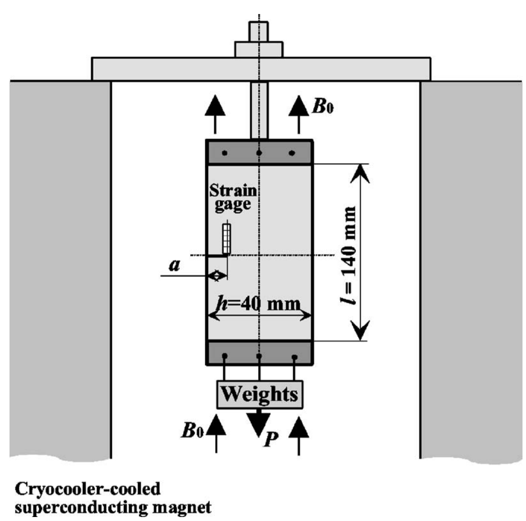


FIG. 2. Experimental setup.

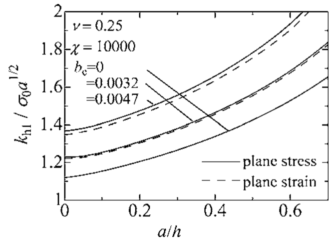


FIG. 3. Stress intensity factor vs $a/h$ ($\nu$=0.25, $\chi$=10 000).

zero since $t_N$ is the closest of the $t_k$ to $-1$. This assumption is in fact true as $N \to \infty$.[7] Eliminating this unknown yields $N-1$ equations for the $N-1$ remaining unknowns. The linear algebraic system given in Eq. (61), where the sum is taken only to $N-1$, is solved using Gaussian elimination for $N-1$ values of $G(t_k)$.

The magnetic stress intensity factor is obtained as

$$k_{h1} = \lim_{x \to a^+} [2(x-a)]^{1/2} t_{yy}^c(x,0) = \frac{a_1}{y_0} \sigma_0 a^{1/2} G(1), \tag{62}$$

where

$$a_1 = 1 + \frac{\chi}{2(\kappa + \mu)} [(\kappa + 2\mu) + \chi(2\kappa + 5\mu)] \left( \frac{b_c}{\mu_r} \right)^2, \tag{63}$$

$$t_{yy}^c(x,0) = t_{yy}(x,0) + t_{yy}^M(x,0). \tag{64}$$

## IV. EXPERIMENTAL PROCEDURE

Tensile tests were performed on nickel–iron TMC-V($E$ =182 GPa, $\nu$=0.146, $\mu_r$=27 900), TMH-B($E$=203 GPa, $\nu$ =0.279, $\mu_r$=10 690) and TMB($E$=146 GPa, $\nu$=0.228, $\mu_r$ =9030), soft magnetic materials (NEC/Tokin Co. Ltd.). Figure 2 shows the specimen and setup for the experiment. The specimen geometry was a plate specimen containing a single-edge crack. The edge-cracked specimen has a length of 140 mm, a thickness of 1 mm, a width, $h$, of 40 mm, and a crack length, $a$, of 4, 10, 15 mm. An initial through-the-thickness notch was machined using electro-discharge machining. The specimen was fatigue precracked and then annealed to obtain the optimum magnetic properties.

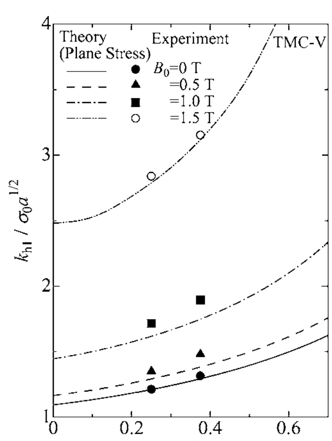


FIG. 4. Stress intensity factor vs $a/h$ (TMC-V).

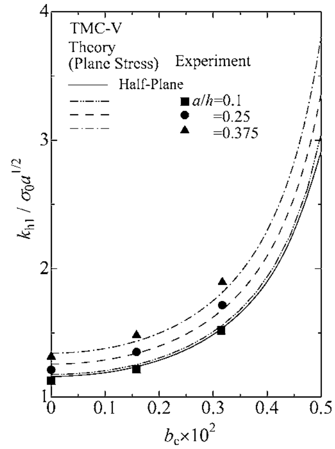


FIG. 5. Stress intensity factor vs magnetic field (TMC-V).

A simple strain gauge method is very suitable to determine the magnetic stress intensity factor.[2,8] A five element strip gauge (KFG-1-120-D19-16N10C2 from Kyowa Electronic Instruments Co. Ltd.) was installed along the 90° line and the center point of the element closest to the crack tip was 2 mm. The strain sensors have an active length of 1 mm.

Tensile load and a magnetic field were simultaneous applied to the edge-cracked plate specimens at room temperature. A 10 T (T: Tesla) cryocooler-cooled superconducting magnet with a 100-mm-diameter working bore was used to create a static uniform magnetic field of magnetic induction $B_0$ normal to the crack surface. The specimen was loaded by $P$=29.4 N load that consisted of weight. The strains near the crack tip were recorded as a function of $B_0$.

For the plane stress case, the strain $\varepsilon_{yy}$ near the crack becomes

$$\begin{aligned} E\varepsilon_{yy} = \frac{k_{h1}}{(2r)^{1/2}} \left\{ \frac{\chi}{2(1+\nu)\chi + \{2\nu + (5-\nu)\chi\}(\chi b_c/\mu_r)^2} \right\} \\ \times \cos\frac{\theta}{2} \left\{ (1-\nu)\left[ 2(1+\nu) + (3+\nu)\left(\frac{\chi b_c}{\mu_r}\right)^2 \right] \right. \\ \left. + (1+\nu)\left[ 2(1+\nu) + (3-\nu)\left(\frac{\chi b_c}{\mu_r}\right)^2 \right] \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \right\} \\ + A_0 + O(r^{1/2}), \end{aligned} \tag{65}$$

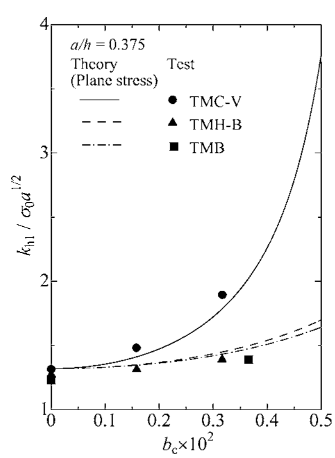


FIG. 6. Stress intensity factor vs magnetic field (TMC-V, TMH-B, TMB).

where $A_0$ is the unknown coefficient. When all the magnetic field quantities are made to vanish, Eq. (65) reduces to the strain near the crack tip in an elastic plane body.[5] Setting $\theta = \pi/2$ gives

$$c_0 E \varepsilon_{yy} r^{1/2} = k_{h1} - 2c_0 A_0 r^{1/2} + \dots , \tag{66}$$

where

$$c_0 = \frac{4}{\chi}\left\{ \frac{2(1+\nu)\chi + \{2\nu + (5-\nu)\chi\}(\chi b_c/\mu_r)^2}{2(3+2\nu-\nu^2) + (9-2\nu-3\nu^2)(\chi b_c/\mu_r)^2} \right\}. \tag{67}$$

From Eq. (66), a plot of $c_0 E\varepsilon_{yy} r^{1/2}$ vs $r^{1/2}$ is linear for small values of $r$ and the intercept at $r=0$, at the crack tip, gives the stress intensity factor $k_{h1}$.

## V. RESULTS AND DISCUSSION

To examine the effect of magnetic field on the stress intensity factor, the solution of a singular integral equation has been computed numerically. Figure 3 shows the normalized stress intensity factor $k_{h1}/\sigma_0 a^{1/2}$ as a function of the strip-width to crack-length ratio $a/h$ under various values of normalized magnetic field $b_c$ for $\nu=0.25$ and $\chi=10\,000$. The data are obtained from the plane stress and plane strain analyses. For $\mu=80$ GPa, $b_c=0.0032$ and 0.0047 correspond to the magnetic induction of $B_0=1.0$ and 1.5 T, respectively. The curves obtained for $b_c=0$ coincide with the purely elastic plane stress and plane strain cases. The normalized stress intensity factor increases slowly as $a/h$ increases and tends to the result of the infinite solid as $a/h\to 0$. The values of $k_{h1}/\sigma_0 a^{1/2}$ for $a/h\to 0$ are found to be $k_{h1}/\sigma_0 a^{1/2}=1.230$, 1.369 ($b_c=0.0032$, 0.0047) for plane stress and $k_{h1}/\sigma_0 a^{1/2}=1.223$, 1.348 ($b_c=0.0032$, 0.0047) for plane strain. Application of the magnetic field increases the stress intensity factor depending on $a/h$. The variations of calculated $k_{h1}/\sigma_0 a^{1/2}$ against $a/h$ for TMC-V under various values of $B_0$ in the plane stress case are compared with the experimental data in Fig. 4. The agreement between the two is good. A larger value of $B_0$ tends to increase the stress intensity factor, and this trend may be more clearly observed in Fig. 5 for $a/h=0.1, 0.25$ and 0.375. Theoretical predictions of the stress intensity factor are in agreement with experimental values. The calculated results for TMC-V, TMH-B and TMB vs $b_c$ are plotted together with the experimental data in Fig. 6 for $a/h=0.375$. The effect of the magnetic field on the stress intensity factor is more pronounced with increasing the specific magnetic permeability.

## VI. CONCLUSIONS

The magnetic fracture behavior was investigated both analytically and experimentally for a soft ferromagnetic strip with a single-edge crack. Based on the results of this study, the following conclusions may be drawn:

1. The magnetic field effect can increase the values of the stress intensity factor, and depends on the geometry and material properties of the soft ferromagnetic solids.
2. The effect of magnetic field on the stress intensity factor becomes more pronounced as the specific magnetic permeability takes on higher values.
3. The applicability of the strain gauge method for magnetic fracture testing was established. A comparison between theoretical and experimental values of the stress intensity factor shows good agreement, and the data verify the validity of the linear theory for magnetoelastic interactions in cracked soft ferromagnetic materials.

## APPENDIX A

**C** and **D** in Eq. (55) are given by

$$\mathbf{C} = \begin{bmatrix} c_{1,1}(\alpha) & c_{1,2}(\alpha) & c_{1,3}(\alpha) & c_{1,4}(\alpha) & c_{1,5}(\alpha) & c_{1,6}(\alpha) & c_{1,7}(\alpha) & c_{1,8}(\alpha) \\ c_{2,1}(\alpha) & c_{2,2}(\alpha) & c_{2,3}(\alpha) & c_{2,4}(\alpha) & c_{2,5}(\alpha) & c_{2,6}(\alpha) & c_{2,7}(\alpha) & c_{2,8}(\alpha) \\ c_{3,1}(\alpha) & c_{3,2}(\alpha) & c_{3,3}(\alpha) & c_{3,4}(\alpha) & c_{3,5}(\alpha) & c_{3,6}(\alpha) & c_{3,7}(\alpha) & c_{3,8}(\alpha) \\ c_{4,1}(\alpha) & c_{4,2}(\alpha) & c_{4,3}(\alpha) & c_{4,4}(\alpha) & c_{4,5}(\alpha) & c_{4,6}(\alpha) & c_{4,7}(\alpha) & c_{4,8}(\alpha) \\ c_{5,1}(\alpha) & c_{5,2}(\alpha) & c_{5,3}(\alpha) & c_{5,4}(\alpha) & c_{5,5}(\alpha) & c_{5,6}(\alpha) & c_{5,7}(\alpha) & c_{5,8}(\alpha) \\ c_{6,1}(\alpha) & c_{6,2}(\alpha) & c_{6,3}(\alpha) & c_{6,4}(\alpha) & c_{6,5}(\alpha) & c_{6,6}(\alpha) & c_{6,7}(\alpha) & c_{6,8}(\alpha) \\ c_{7,1}(\alpha) & c_{7,2}(\alpha) & c_{7,3}(\alpha) & c_{7,4}(\alpha) & c_{7,5}(\alpha) & c_{7,6}(\alpha) & c_{7,7}(\alpha) & c_{7,8}(\alpha) \\ c_{8,1}(\alpha) & c_{8,2}(\alpha) & c_{8,3}(\alpha) & c_{8,4}(\alpha) & c_{8,5}(\alpha) & c_{8,6}(\alpha) & c_{8,7}(\alpha) & c_{8,8}(\alpha) \end{bmatrix} \tag{A1}$$

$$\mathbf{D} = \{D_1(\alpha), D_2(\alpha), D_3(\alpha), D_4(\alpha), D_5(\alpha), D_6(\alpha), D_7(\alpha), D_8(\alpha)\}^T, \tag{A2}$$

where

$$c_{1,1}(\alpha) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left[\alpha h \cosh(\alpha h) - \frac{\kappa+3\mu}{\kappa+\mu}\sinh(\alpha h)\right],$$

$$c_{1,2}(\alpha) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left[\alpha h \sinh(\alpha h) - \frac{\kappa+3\mu}{\kappa+\mu}\cosh(\alpha h)\right],$$

$$c_{1,3}(\alpha) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\alpha \sinh(\alpha h),$$

$$c_{1,4}(\alpha) = \left(\frac{\chi B_0}{\mu_0 \mu_r}\right)\alpha \cosh(\alpha h),$$

$$c_{1,5}(\alpha) = \left[2\left(\frac{\chi b_c}{\mu_r}\right)^2\left(\frac{\mu}{\kappa+\mu}\right) - \mu_r\right]\alpha \sinh(\alpha h),$$

$$c_{1,6}(\alpha) = \left[2\left(\frac{\chi b_c}{\mu_r}\right)^2\left(\frac{\mu}{\kappa+\mu}\right) - \mu_r\right]\alpha \cosh(\alpha h),$$

$$c_{1,7}(\alpha) = \alpha e^{-\alpha h},$$

$$c_{1,8}(\alpha) = 0, \tag{A3}$$

$$c_{2,1}(\alpha) = c_{2,2}(\alpha) = c_{2,3}(\alpha) = c_{2,4}(\alpha) = c_{2,8}(\alpha) = 0,$$

$$c_{2,5}(\alpha) = \alpha \cosh(\alpha h),$$

$$c_{2,6}(\alpha) = \alpha \sinh(\alpha h), \tag{A4}$$

$$c_{2,7}(\alpha) = \alpha e^{-\alpha h},$$

$$c_{3,1}(\alpha) = \alpha h \cosh(\alpha h) - \frac{\mu}{\kappa+\mu}\sinh(\alpha h),$$

$$c_{3,2}(\alpha) = \alpha h \sinh(\alpha h) - \frac{\mu}{\kappa+\mu}\cosh(\alpha h),$$

$$c_{3,3}(\alpha) = \alpha \sinh(\alpha h),$$

$$c_{3,4}(\alpha) = \alpha \cosh(\alpha h), \tag{A5}$$

$$c_{3,5}(\alpha) = \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\alpha \sinh(\alpha h),$$

$$c_{3,6}(\alpha) = \left(\frac{\chi B_0}{2\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\alpha \cosh(\alpha h),$$

$$c_{3,7}(\alpha) = c_{3,8}(\alpha) = 0,$$

$$c_{4,1}(\alpha) = \alpha h \sinh(\alpha h) - \frac{\kappa+2\mu}{\kappa+\mu}\cosh(\alpha h),$$

$$c_{4,2}(\alpha) = \alpha h \cosh(\alpha h) - \frac{\kappa+2\mu}{\kappa+\mu}\sinh(\alpha h),$$

$$c_{4,3}(\alpha) = \alpha \cosh(\alpha h),$$

$$c_{4,4}(\alpha) = \alpha \sinh(\alpha h), \tag{A6}$$

$$c_{4,5}(\alpha) = \left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\alpha \cosh(\alpha h),$$

$$c_{4,6}(\alpha) = \left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\alpha \sinh(\alpha h),$$

$$c_{4,7}(\alpha) = c_{4,8}(\alpha) = 0,$$

$$c_{5,1}(\alpha) = c_{5,2}(\alpha) = c_{5,3}(\alpha) = c_{5,4}(\alpha) = c_{5,6}(\alpha) = c_{5,7}(\alpha) = 0,$$

$$c_{5,5}(\alpha) = -\alpha, \tag{A7}$$

$$c_{5,8}(\alpha) = \alpha,$$

$$c_{6,1}(\alpha) = c_{6,3}(\alpha) = c_{6,5}(\alpha) = c_{6,7}(\alpha) = 0,$$

$$c_{6,2}(\alpha) = -\left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+3\mu}{\kappa+\mu}\right),$$

$$c_{6,4}(\alpha) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\alpha, \tag{A8}$$

$$c_{6,6}(\alpha) = \left[2\left(\frac{\chi b_c}{\mu_r}\right)^2\left(\frac{\mu}{\kappa+\mu}\right) - \mu_r\right]\alpha,$$

$$c_{6,8}(\alpha) = \alpha,$$

$$c_{7,1}(\alpha) = -\left(\frac{\kappa+2\mu}{\kappa+\mu}\right),$$

$$c_{7,2}(\alpha) = c_{7,4}(\alpha) = c_{7,6}(\alpha) = c_{7,7}(\alpha) = c_{7,8}(\alpha) = 0,$$

$$c_{7,3}(\alpha) = \alpha, \tag{A9}$$

$$c_{7,5}(\alpha) = \left(\frac{\chi B_0}{\mu\mu_r}\right)\left(\frac{\kappa+2\mu}{\kappa+\mu}\right)\alpha,$$

$$c_{8,1}(\alpha) = c_{8,3}(\alpha) = c_{8,5}(\alpha) = c_{8,7}(\alpha) = c_{8,8}(\alpha) = 0,$$

$$c_{8,2}(\alpha) = -\left(\frac{\mu}{\kappa+\mu}\right),$$

$$c_{8,4}(\alpha) = \alpha, \tag{A10}$$

$$c_{8,6}(\alpha) = \left(\frac{\chi B_0}{\mu_r}\right)\left(\frac{1}{\kappa+\mu}\right)\alpha,$$

and

$$D_1(\alpha,\xi) = \left(\frac{\chi B_0}{\mu_0\mu_r}\right)\left\{\left(\frac{\kappa+\mu}{\kappa+2\mu}\right)\left[1+\frac{1}{2}\left(\frac{\chi b_c}{\mu_r}\right)^2\left(\frac{\kappa+3\mu}{\kappa+\mu}\right)\right] \times \{(1-\alpha h)\sinh(\alpha\xi) + \alpha\xi\cosh(\alpha\xi)\} - \left[1+\mu_r+\left(\frac{\chi b_c}{\mu_r}\right)^2\right]\sinh(\alpha\xi)\right\}e^{-\alpha h}, \tag{A11}$$

$$D_2(\alpha,\xi) = -\left(\frac{\chi B_0}{\mu_0\mu_r}\right)\sinh(\alpha\xi)e^{-\alpha h}, \tag{A12}$$

$$D_3(\alpha,\xi) = \left\{ \left( \frac{\kappa+\mu}{\kappa+2\mu} \right) \left[ 1 + \frac{1}{2} \left( \frac{\chi b_c}{\mu_r} \right)^2 \left( \frac{\kappa+3\mu}{\kappa+\mu} \right) \right] \right.$$
$$\left. \times [(1-\alpha h)\sinh(\alpha\xi) + \alpha\xi\cosh(\alpha\xi)] \right\} e^{-\alpha h}, \tag{A13}$$

$$D_4(\alpha,\xi) = -\left\{ \left( \frac{\kappa+\mu}{\kappa+2\mu} \right) \left[ 1 + \frac{1}{2} \left( \frac{\chi b_c}{\mu_r} \right)^2 \left( \frac{\kappa+3\mu}{\kappa+\mu} \right) \right] \right.$$
$$\times \{(1-\alpha h)\sinh(\alpha\xi) + \alpha\xi\cosh(\alpha\xi)\}$$
$$- \frac{\mu}{\kappa+2\mu} \left[ \left( \frac{\kappa+\mu}{\mu} \right) \right.$$
$$\left. \left. + \frac{1}{2} \left( \frac{\chi b_c}{\mu_r} \right)^2 \right] \sinh(\alpha\xi) \right\} e^{-\alpha h}, \tag{A14}$$

$$D_5(\alpha,\xi) = -\left( \frac{\chi B_0}{\mu_0 \mu_r} \right) e^{-\alpha\xi}, \tag{A15}$$

$$D_6(\alpha,\xi) = 0, \tag{A16}$$

$$D_7(\alpha,\xi) = \left\{ \left( \frac{\kappa+\mu}{\kappa+2\mu} \right) \left[ 1 + \frac{1}{2} \left( \frac{\chi b_c}{\mu_r} \right)^2 \left( \frac{\kappa+3\mu}{\kappa+\mu} \right) \right] \right.$$
$$\times \{\sinh(\alpha\xi) + \alpha\xi\cosh(\alpha\xi)\}$$
$$\left. - \frac{\mu}{\kappa+2\mu} \left[ \left( \frac{\kappa+\mu}{\mu} \right) + \frac{1}{2} \left( \frac{\chi b_c}{\mu_r} \right)^2 \right] e^{-\alpha\xi} \right\}, \tag{A17}$$

$$D_8(\alpha,\xi) = 0. \tag{A18}$$

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