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Analysis of the resonance characteristics of a cantilever vibrated photothermally in a liquid

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The resonance characteristics of stainless-steel cantilevers vibrated photothermally in a liquid have been investigated using an optical detection system. Resonance frequencies of vibrating cantilevers in liquids are lower than those in air as a result of the action of reaction forces in the liquid. It is shown that experimental values of the resonance frequencies for several types of cantilevers in water agree well with those calculated. The half power width at resonance broadens with increasing viscosity of the liquid. The possibility for using this photothermal vibration method as an optical sensing system for the density or viscosity of liquids is described.

I. INTRODUCTION

Photoacoustic or photothermal vibration can be used to measure the vibration parameters of a solid resonator in a medium, such as its resonance frequency or mechanical Q factor. Since this technique has a significant advantage of being noncontact, it is useful and powerful not only for spectroscopy but also for material characterization.[1] As an application of this technique, the influence of thermoelastic bending (drum effect[2]) on the photoacoustic signal was examined theoretically and experimentally in connection with thermal diffusivity measurements of materials.[3] Other methods utilizing thermoelastic bending have been adopted for detecting or probing delamination in layered materials,[4,5] and platelike samples.[6] Imaging techniques using thermoelastic vibration for platelike samples have also been reported.[7] Moreover, this technique can be used in the inspection of solder joints,[8] tension measurement,[9] and in the improvement of the sensitivity of spectroscopic studies of GaAs wafers.[10]

In addition to the above investigations in nondestructive diagnostics, pressure sensors have been recently proposed in which vibrating plates interact with environmental gases, with the shift of resonance frequency of each plate detected as a function of gas pressure. For example, two types of optical vacuum gauges for the low vacuum region[11] (from 1 to 760 Torr) and medium vacuum region[12–15] (from 10^{-3} to 1 Torr) have been developed. There have been many such reports on the photothermal vibration technique. However, test media were mainly restricted to solids or gases. It is also of interest to investigate the behavior of photothermal vibrations in liquids, especially since there are so few such investigations. In our previous work, it was found that when a resonator of the cantilever structure was vibrated photothermally in water, the resonance vibration could be observed but the resonance frequency and the Q factor were very small compared with those in air.[16] The small value of the resonance frequency was explained as a result of acoustic emission from the resonator, and also it was pointed out that the resonance frequency depends on the liquid density.[17]

In this work, resonance characteristics of several types of rectangular cantilevers in several kinds of liquids have been measured, and values of resonance frequencies have been compared with theoretically estimated ones. The density and viscosity of the test liquid can be determined from the resonance frequency and the resonance width, respectively. Since the resonance frequency contains information relating to both density and viscosity, the influence of viscosity on determining density from the measurement of resonance frequency should be considered. For this purpose, we prepared two kinds of liquids of equal density but with different coefficients of viscosity. As a result of this work, the possibility of an optical viscosity sensor for liquids based on the principle of photothermal vibration is described. The optical detection system adopted in this work seems to be very useful in as much that (1) density measurement is possible with a high response time and with a very small amount of liquid, and (2) measurement under inaccessible conditions is possible, such as in deep sea applications. Moreover, although frequency measurement is important in this method, it has the advantages of high precision, fast time response, high sensitivity, and noise immunity.

II. EXPERIMENT

Figure 1 shows a schematic drawing of a typical cantilever made of stainless steel (type 304). The cantilever, structure of length L, width W, and thickness T, was prepared in four rectangular shapes (taking a set of $L\times W$ as 1.0×0.2, 1.5×0.3, 2.0×0.5, and 3.0×0.45 mm^2, and varying in T from 0.01 to 0.05 mm) by the chemical etching technique. One side end of it was clamped and the other end was free.

An experimental apparatus is schematically shown in Fig. 2. The test membrane was placed in a depth of 5 cm under the surface level of every test liquid. The liquid tem-

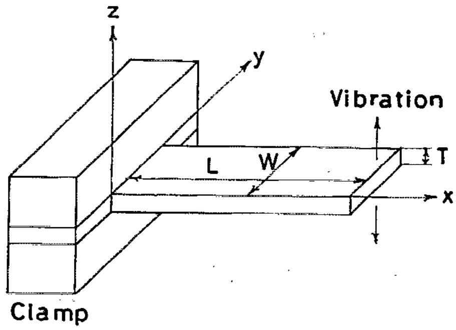

FIG. 1. Schematic drawing of the vibrating cantilever.

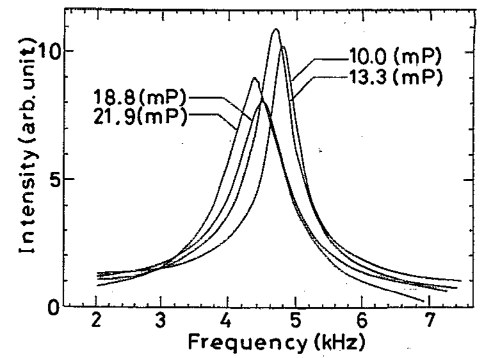

FIG. 3. An example of resonance curves for the vibration amplitude as a function of the frequency. A $1.5\times0.3\times0.02$ (mm^3) cantilever was used. The coefficient of viscosity was varied by mixing glycerol with water.

perature was normally kept to 20±0.2 °C. Four kinds of liquid samples for the experiment were prepared: ethanol, water, a mixed solution of ethanol and water, and a solution of NaCl in water (concentration is less than 20 wt %). Several kinds of mixed solutions of water and glycerol, in which each coefficient of viscosity was varied in stages, were also prepared and used to test the viscosity dependence of the frequency response. A laser diode (output power of the laser, 14.5 mW, and laser wavelength, 833.0 nm) was operated by chopping electronically at constant frequency, and focused on the cantilever to excite photothermal vibrations. The typical spot size of the laser beam on the cantilever was about 1.0 mm in diameter. The vibrating motion was observed by detecting the reflection of a probing He-Ne laser beam. The reflected beam from the cantilever was passed over a knife edge and incident on a photomultiplier. The optical path length of the He-Ne laser beam from the cantilever to the photomultiplier was typically about 50 cm. The vibration amplitude of this cantilever was typically about 0.2 μm or less at low frequencies of less than 1 Hz. The characteristic frequency at which the intensity of the resonance curve for amplitude of a function $z=P(t)$ on the position of the vibrating cantilever becomes a maximum was somewhat less than the resonance frequency due to the fact that the damping force acting on the moving cantilever in liquid was not negligible. There are two methods for eliminating influence from the damping force for determining the resonance frequency. In an earlier work, the resonance frequency was determined by measuring both phases of the laser diode light and the vibration of the cantilever. However, for vibrations at high frequency, the actual phase of the vibrating cantilever might be delayed from that of the laser diode light. Therefore, the phase detector, such as a lock-in amplifier, was not used for determining the resonance frequency. In a harmonic oscillation, the resonance frequency can be determined from the resonance curve for amplitude of a function dz/dt on the velocity of the vibrating cantilever without influence from the damping force.[18] In this work, the signal from the photomultiplier was differentiated electrically. The frequency dependence of the signal of the photomultiplier and that of its differential signal were measured. There is another possible method to measure the velocity, using a self-mixing method with a laser diode,[19] but here it was not applied because of its relative complexity.

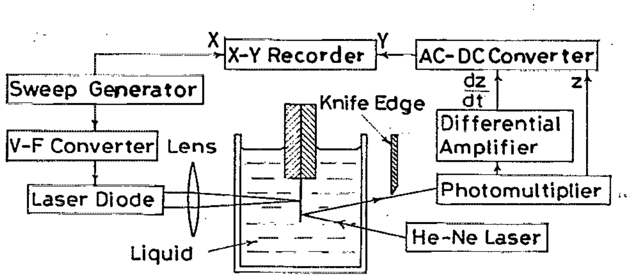

FIG. 2. Schematic figure of the experimental apparatus. The frequency dependences of the vibration amplitude and the vibration velocity were measured.

The vibration amplitude was converted to a dc voltage with an ac-dc converter (National VP-9631A). The operating frequency of the laser diode was scanned by a sweep generator (NF FG-121B) through a voltage-frequency converter (Kenwood FG-273). The resonance curve was obtained on a X-Y recorder (Graphtek WX-1200). For the calibration of this optical system, the density or the coefficient of viscosity of a test liquid was determined from measurements of volume and weight or by using a dip viscosity flow cup (Erichsen Model-321), respectively.

III. RESULTS AND DISCUSSION

A. Resonance frequency of cantilevers

Figure 3 shows the resonance curve of the vibration amplitude of the cantilevers of dimensions $1.5\times0.3\times0.02$ (mm^3) as $L\times W\times T$ shown in Fig. 1, which was measured in water and in a mixed solution of glycerol and water. The resonance frequency and the mechanical Q factor are very low compared with those in air or vacuum, and, as discussed in the next section, these values are dependent on the density and viscosity of the test liquid.

The resonance frequency, f_{vac}, of the cantilever shown in Fig. 1 in vacuum can be described by the following equation:[20]

$$f_{vac}=\frac{CT}{2\pi L^2}\sqrt{\frac{E}{12\rho_0(1-\nu^2)}}. \tag{1}$$

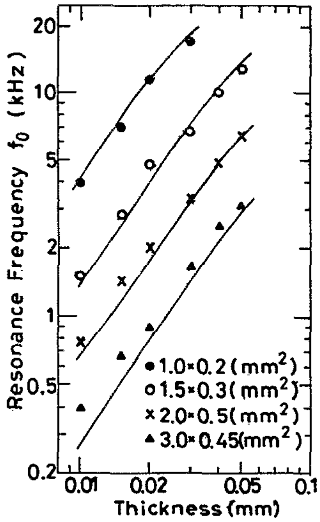

FIG. 4. Resonance frequency f_0 for several cantilevers in water as a function of its thickness. Solid curves are calculated values from Eqs. (1) and (2).

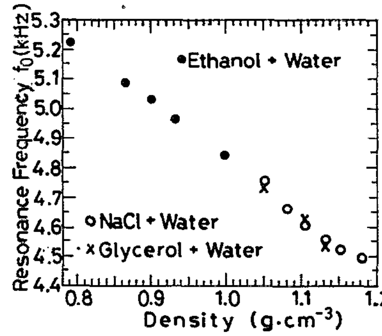

FIG. 5. Resonance frequency f_0 as a function of liquid density. The liquids used for the density less than 1 g/cm^3 are ethanol and its mixture with water, and those more than that value are a solution of NaCl and a solution of glycerol with water. The error of the determination of the resonance frequency is about 40 Hz, which corresponds to the value of 20 mg/cm^3 for the accuracy of the measurement for liquid density.

Here E, ν, and ρ_0 are Young's modulus, Poisson's ratio, and cantilever density, respectively. The value of C depends on the vibrating mode and the ratio W/L. For the quantitative evaluation of the resonance frequency, the kinetic motion for the vibrating cantilever must be considered, and it is important to take into account the inertial part of viscosity drag[21] in the equation of motion and the reaction force from the liquid. Ito and Nakazawa[22] have recently proposed the following equation to describe the resonance frequency, f_0, of a rectangular cantilever vibrating in a gas or liquid having density ρ and coefficient of viscosity η:

$$\frac{f_{\text{vac}}}{f_0}=\sqrt{1+\frac{1}{\rho_0 W}\sqrt{\frac{2\eta\rho}{2\pi f_0}}+\frac{\rho}{\rho_0}\frac{L}{T}F(W/L)\frac{A^2}{B}}. \tag{2}$$

Here, $F(W/L)$ is a function of the aspect ratio for rectangular cantilever. The second term in the root of the right-hand side of Eq. (2) is related to the inertial part of viscosity drag. The last term, which was derived by Greenspon,[23] is due to the reaction force from the fluid and can be obtained from the acoustic reactance for a rectangular piston calculated by Stenzel.[24] The A^2/B term is a constant that is determined from the mode of vibration as follows:

$$A=\frac{\int u(x)u(y)dx\,dy}{LW}, \tag{3}$$

$$B=\frac{\int u(x)^2u(y)^2dx\,dy}{LW}, \tag{4}$$

where $u(x)$ and $u(y)$ in $z=u(x)u(y)$ are deflection functions of the vibrating cantilever.[25] Assuming that $u(y)$ is constant, a numerical value of $A^2/B=0.613$ is typical for the cantilever in Fig. 1.

Figure 4 shows the measured resonance frequencies in four types of cantilevers as a function of thickness. Photothermal vibration for the cantilever thicker than 0.06 mm was not observed clearly with this optical detection system. Solid curves are calculated using Eqs. (1) and (2). In this numerical estimation, the second term in the root of the right-hand side of Eq. (2) gives a very small value, $<1\times10^{-2}$, compared with other two terms. The estimated values of resonance frequencies agree well with the experiment.

Equations (1) and (2) and Fig. 4 show that it is effective to miniaturize the size of cantilever to obtain a higher sensitivity for the measurement of liquid density or viscosity. In fact, a micromachined resonator has been recently tested by us as a pressure sensor in vacuum and resulted in demonstration of a highly sensitive pressure response.[11]

B. Liquid density sensing

According to Eq. (2), the resonance frequency depends on the density of the liquid. In order to develop a liquid density sensor by frequency measurement, the influence from the viscosity drag must be considered. As was shown above, the influence on the resonance frequency of the inertial part of the viscosity drag is negligible, and that of the dissipative part of the viscosity drag (damping force) can be eliminated by determining the resonance frequency from the resonance curve for the vibration velocity rather than for the vibration amplitude. Resonance frequencies, f_0, were measured for several kinds of liquids, and are plotted as a function of density in Fig. 5. The frequency decreased monotonically with an increase in density. The measurement error of the resonance frequency was about 40 Hz, caused mainly by the relatively low Q factor. The measured frequency responds only to the density, independent of the viscosity. The measuring accuracy of density with this method is estimated to be about 20 mg/cm^3 from the result of Fig. 5.

C. Viscosity sensing

By considering a model of harmonic oscillation with an object of mass m, spring constant k, and damping co-

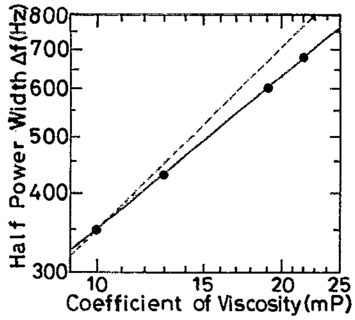

FIG. 6. The half power width that gives the value $1/\sqrt{2}$ of the maximum value of the resonance curve for the vibrational velocity as a function of the coefficient of viscosity for a solution of glycerol with water. The dashed line shows calculated values of Δf as a function of the coefficient of viscosity for virtual liquid having a density of 1.0 g/cm^3.

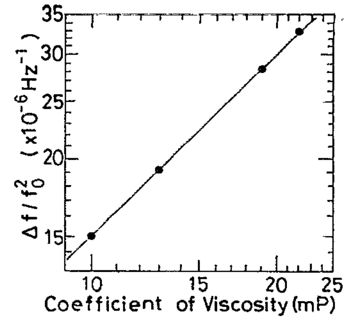

FIG. 7. $\Delta f/f_0^2$ as a function of the coefficient of viscosity. The solid line of unit slope agrees well with experimental values.

efficient $R(\eta)$ (Stokes term), in which the mass is vibrated by an external force, we can derive a simple formula for the half power width Δf as follows:

$$\Delta f = \frac{R(\eta)}{2\pi M}. \tag{5}$$

Here, M includes the mass of the cantilever and an additional mass loading that is caused by the reaction force from the liquid. The additional mass depends on the liquid density as it is seen in Eq. (2). In order to obtain the viscosity of a liquid with the above formula, the mass dependency must be considered. Figure 6 shows measured Δf as a function of the coefficient of viscosity on a log-log plot. It increases monotonically with increase in viscosity. By referring to the textbook of Lamb[26] and the Stokes term $R(\eta)$ for a sphere, a revolving ellipse or a disk is proportional to the coefficient of viscosity. However, the value of the slope of the solid line on the log-log plot in Fig. 6 is 0.85. The dashed line shows calculated values of Δf as a function of the coefficient of viscosity for virtual liquid having a density of 1.0 g/cm^3. The difference between the solid line and dashed line suggests that the mass dependence on Δf in Eq. (5) cannot be neglected. For example, the difference of the liquid density by 10% causes the error of 15% in determining the coefficient of viscosity. The influence from difference of liquid density on the measurement of the coefficient of viscosity can be eliminated by the following equation, in which the mass term is not included:

$$\frac{\Delta f}{f_0^2} = \frac{2\pi R(\eta)}{k}. \tag{6}$$

Figure 7 shows value of $\Delta f/f_0^2$ as a function of the coefficient of viscosity on a log-log plot. Each value fits very well to a straight line with the slope of unity. The linear relationship shows that the Stokes term $R(\eta)$ for the motion of the cantilever in the liquid is proportional to the coefficient of viscosity.

For the four kinds of test liquids, the typical measurement error for the coefficient of viscosity was estimated to be about 5%, which arised mainly from the audio frequency (af) measurement.

IV. CONCLUSIONS

Resonance frequencies for several types of the rectangular cantilever vibrated photothermally were measured in water, and were compared with theoretical values considering the effect of reaction forces in the liquid. It was shown that the optical sensing of both liquid density and the coefficient of viscosity was possible. The accuracy of the determination of these values depends mainly on the mechanical Q factor. The miniaturization of the size of the cantilever is effective in obtaining a higher sensitivity. We believe that this system will be useful for liquids in which only a very small amount is available or under inaccessible conditions.

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