Observation of millimeter-wave radiation generated by the interaction between an electron beam and a photonic crystal

doi: 10.1103/PhysRevE.69.045601

PHYSICAL REVIEW E **69**, 045601(R) (2004)

Observation of millimeter-wave radiation generated by the interaction between an electron beam and a photonic crystal

K. Yamamoto,[1,*] R. Sakakibara,[2] S. Yano,[1] Y. Segawa,[1,2] Y. Shibata,[3] K. Ishi,[3] T. Ohsaka,[3] T. Hara,[4] Y. Kondo,[4] H. Miyazaki,[4] F. Hinode,[5] T. Matsuyama,[5] S. Yamaguti,[6] and K. Ohtaka[7]

[1]*Photodynamics Research Center, The Institute of Physical and Chemical Research (RIKEN), Sendai 980-0845, Japan*

[2]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

[3]*Institute of Multidisciplinary Research for Advanced Materials, Tohoku University, Sendai 980-8577, Japan*

[4]*Department of Applied Physics, Tohoku University, Sendai 980-8579, Japan*

[5]*Laboratory of Nuclear Science, Tohoku University, Sendai 982-0826, Japan*

[6]*Graduate School of Science and Technology, Chiba University, Chiba 263-8522, Japan*

[7]*Center for Frontier Science, Chiba University, Chiba 263-8522, Japan*

(Received 7 November 2003; published 29 April 2004)

We observed directional light emission in the millimeter-wave region when a high-energy (150 MeV) electron beam passes just above a photonic crystal made of polytetrafluoroethylene beads ($\simeq$3.2 mm in diameter). The relation between the momentum and the energy of the emitted photons strongly suggests that the observed light is generated by the umklapp scattering process that changes the evanescent waves emitted by the electron beam into observable ones. By comparing the observed spectra with calculated ones based on the photonic band structure, we found that generated photons excite the photonic band modes making them observable as enhanced fine structures in the emission spectra.

DOI: 10.1103/PhysRevE.69.045601 PACS number(s): 42.70.Qs, 41.60.−m

Photonic crystals (PhCs) have drawn much attention due to their potential applications to new types of optical materials. Owing to their multidimensional periodic dielectric structures, they show unique optical properties arising from a so-called photonic band gap (PBG) and a photonic band (PB) [1–4]. In these days, light emission phenomena involving the excitation of PB modes have become one of the hottest issues in this field of investigation because they are expected to show unique features; sharply enhanced Smith-Purcell radiation (Refs. [5–7]) and backward-pointing Cherenkov radiation (Ref. [8]) are such examples. These features obviously come from the peculiar energy dispersion of photon (PBs) in PhCs. Recently, an electron energy-loss spectroscopy (EELS) study has reported the Cherenkov effect of a PhC [9]. However, to the best of our knowledge, there is no study on direct observation of the radiation involving the excitation of PB modes. In this paper we would like to report an experimental study of one of the radiation phenomena, which was based on the theoretical prediction in Refs. [5–7], to demonstrate that the light emission involving the excitation of PB modes can indeed be observed.

Let us briefly explain the formulation of the light emission mechanism. An electron running at a constant velocity in the x direction [Fig. 1(a)] emits an evanescent light whose wave vector is defined by

$$\mathbf{k}_i = (k_x, q_y, \Gamma_i). \tag{1}$$

The suffix i of $\mathbf{k}_i$ shows that this evanescent light is taken to be an incident light on the PhC. We define the direction that is perpendicular to the PhC as the z direction [Fig. 1(d)]. Note that the x component of $\mathbf{k}_i$ is fixed at the value $k_x = \omega/v$, where ω is the frequency of the emitted light that varies from zero to infinity, and v is the velocity of the running charge. The evanescent wave can be expressed by the superposition over q_y values. Γ_i is the quantity that governs the propagation in the z direction of the evanescent light and which is defined by

$$\Gamma_i = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - q_y^2} \equiv \sqrt{\frac{\omega^2}{c^2} - \frac{\omega^2}{v^2} - q_y^2}. \tag{2}$$

Since v is always smaller than c, Γ_i is always an imaginary number. This is why the direct light from the charge is evanescent.

The umklapp process in the subsequent light scattering of the evanescent light by a periodic structure (a grating or a PhC) converts it into a light which can be observed at a far-field observation point. In a triangular lattice of a two-dimensional (2D) periodic array of beads [see Fig. 1(a)], which is the PhC used in the present study, basis vectors of the reciprocal space are [see Fig. 1(b)]

$$\mathbf{K}_1 = \frac{4\pi}{d\sqrt{3}}(1,0) \equiv (K_{1x}, K_{1y}), \tag{3}$$

$$\mathbf{K}_2 = \frac{4\pi}{d\sqrt{3}}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \equiv (K_{2x}, K_{2y}), \tag{4}$$

where d is the diameter of each bead composing the PhC. The radiation due to the umklapp process can then be described by a propagating plane wave whose wave vector is

$$\mathbf{k}_s = (k_x + mK_{1x} + nK_{2x}, q_y + mK_{1y} + nK_{2y}, \Gamma_{mn}), \tag{5}$$

*Present address: Institute of Physics, University of Tsukuba, Tsukuba 305-8571, Japan.

with two integers m and n. By the energy-momentum relation, Γ_{mn} is given by

$$\Gamma_{mn}=\sqrt{\frac{\omega^2}{c^2}-\left(\frac{\omega}{v}+mK_{1x}+nK_{2x}\right)^2-(q_y+mK_{1y}+nK_{2y})^2}. \tag{6}$$

In the (k,ω) space, therefore, the dispersion line of the evanescent light is shifted by $m\mathbf{K}_1+n\mathbf{K}_2$, from the outside to the inside region of the light cone whose boundary is the light line $(\omega=ck)$ [see Fig. 1(c)]. In the following, we call these shifted evanescent light dispersion lines $\mathbf{H}_{m,n}$ lines, using a set of (m,n) that defines the wave vector of the propagating light after the scattering.

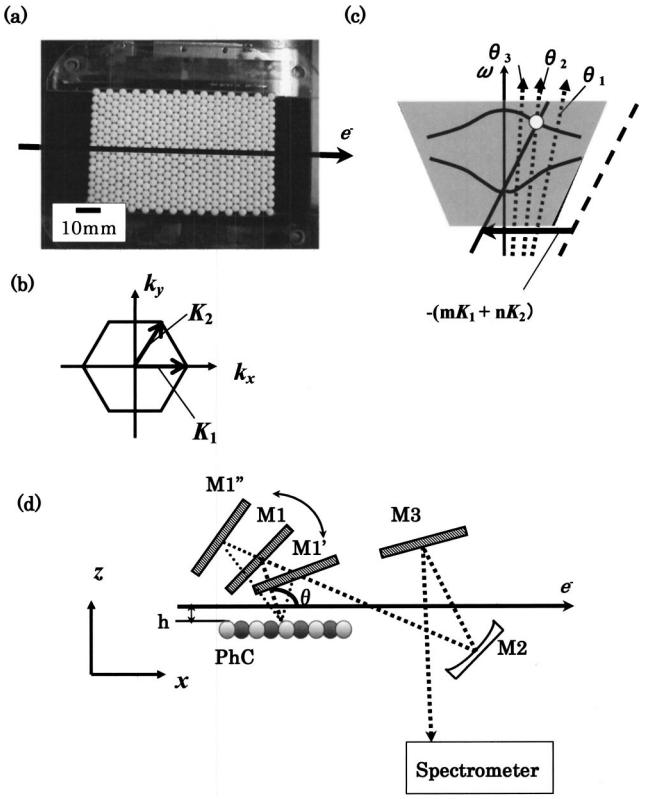

FIG. 1. (a) A picture of monolayer triangular lattice of PTFE spheres used in the present study as a PhC sample. The charge current, which is shown by a solid arrow, runs in the x direction. (b) Two basis vectors ($\mathbf{K}_1$ and $\mathbf{K}_2$) in the reciprocal lattice space of the PhC used in the present study. The Brillouin zone is enclosed by thick solid lines. (c) An illustration of the umklapp scattering process in the $((k_s)_x,\omega)$ plane. The dispersion line of the evanescent light (broken line) generated by the running charge goes into the light cone (shaded area) to become an $\mathbf{H}_{m,n}$ line (solid lines) by the umklapp momentum transfer in the course of the scattering (see the text). The dispersion curves of PBs are schematically shown by solid curves. The θ-constant spectra obtained in the experiment are along the dotted arrows in the $((k_s)_x,\omega)$ plane. The point of intersection between an $\mathbf{H}_{m,n}$ line and a PB dispersion curve and an observation line $[(k_s)_x=(\omega/c)\cos\theta_2]$ is shown by an open circle. (d) Experimental setup. M1, M3: plane mirrors; M2: concave mirror; h: beam height. The position of mirror M1 can be changed from M1′ to M1″.

If we use a metallic grating as a periodic structure, the effect formulated above is known as Smith-Purcell radiation (SPR) [10,11]. The difference from SPR arises when we use PhCs in place of a metallic grating; the photons on the $\mathbf{H}_{m,n}$ lines are expected to excite PB modes. It is predicted theoretically that a strongly enhanced sharp light emission will be generated from PhCs [5–7]. Such a strongly enhanced light emission reflects the confinement effect of electromagnetic energies, which results from the high Q value of an excited PB [12]. The purpose of the present study is to observe such resonant emission of photons by exciting a PB mode (REPEP).

We prepared the experimental setup illustrated in Fig. 1(d) at the Laboratory of Nuclear Science (LNS), Tohoku University. Electrons with the energy of 150 MeV from the *S*-band linear accelerator passed near the PhC in Fig. 1(d). The duration of a burst of electron pulses was 1.8 μs, whose repetition rate was 16.67 Hz. The average beam current was typically 1.0 μA. The cross section of the beam was about 10×12 mm^2. The height of the beam shown in Fig. 1(d) was 10 mm. In order to obtain emission angle dependence, we varied the angle θ of Fig. 1(d) from 60° to 112° by moving the mirror M1. The acceptance angle of the optical system was $\pm0.75°$ in the xz plane, and $\pm1.4°$ in the yz plane. The radiated light was directed to a Martin-Puplett type Fourier-transform spectrometer equipped with a liquid helium-cooled Si bolometer. The resolution of the spectrometer was 3.75 GHz.

The PhC used in this study [Fig. 1(a)] is a single layer of polytetrafluoroethylene (PTFE) beads periodically arrayed in a 2D triangular lattice. The diameter of a bead is 1/8 inch ($\simeq$3.2 mm). In a single layer of beads arrayed in a 2D lattice structure, each whispering gallery (WG) mode of a bead, specified by the angular momentum index (l,m), couples with WG modes of other beads to form tight binding bands with 2D dispersion. The $(2l+1)$-fold degeneracy of the WG mode is partly lifted in a lattice of beads due to this mode mixing, producing the densely populated 2D PB structure. The existence of such PB structures in PhCs of single layered beads has already been probed by the incident-angle-dependent transmission spectra of them, and very good agreement with the calculated dispersion relation was confirmed [13–15].

Let us turn our eyes on the experimental result and discussion. Figures 2(b) and 2(c) show spectral changes in the emitted light as θ is varied. We can see sharp peaks with a band width [full width at half maximum] (FWHM) typically of about 5 GHz. The spectrum is thus characterized by the presence of monochromatic light emissions. It should be noted that these emission peaks present strong θ dependence. For example, the peaks indexed as $(-1,0)$, $(-2,0)$, and $(-3,0)$ change their intensities drastically within the angle variation of 5° in Figs. 2(b) and 2(c).

To determine the origin of the directional light emission, the measured intensity data in the (θ,ω) plane was converted to those of $((k_s)_x,\ \omega)$, where $(k_s)_x$ is the x component of $\mathbf{k}_s$ defined by Eq. (5). The values of $(k_s)_x$ are derived from θ by using the relation $(k_s)_x=(\omega/c)\cos\theta$. The y component of $\mathbf{k}_s$ can be considered to be zero $[(k_s)_y=0]$ because the mirror

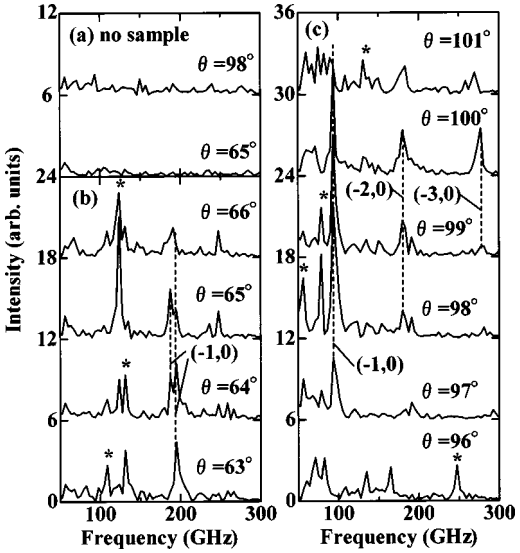

FIG. 2. Emission-angle (θ) dependence of the spectra of emitted light [panels (b) and (c)]. The emission peaks on the $\mathbf{H}_{m,n}$ lines are indexed as (m,n) (see text). The dotted lines are a guide for the eyes. For comparison, panel (a) shows the spectrum with no PhC.

M1 in our measurement system [Fig. 1(d)], which stands perpendicular to the *xz* plane, always moves within the *xz* plane. The contour map of the intensity thus obtained is shown in Fig. 3. We can see that intense light emission indeed occurs around the $\mathbf{H}_{-1,0}$ line, and even around the $\mathbf{H}_{-2,0}$ and $\mathbf{H}_{-3,0}$ lines (Fig. 3). This indicates that the emitted photons corresponding to these parts are generated through the PhC and emitted into the direction determined by Eq. (5).

In the emission spectra, however, there are some peaks that are not on the $\mathbf{H}_{m,n}$ lines: such emission peaks are indicated by asterisks in Figs. 2(b) and 2(c). These emission peaks correspond to the part with strong emission intensity below the $\mathbf{H}_{-1,0}$ line in the $((k_s)_x,\ \omega)$ plane (Fig. 3). One of

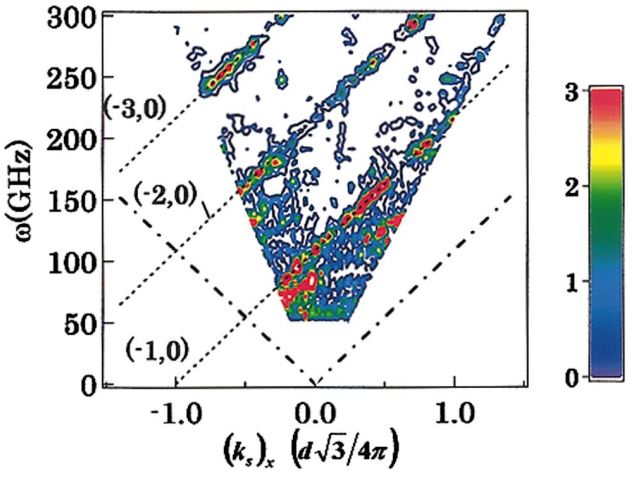

FIG. 3. (Color) Contour map of the light emission intensity in the $[(k_s)_x, \omega]$ plane. The dotted–broken lines show the boundary of the light cone. The dotted lines show the $\mathbf{H}_{-1,0}$, $((-1,0))$, the $\mathbf{H}_{-2,0}$, $((-2,0))$, and the $\mathbf{H}_{-3,0}$ $((-3,0))$ lines.

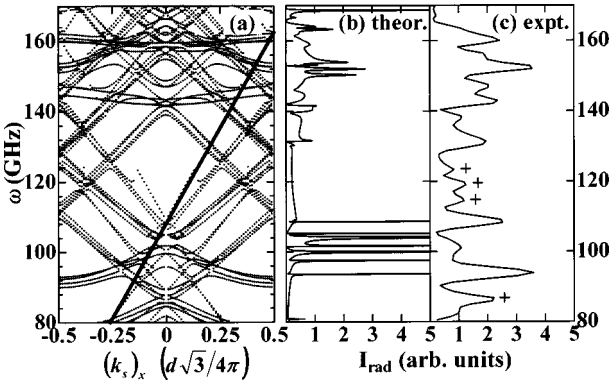

FIG. 4. Calculated PB structure of the single layer PTFE PhC (a), and a comparison between the theoretical (b) and experimental (c) emission spectra. Dotted curves and a solid line in (a) correspond to the PBs and $\mathbf{H}_{-1,0}$ line, respectively. The emission spectra in (b) and (c) are those along the $\mathbf{H}_{-1,0}$ line (see the text). As for the peaks indicated by crosses, see Ref. [20].

the possible origins is an excitation of PBs by the transition radiation, because the area of the light emission in the $((k_s)_x,\ \omega)$ plane almost coincides with the calculated PB dispersion curves. Since we cannot clearly identify the origin of these light emissions at this stage, we will focus only on the light emission peaks on the $\mathbf{H}_{m,n}$ lines in the following discussion.

Next, let us discuss the origin of the strong θ dependence of the emission intensity along the $\mathbf{H}_{m,n}$ lines. If it were not for any dispersion curve of special modes in the $((k_s)_x,\ \omega)$ plane, the emission intensity would show a smooth variation on θ. This is the case in a grating system, where the SPR spectrum has generally a smooth change except the fine structures brought about by Wood anomaly [16,17]. Moreover, fine structures of SPR spectra have their θ dependence much smaller than those observed here [7]. The strong θ dependence observed here therefore proves that the emission is indeed of REPEP origin, which is schematically explained in Fig. 1(c). By varying ω of the observed photons with θ kept fixed, we can experimentally sweep $((k_s)_x,\ \omega)$ space along the line $\omega = c(k_s)_x/\cos\theta$ (called the observation line). An enhanced light emission occurs only when all three curves, the observation line, the $\mathbf{H}_{m,n}$ line, and the dispersion curve of a PB, meet together in the $((k_s)_x,\ \omega)$ space. In Fig. 1(c), three observation lines are drawn to show that only the θ_2 line has an enhanced signal. By varying θ, we can tilt the observation line to spoil the matching. This is the origin of the rapid θ dependence on the $\mathbf{H}_{m,n}$ line. Since PBs have finite lifetimes, a dispersion curve has a finite width, which determines the sharpness of the θ dependence of REPEP signal.

If the peaks of the spectrum indeed arise from the excitation of PB modes along the $\mathbf{H}_{m,n}$ line, they should reflect the PB structure. In order to confirm this point, we calculated the PB structure and the spectrum of the emitted light by using the vector Koringa-Kohn-Rostoker (KKR) formalism [1,18,19], which is the same method as that presented in Ref. [7]. The value of the complex refractive index of the PTFE

beads used in the calculation of the emission spectrum was $1.437+0.0002i$ $(i^2=-1)$ [13]. We can find a good correspondence between the calculated emission peak positions [Fig. 4(b)] and the position where the PBs cross with the $\mathbf{H}_{-1,0}$ line [Fig. 4(a)].

Then let us compare the calculated emission spectrum [Fig. 4(b)] and the observed one [Fig. 4(c)]. The spectrum was calculated along with a dispersion line $\mathbf{H}_{-1,0}$ [Fig. 4(a)] in the $((k_s)_x, \omega)$ plane. Since the intensity of the experimental results shown in Fig. 4(c) is not corrected by the difference in intensities arising from the difference in optical paths between each θ-const spectrum [see Fig. 1(d)], we can only discuss the peak positions of the observed light emission. Besides the peaks indicated by crosses [20], the peak positions in the frequency ranges of 140–170, 125–140, and 90–110 GHz in the observed spectrum correspond well to those in the calculated one [21]. This provides strong evidence that the observed light emission reflects a PB structure, which is characteristic of the REPEP.

In conclusion, we observed REPEP from a PhC using an electron beam running just above the PhC. The observed light emission spectrum consists of many sharp peaks. The rapid variation in the emission peaks with change in the direction of observation shows that the peaks indeed arise accompanying excitation of the PB modes. The excitation of the PB modes was also confirmed by a comparison of experimentally obtained spectrum and the calculated one reflecting the PB structure.

We thank Mr. T. Tsutaya, Mr. N. Ohara, Mr. M. Kanbe, Dr. N. Horiuchi, and the staff of LNS, Tohoku University, for their help in the experiment. The present work was supported by a Grant-In-Aid for Scientific Research and Special Coordination Funds from the Ministry of Education, Culture, Sports, Science and Technology, Japan, and by the Special Postdoctoral Research Program of RIKEN, Japan.

[1] K. Ohtaka, Phys. Rev. B **19**, 5057 (1979).
[2] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[3] E. Yablonovitch and T.J. Gmitter, Phys. Rev. Lett. **63**, 1950 (1989).
[4] E. Yablonovitch, J. Phys.: Condens. Matter **5**, 2443 (1993).
[5] K. Ohtaka and S. Yamaguti, Opt. Spectrosc. **91**, 477 (2001).
[6] K. Ohtaka and S. Yamaguti, Opt. Quantum Electron. **34**, 235 (2002).
[7] S. Yamaguti, J. Inoue, O. Haeberlé, and K. Ohtaka, Phys. Rev. B **66**, 195202 (2002).
[8] C. Luo, M. Ibanescu, S.G. Johonson, and J.D. Joannopoulos, Science **299**, 368 (2003).
[9] F.J. García de Abajo, A.G. Pattantyus-Abraham, N. Zabala, A. Rivacoba, M.O. Wolf, and P.M. Echenique, Phys. Rev. Lett. **91**, 143902 (2003).
[10] S.J. Smith and E.M. Purcell, Phys. Rev. **92**, 1069 (1953).
[11] G. Doucas, J.H. Mulvey, M. Omori, J. Walsh, and M.F. Kimmitt, Phys. Rev. Lett. **69**, 1761 (1992).
[12] The optical density of states (ODOS) spectrum of a PhC has fine structures reflecting the PB structure. The bandwidths of the fine structures in the ODOS spectrum reflect the *Q* values of the PBs. We confirmed by calculation (not shown) that the light emission intensity correlates quite well to the bandwidths of the fine structures in the ODOS spectrum of the PhC, which means that strong light emission results from the high *Q* values of PBs.
[13] T. Kondo, thesis, Osaka University, Japan, 2002.
[14] K. Ohtaka, Y. Suda, S. Nagano, T. Ueta, A. Imada, T. Koda, J.S. Bae, K. Mizuno, S. Yano, and Y. Segawa, Phys. Rev. B **61**, 5267 (2000).
[15] T. Kondo, M. Hangyo, S. Yamaguchi, S. Yano, Y. Segawa, and K. Ohtaka, Phys. Rev. B **66**, 033111 (2002).
[16] O. Haeberlé, P. Rullhusen, J.-M. Salomé, and N. Maene, Phys. Rev. E **49**, 3340 (1994); *ibid.* **55**, 4675 (1997).
[17] Y. Shibata, S. Hasebe, K. Ishi, S. Ono, M. Ikezawa, T. Nakazato, M. Oyamada, S. Urasawa, T. Takahashi, T. Matsuyama, K. Kobayashi, and Y. Fujita, Phys. Rev. E **57**, 1061 (1998).
[18] K. Ohtaka, J. Phys. C **13**, 667 (1980).
[19] K. Ohtaka and Y. Tanabe, J. Phys. Soc. Jpn. **65**, 2265 (1996).
[20] Although we cannot clarify the origins of the peaks, which are absent in the calculated spectrum, we can speculate that these peaks also arise from the excitation of the photonic band modes. Some of the PBs and the $\mathbf{H}_{-1,0}$ line may cross at the energy of these peaks due to a slight difference in the PB structure from what is calculated. A possible reason for the change in the PB structures is a slight deformation of the PhC sample.
[21] The difference in the emission intensity ratios among the peaks between the observed and calculated spectra might partly be caused by the damping effect due to the slight distortion of the PhC lattice structure. Since these modes are strongly confined and stay a longer time within the PhC, they are affected more substantially by the damping effect while they stay within the PhC. According to our calculation, the sharp peaks are easily affected by the damping effect, and weaken their intensity. This may especially be the case for the peaks at around 100 GHz.