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著者	宮嵜　博司
journal or publication title	Physical review. A
volume	53
number	4
page range	2877-2880
year	1996
URL	http://hdl.handle.net/10097/35677

doi: 10.1103/PhysRevA.53.2877

PHYSICAL REVIEW A VOLUME 53, NUMBER 4 APRIL 1996

Multiphotonic lattices and Stark localization of electromagnetic fields in one dimension

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(Received 19 July 1994; revised manuscript received 25 July 1995)

By extending the concept of a photonic lattice (PL), we have studied the one-dimensional multiphotonic lattice (MPL) composed of a sequence of different types of PL's. Each PL plays the role of either a well or a barrier for the electric field due to the difference of photonic band gaps. As a result, bound states of the electric field appear near the band edges, which are strikingly reminiscent of electrons or holes in semiconductor quantum wells. This analogy is confirmed by studying the Stark localization of electric field in the MPL composed of alternate stacks of well-like and barrierlike PL's.

PACS number(s): 42.50.Hz, 41.20.Jb, 73.20.Fz, 77.55.+f

The recent proposal and subsequent investigation of photonic lattices (PL's) have opened up an exciting field of research in quantum electrodynamics [1,2]. A one-dimensional (1D) PL, for example, is composed of alternate stacks of dielectric slabs having different dielectric constant and thickness. Periodic modulation of the propagation of the electromagnetic field within the PL yields photonic band gaps at the center and the edges of the Brillouin zone [3,4]. Research on PL's seems to be oriented in several directions: one is to find 2D and 3D PL's having complete band gaps. By introducing defects into these systems, one finds within the band gaps the isolated defect modes that can be utilized to manufacture semiconductor microlasers with almost perfect quantum efficiency and extremely low threshold [1–3]. Another direction of research is to study Anderson localization of photons in the PL's [5]. This is also a fascinating problem since photons in the PL of linear optical elements have no interaction with each other so that one can study Anderson localization without many-body effects. Atomic injection into the PL's [6] is also interesting in that the spontaneous emission from the injected atoms is strongly suppressed if the emitted photon energy falls within the band gaps of the PL's. This enables us to control the lifetime of the excited atoms, making it possible to very accurately determine their energy levels. One can also use the atomic injection to control the photon states of PL's and to create new states such as squeezed or Schrödinger cat states [7].

The band gaps of the PL appear as a result of the periodic modulation of the dielectric constant [8]. This is in close analogy with semiconductor multiple quantum wells (MQW's) or superlattices [9] in which electron or hole wave functions are modulated by the periodic sequence of barriers and wells, yielding mini-bands. Because of the recent development of microfabrication technology, the quantum well (QW) offers an enormous field of fundamental as well as applied research into resonant tunneling [10], Bloch oscillation [11], and Stark localization [12] of the electron and hole in 1D MQW's by a static electric field, 2D electron gas at the interface and 1D quantum wires [13] and 0D quantum dots and their application to single-electron transistors [14]. It would be interesting to mimic a variety of these phenomena by the electromagnetic field in PL's.

A key step in this direction is to note the importance of the band gaps and the dispersion relation. In MQW's much of the interesting properties relies on the band offset and the quadratic dispersion of the electrons or holes. In this respect, a direct way to relate the PL's with the MQW's is to regard each dielectric slab in the PL as a constituent atomic layer of a MQW such as Ga, Al_xGa_{1-x} or As layers. This correspondence has already been pointed out by Yeh, Yariv, and Hong [15] and the concept of photonic band gaps has been known for a long time as stop bands in the theory of the optical properties of thin-film multilayer dielectric stacks [16]. According to this correspondence, the photonic version of the MQW is a sequence of finite-size 1D PL's, each of which has a different photonic band structure. This system may be called a MPL or a photonic superlattice. Because of the band-gap difference of the constituent PL's, each PL may play the role of a well or a barrier for the electromagnetic field propagating within the multiphotonic lattice (MPL). In addition, the frequency dispersion of each PL near the band-gap region is quadratic. In this paper, we study the bound states of the electromagnetic field near the band-gap region of the well in the MPL and show that it is well described by the envelope function formalism used in semiconductor QW theory. In addition, it is shown that the Stark localization of the electromagnetic field can be realized in the MPL's.

Let us consider a 1D MPL composed of a β PL sandwiched by two α PL's. The α (β) PL is composed of stacks of alternating slabs having dielectric constants $\{\varepsilon_a,\varepsilon_b\}$ ($\{\varepsilon_c,\varepsilon_d\}$) and thicknesses $\{d_a,d_b\}$ ($\{d_c,d_d\}$). The system is in vacuum and the electromagnetic field is incident normally from the left-hand side. It should be noted here that the photonic band structure generally depends on the direction of the propagation of the electromagnetic field because of the inherent transverse vector nature of the photon field [2]. Since we are interested in the analogy between photons and electrons or holes, we will concentrate in this paper on the case of normal incidence. This reduces the vector nature of the photon field to the scalar one. We choose α and β to be composed of 20 pairs of dielectric slabs having $\{\varepsilon_a=3, \varepsilon_b=1, d_a=d_b=1$ cm$\}$ and 10 pairs of slabs having $\{\varepsilon_c=4, \varepsilon_d=1, d_c=d_d=1$ cm$\}$, respectively. The upper band edge of the lowest band gap of α and β turns out to be $\omega_\alpha^0=6.3225$ GHz and $\omega_\beta^0=5.8733$ GHz. Therefore, α and β PL's are ex-

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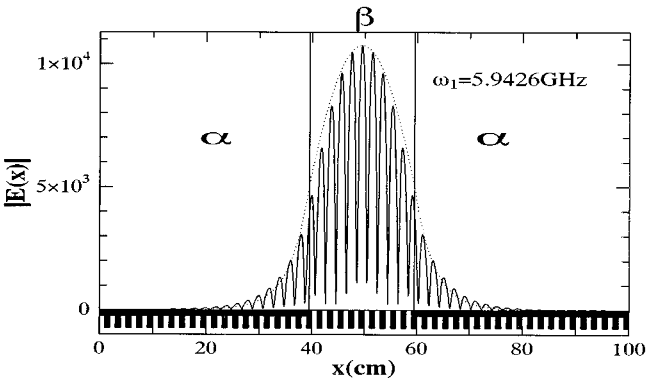

FIG. 1. Absolute amplitude $|E(x)|$ of the electric field of the symmetric bound state in the MPL composed of a well-like β PL sandwiched by two barrier-like α PL's. $|E(x)|$ is normalized to the amplitude of the light incident from the left-hand side. Vertical lines represent the interfaces between α and β PL's and the dotted line shows the envelope function $F(x)$ normalized to the maximum amplitude of the exact numerical calculation. Below the zero line of the figure we plot a schematic shape of $\varepsilon(x)$ as an array of the white vertical bars. The base white part gives the region of $\varepsilon=1$ and short or long vertical white bars represent slabs with $\varepsilon=3$ or 4, respectively.

pected to play the role of a barrier and a well for the electric field of frequency ω between ω_β^0 and ω_α^0, respectively. The electric field of the frequency ω inside the system is obtained by the method of optical matrix transfer for multilayer thin films [16]. By scanning ω between ω_β^0 and ω_α^0, we find that a perfect transmission occurs at $\omega_1=5.9426$ GHz, $\omega_2=6.1171$ GHz, and $\omega_3=6.3149$ GHz. Figure 1 shows the absolute amplitude of the total electric field $E(x)$ for ω_1 as a function of position x inside the system. The left end of the system is taken as the origin of x. We have also shown in the bottom of each figure the schematic shape of $\varepsilon(x)$ as an array of the vertical white bars. As seen from the figure, this mode is a symmetric ground-state mode within the β region and decays exponentially in the α regions. This form of $E(x)$ could never be observed if one replaces β by a uniform dielectric slab. One should also note that the envelope of this ground-state mode is very similar to that of electron or hole wave functions in the single QW.

While the present system can be treated exactly by the matrix transfer method, it would be worthwhile to deal the problem within the framework of the envelope function formalism in the QW theory [17]. Within single photonic lattice, say, α, the Maxwell's equations can be written as

$$H(x)E(x)=-\frac{c^2}{\varepsilon(x)}\frac{\partial^2}{\partial x^2}E(x)=\omega^2E(x), \tag{1}$$

where $\varepsilon(x)$ is the dielectric constant with period $d=d_a+d_b$. Since $\varepsilon(x)$ is periodic, the solution to Eq. (1) with the eigenfrequency $\omega=\omega_n(k)$ satisfies the Bloch theorem

$$E_n^k(x+d)=\exp(ikd)E_n^k(x), \tag{2}$$

where n is the band index and k is the wave number. When the PL is finite, the electric field within the PL can be expanded, in general, in terms of $\{E_n^k(x)\}$ and the Fourier transform of the expansion coefficients $\{A_n(k)\}$ defines the envelope function $F_n(x)$ for each band index n. It is easy to show that $F_n(x)$ satisfies the following equation within the one-band approximation [18]:

$$\{\omega_n(k-k_0\Rightarrow -id/dx)-\omega\}F_n(x)=0, \tag{3}$$

where k_0 gives the maximum or minimum of $\omega_n(k)$. The $\omega_n(k)$ of the α and β PL's near the band edge $k_0=\pi/d$ is expressed as a quadratic function of $q=k-k_0$ as $\omega_n(k)=\omega_n^0+K_nq^2/2$. In the present case we have $K_\alpha=13.6086$ GHz cm^2 and $K_\beta=10.1215$ GHz cm^2. Hence $F(x)$ satisfies exactly the same equation for the electron wave function of the QW. To connect $F_\alpha(x)$ and $F_\beta(x)$ at the interfaces x_0, we make use of the similarity between the present case and that of the QW and adopt $F_\alpha(x_0)=F_\beta(x_0)$ and $K_\alpha F'_\alpha(x_0)=K_\beta F'_\beta(x_0)$. We also assume the α PL's to be semi-infinite.

The results of calculation using the envelope function formalism give $\omega_1^{app}=5.9524$ GHz, in excellent agreement with that of the numerical calculation. For ω_2 we obtain $\omega_2^{app}=6.1763$ GHz, in not as good agreement because of the deviation of $\omega_n(k)$ from the quadratic form. The dotted line of Fig. 1 shows the envelope function normalized to the maximum amplitude of the exact calculation. The boundaries between α and β are taken to be at 39.5 and 59.5 cm.

Thus we have shown that each PL serves as a well or a barrier for the electric field just like those for electrons or holes in the MQW. We call these a photon well and a photon barrier hereafter. It would then be an interesting problem to see if the Stark localization of the electric field could occur in the MPL's similarly to the case of MQW's under the static electric field [12]. The Stark localization of the electron or hole occurs when the electrostatic potential drop ΔV between the adjacent wells exceeds the interaction J between them. The degree of localization is mainly determined by $\tau=\Delta V/J$ [19]. Larger values of τ destroy the coherent transfer of the electron or hole through the MQW and result in the stronger Stark localization.

To study the Stark localization of the electric field in the MPL's we adopt a system composed of six photon wells $\{\beta_1,\beta_2,...,\beta_6\}$ and five photon barriers $\{\alpha_1,\alpha_2,...,\alpha_5\}$ sandwiched by substrate photon barriers γ_L and γ_R, as shown in Fig. 2. For simplicity, all the dielectric slabs have the common thickness of 1 cm. Each length of α_n and β_m is 10 cm and that for γ_L and γ_R is 20 cm. As for $\varepsilon(x)$ we take $\varepsilon=1$ for every right-hand side slab of pairs in α_n, β_m, γ_L, and γ_R. Those of the left-hand side slabs in the photon wells (barriers) start with $\varepsilon_w^1(\varepsilon_b^1)$ for $\beta_1(\alpha_1)$ and end with $\varepsilon_w^6(\varepsilon_b^5)$ for $\beta_6(\alpha_5)$ with a step decrease of $\Delta\varepsilon_w(\Delta\varepsilon_b)$. The dielectric constant ε_L and ε_R of the left-hand side slabs of γ_L and γ_R are chosen so that the field is trapped within the MPL. The system is in vacuum and the incident light comes in normal from the left-hand side.

In the numerical calculation, we adopt common values of $\varepsilon_w^1=5.0$, $\varepsilon_b^1=4.0$, and $\varepsilon_L=\varepsilon_R=3.0$ and choose two cases of $\Delta\varepsilon_w$ and $\Delta\varepsilon_b$: $\Delta\varepsilon_w=\Delta$, $\varepsilon_b=0.05$ and Δ $\varepsilon_w=\Delta$, $\varepsilon_b=0.2$. In both cases the transmission spectra show several sharp peaks between $\omega=5.4970$ and 6.3225 GHz, which are the upper band edge of γ_L $(=\gamma_R)$ and β_1, respectively. Figures 2(a) and 2(b) show the electric field $|E(x)|$ of the third lowest bound

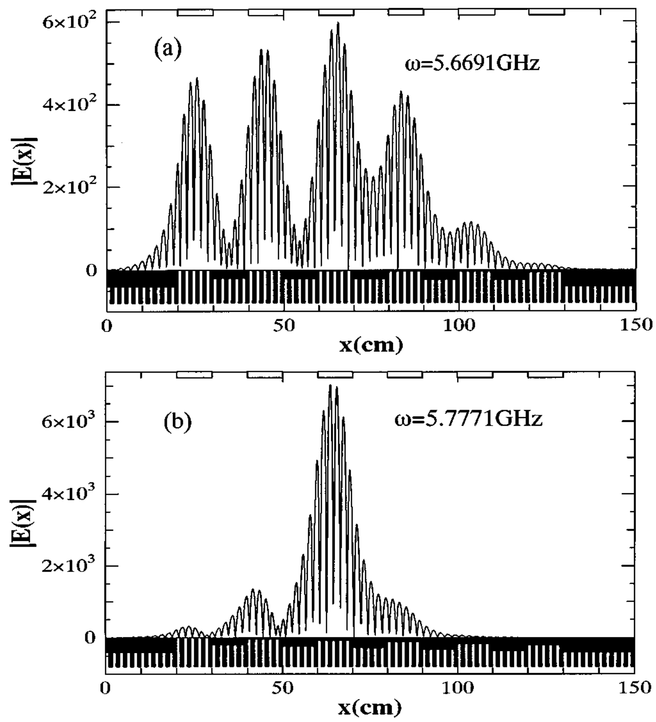

FIG. 2. Absolute amplitudes $|E(x)|$ of the electric field of the third bound state in the two types of MPL's composed of six photon wells and five photon barriers. $|E(x)|$ is normalized to the amplitude of the light incident from the left-hand side. The white rectangular boxes in the upper coordinate show the well regions. For values of parameters in (a) and (b) see the text. Below the zero line of the figure we plot a schematic shape of $\varepsilon(x)$ as an array of the white vertical bars. The base white part gives the region of $\varepsilon=1$ and the left and right ten white bars represent the slabs with $\varepsilon=3$. In the middle region of six photon wells or five photon barriers ε starts from 5 or 4 and decrease with (a) $\Delta\varepsilon_w=\Delta$, $\varepsilon_b=0.05$ and (b) $\Delta\varepsilon_w=\Delta$, $\varepsilon_b=0.1$, respectively.

state for $\Delta\varepsilon_w=\Delta$, $\varepsilon_b=0.05$ and $\Delta\varepsilon_w=\Delta$, $\varepsilon_b=0.2$, respectively. As can be seen from the figure, the electric field tends to localize around the third photon well β_3 as $\Delta\varepsilon_w$ and $\Delta\varepsilon_b$ are increased. This behavior can be explained from the analogy to the Stark localization of electrons in the MQW. The decrease of ε_w^n or ε_b^n generally pushes up the upper band-edge frequency ω_n^0 [3,15]. This increase of ω_n^0 has qualitatively the same effect as the increase of the potential bottom in the MQW due to the static electric field. In the MPL, ΔV is given by the difference of ω_n^0 between the adjacent wells or barriers. ΔV for wells (barriers) is estimated on average to be 14.7 and (16.7 MHz) and 62.7 and (70.5 MHz) in Figs. 2(a) and 2(b), respectively. The interaction J between the adjacent photon wells is roughly given by 9.5 MHz for a system of double photon wells, each composed of five pairs of slabs with $\varepsilon_a=5.0$, $\varepsilon_b=1.0$, and $d_a=d_b=1$ cm separated by a photon barrier of five pairs of slabs with $\varepsilon_a=4.0$, $\varepsilon_b=1.0$, and $d_a=d_b=1$ cm. By assuming J to be almost constant within the present choice of parameters, $\tau=\Delta V/J$ is given as 1.5~1.7 and 6.6~7.4 for Figs. 2(a) and 2(b), respectively. Thus we can see that the increase of $\Delta\varepsilon_w$ and $\Delta\varepsilon_b$ results in the stronger Stark localization as a result of the increase of τ.

One might think that the localized nature of the bound states is mostly due to the substrate photon barriers γ_L and γ_R. The calculation of the same system without γ_L and γ_R, however, shows qualitatively the same behavior, though the lowest state at β_1 is missing and the amplitude of the electric field of each bound state is much reduced. It should further be noticed that when the number of the pair slabs is decreased from 5 to 1 in the same system without γ_L and γ_R, the internal electric field shows almost no evidence of localization [20]. Therefore, the Stark localization of the present system is attributed to the presence of the band edge of each photon well and barrier. We would also like to point out that the amplitude of the internal electric field of the first two lowest bound states is much reduced for the right incidence of light because of the asymmetry of the present system [20].

The important parameters of the MPL's are the band-edge frequency ω_n^0 and the coefficient K_n^0 of the quadratic dispersion of each PL near the band edge. To get a better simulation of the Stark localization, it is desirable to have a linear variation of ω_n^0 and a common value of K_n. For this purpose one can change the spacing of each of the dielectric slabs linearly instead of their dielectric constants. This would be quite favorable for the experiments. One can also introduce another type of PL whose unit cell is composed of three dielectric slabs. This will widen the choice of parameters. The connection rule of the envelope functions at the interfaces deserves further study since even in the MQW's this problem is still under study [21].

Having made a correspondence between the MPL's and the MQW's, it would be interesting to simulate a variety of phenomena observed in the MQW's by using the MPL's. In the MQW's, for example, various cases of heterojunctions are under study [9], but the problem is always accompanied by the uncertainties of the actual value of the band offset. In the MPL's this is not the case, since the band gap can always be determined accurately by the numerical calculation. One can also study the transient response of the pulse injection into the MPL's [20,22]. It is easily verified that the time dependence of the envelope function is described by an equation analogous to the time-dependent Schrödinger equation in the QW. The input pulse may be considered as a wave packet of electrons or holes so that the Bloch oscillation [11] of the input pulse is expected. In addition, one could expect the periodic emission of pulses from the system during the Bloch oscillation. The time dependence of the envelope function in the double photon well can also be used to study the tunneling problem of electrons in the MQW [10]. Another interesting problem is to introduce the interaction between the bound states either by the injection of atoms or by the use of nonlinear dielectric slabs [23]. The idea of MPL's may be extended to 1D photon wires or 0D photon dots in analogy to the quantum wires and quantum dots. Since the vector nature of the electromagnetic field is especially important in these cases [2], it would be interesting to see whether the present analogy still survives for these cases [22]. Finally, it would be very intriguing to study the true particle-like character of photons in the MPL's, since the present study uses only the wavelike character of photons.

There would be at least three possibilities of realizing the MPL's from the thin-film multilayer structures. One is the rugate filter in which the refractive index is modulated sinusoidally [24]. If the modulation differs spatially this would serve as a MPL. The second one is a MQW semiconductor laser sandwiched by the distributed Bragg reflector (DBR) [25]. While the DBR is literally a PL, the central MQW may not be considered as a PL because of its small size compared with the wavelength of the relevant electric field. This would rather be classified as a Fabry-Pérot resonator. The third one is a high transmission comblike filter in which various PL's of finite size are stacked in one direction [26]. While the comblike filter is introduced as a multipurpose wideband filter, it can be considered as the MPL considered in this paper if we limit our attention to the frequency region near the band edges.

In summary, we have introduced a MPL and have shown that each PL plays the role of either a well or a barrier for electromagnetic field due to the difference of the photonic band gaps. We have pointed out that there is a striking analogy of the MPL's to the semiconductor MQW's if one considers the envelope function of the electromagnetic field within the MPL's. This analogy was further confirmed by the evidence of the Stark localization of the electric field in the MPL's. Various possibilities of research using the MPL's have been suggested.

This work is supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture, Japan.

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