Generalization of Computer Simulation of Dislocation Emission under Constant Rate of Stress Application

doi: 10.1063/1.322174

Generalization of computer simulation of dislocation emission under constant rate of stress application

A. T. Yokobori, Jr., T. Yokobori, and A. Kamei

Department of Mechanical Engineering II, Tohoku University, Sendai, Japan
(Received 10 March 1975; in final form 16 May 1975)

Generalization of computer simulation of dislocation groups emission with the dislocation velocity-stress relation of power type: $v = M\tau^m$ under constant rate of stress application has been attempted, such as for the value of m ranging between 1 and 6.4. For metals in which the velocity of an isolated dislocation is given by a thermally activated process, the number of dislocations emitted up to time t is expressed in terms of a single apparent rate process and the activation energy is a constant multiple [such as $(m+1)/(m+2)$ or $m/(m+2)$] of the activation energy for the motion of an isolated dislocation. The mean mobile dislocation density along one slip plane was obtained, and its temperature and strain rate dependence are in good agreement with the semiexperimental results in literature.

PACS numbers: 61.70.M, 66.30.K

I. INTRODUCTION

In a previous paper[1] the computer simulation of the dynamic behavior under constant rate of stress application for straight parallel coplanar edge dislocations was carried out by using the stress-velocity relation of power type: $v = M\tau^m$ for the case $m = 3$ (pure iron). In the present paper these computations have been repeated for several other metals with values of m ranging between 1 and 6.4.

In the model the equation of the velocity of an isolated dislocation is used as the experimental formula. That is, the model does not require that the velocity is given as a thermally activated process. The case is also considered in which the velocity of an isolated dislocation is given by a thermally activated process. In this case the number of dislocations emitted from the source up to time t is expressed in terms of an apparent single rate process and the activation energy is a constant multiple [such as $(m+1)/(m+2)$ or $m/(m+2)$] of the activation energy for the motion of an isolated dislocation, although the dislocation groups movement is not a single rate process, but consists of a composite rate process.

The mean mobile dislocation density along one slip plane was obtained, and its temperature and strain rate dependences are in good agreement with the semiexperimental results in literature.

II. THE MODEL AND METHOD OF COMPUTATION

The model and method of computation are the same as used in a previous paper.[1] Therefore only two essential equations are repeated. The first one is the power relation between the stress τ and velocity v for each individual dislocation in a linear array:

$$v_i = M\tau_{\mathrm{eff},i}^m, \tag{1}$$

where m is the material constant, $M = v_0(1/\tau_0^*)^m$, τ_0^* is a constant representing the stress required to give a dislocation velocity $v = 1$ cm/sec (the resistant stress against the dislocation motion), $v_0 = 1$ cm/sec, i is the index number of dislocation in order of the emission from the source, and $\tau_{\mathrm{eff},i}$ is the effective stress on individual dislocations in terms of shear stress. The second equation is the nondimensional equation of motion of the ith dislocation in the array:

$$\frac{dS_i}{d\theta} = (m+1)\left(\frac{\dot\tau}{\dot\tau_0}\theta + \frac{Gb}{2\pi(1-\nu)\dot\tau_0 t_0 l}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{S_i - S_j}\right)^m, \tag{2}$$

where $\dot\tau \equiv d\tau/dt$ is the constantly increasing rate of applied stress τ, G is the shear modulus, ν is Poisson's ratio, and b is the Burgers vector. l and t_0 are some specified constants, and $\dot\tau_0$ is the specified rate of stress application. x_i is the position of the ith dislocation, and t is the time. $S_i \equiv x_i/l$, and $\theta = t/t_0$.

III. RESULTS OF CALCULATIONS

A. Dynamic behavior

From the results of calculations, it is shown that the dynamic behavior, such as the ratio of the position of, the effective stress on, and the velocity of the individual dislocation to those of an isolated dislocation is determined only by the dynamic factor $\Theta \equiv (\dot\tau/\dot\tau_0)^{(m+1)/(m+2)}\theta$ or $\eta \equiv \dot\tau^{(m+1)/(m+2)}t$ as far as $\dot\tau$ and t is concerned. For example, the dynamic behavior of the fifth dislocation in the array for tungsten ($m = 4.8$) is shown in Table I. It is also shown that the effective stress $\tau_{\mathrm{eff},i}$ on an individual dislocation in the array is nearly constant and almost equal to the applied stress τ_a. The maximum value of $\tau_{\mathrm{eff},1}/\tau_{\mathrm{iso}}$ becomes slightly large as m de-

TABLE I. The dynamic behavior of the fifth dislocation (for $m = 4.8$). (N is the number of dislocations emitted.)

		x_5/x_{iso}		v_5/v_{iso}		$\tau_{\mathrm{eff},5}/\tau_{\mathrm{iso}}$	
$\dot\tau/\dot\tau_0$		10	10^3	10	10^3	10	10^3
Θ	N						
1.004	10	0.6818	0.6824	1.197	1.197	1.038	1.038
1.196	20	1.110	1.110	1.376	1.377	1.069	1.069
1.312	30	1.229	1.229	1.392	1.393	1.071	1.071

TABLE II. The maximum values of $\tau_{eff,1}/\tau_{iso}$, v_1/v_{iso}, and x_1/x_{iso} for various metals.

	m	τ_0^* (kg/mm^2)	$\left(\frac{\tau_{eff,1}}{\tau_{iso}}\right)_{max}$	$\left(\frac{v_1}{v_{iso}}\right)_{max}$	$\left(\frac{x_1}{x_{iso}}\right)_{max}$
Al (edge)	1[a]	8.5×10^{-5}[a]	1.296	1.296	1.317
Fe (screw)	2.6[b]	50[b]	1.193	1.581	1.741
Fe (edge)	3[c]	6[c]	1.179[d]	1.638[d]	1.837[d]
W282-2 (edge)	4[e]	55.9[e]	1.152	1.762	2.067
W (edge)	4.8[e]	31.4[e]	1.137	1.851	2.218
Mo (edge)	6.4[f]	5.5[f]	1.114	1.998	2.575

[a] K. Sumino, Bull. Jpn. Inst. Met. 10, 758 (1971).
[b] H. Saka, K. Noda, and T. Imura, Cryst. Lattice Defects 4, 45 (1973).
[c] A. P. L. Turner and T. Vreeland, Jr., Acta Metall. 18, 1255 (1970).
[d] See Ref. 1.
[e] H. W. Schadler, Acta Metall. 12, 861 (1964).
[f] H. L. Prekel, A. Lawler, and H. Conrad, Acta Metall. 16, 337 (1968).

creases, whereas the maximum values of v_1/v_{iso} and x_1/x_{iso} become large as m increases (Table II). The effect of the value of τ_0^* on $(\tau_{eff,1}/\tau_{iso})_{max}$, $(v_1/v_{iso})_{max}$, and $(S_1/S_{iso})_{max}$ is small.

B. The physical meaning of the dynamic factor

In a previous paper[1] the calculation was made for the case of $N=2$ and from the results the reasoning was made that the number N of dislocations emitted is controlled mainly by the interaction of the source and the last dislocation. Here the reason why N is controlled by the dynamic factor $\Theta \equiv (\dot\tau/\dot\tau_0)^{(m+1)/(m+2)}(t/t_0)$ is studied generally and more precisely.

When a new dislocation is going to be emitted from the source, that is, $\tau_{eff,s}=\tau_s$ becomes zero, we assume that the back stress on the source exerted by all dislocations in the array is a multiple $1/k$ of the back stress due to the last dislocation. Thus a new dislocation is emitted from the source when the following equation is satisfied:

$$\frac{\tau_{eff,s}}{\dot\tau_0 t_0}=\frac{\dot\tau}{\dot\tau_0}\theta-\frac{Gb}{k2\pi(1-\nu)}\frac{1}{\dot\tau_0 t_0 l S_L}=0. \tag{3}$$

In addition, let us put

$$\frac{S_L}{S_{iso}}=a_L, \tag{4}$$

TABLE III. Values of $k(N)$ (for pure iron).

N \ $\dot\tau/\dot\tau_0$	10^3	10^4
10	0.693	0.692
20	0.642	0.642
30	0.619	0.619

TABLE IV. Values of $a_L(N)$ (for pure iron).

N \ $\dot\tau/\dot\tau_0$	10^3	10^4
10	0.0242	0.0243
20	0.00830	0.00850
30	0.00467	0.00468

where S_L is the position of the last dislocation at the time a new dislocation is emitted. For instance, for $m=3$ (for pure iron) the values of k and a_L calculated against N with the parameter of $\dot\tau/\dot\tau_0$ are shown in Tables III and IV. The result is similar for other values of m. In Table V the value of k against N is shown for various metals. From these results, for various values of $\dot\tau/\dot\tau_0$, each value of k and a_L takes the same value corresponding to the same value of N, respectively. Therefore, k and a_L will be expressed as a function of N, that is,

$$k=k(N) \tag{5}$$

and

$$a_L=a_L(N). \tag{6}$$

Thus from Eq. (4), and the relation $S_{iso}=(\dot\tau/\dot\tau_0)^m\theta^{m+1}$ in a previous paper,[1] the following equation is obtained:

$$S_L=a_L(N)(\dot\tau/\dot\tau_0)^m\theta^{m+1}. \tag{7}$$

Eliminating S_L from Eqs. (3) and (7), we obtain

$$\left(\frac{\dot\tau}{\dot\tau_0}\right)^{m+1}\theta^{m+2}=\frac{Gb}{2\pi(1-\nu)}\frac{1}{a_L(N)k(N)\dot\tau_0 t_0 l} \tag{8}$$

and

$$\Theta\equiv\left(\frac{\dot\tau}{\dot\tau_0}\right)^{(m+1)/(m+2)}\theta$$

$$=\left(\frac{Gb}{2\pi(1-\nu)}\frac{1}{a_L(N)k(N)\dot\tau_0 t_0 l}\right)^{1/(m+2)}. \tag{9}$$

The right-hand side of Eq. (9) takes the unique value corresponding to the same value of N, because both $a_L(N)$ and $k(N)$ decrease with an increase of N, independent of $\dot\tau/\dot\tau_0$ as shown in Tables III and IV. Thus by the computer simulation, we can conclude that the number of dislocations emitted is controlled by the dynamic factor Θ.

Furthermore, it can be seen from Table V, the stress exerted by the last dislocation on a new dislocation going to emit at the source is, say 50–80% of the total back stress (the percentage increasing with the increase of m), although the last dislocation plays an important role in dislocation emission.

Nevertheless, note that it seems difficult to predict

TABLE V. Values of $k(N)$ with parameter of m.

N \ m	1.0	3.0	4.0	4.8	6.4
10	0.56	0.69	0.73	0.75	0.78
20	0.48	0.64	0.68	0.71	0.75
30	0.47	0.62	0.66	0.69	0.73

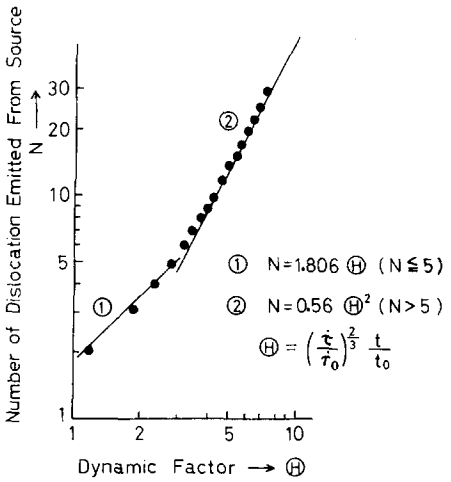

FIG. 1. The representation of the relation between the number N of dislocations emitted from a source and the dynamic factor Θ for pure alminum ($m=1$). Solid circles were calculated by computer.

that without using computer simulation, other dynamic behavior, such as, the ratio of the position of, the effective stress on, and the velocity of the individual dislocation to those of an isolated dislocation is determined only by the dynamic factor Θ.

C. The representative equation of the number N of dislocations emitted

From the reasoning described in Sec. III B, the following simple general formula is justified, which is proposed tentatively for large N in a previous paper[1]:

$$N=\gamma(m)(M/Gb)^{(m+1)/(m+2)}\eta^{m+1}, \tag{10}$$

where

$$\eta\equiv\dot{\tau}^{(m+1)/(m+2)}t$$

and $\gamma(m)$ is the nondimensional value depending on m.

In the present paper N for several metals has been obtained. The physical constants are used from the experimental data in the literature. The calculated results show that for large N, N is quite well approximated by Eq. (10) and for small N also in good approximation expressed by the following simple formula:

$$N=\gamma_0(m)(M/Gb)^{m/(m+2)}\eta^m, \tag{11}$$

where $\gamma_0(m)$ is the nondimensional value depending on m.

TABLE VI. The dynamic factor η and the formula for the number N of dislocations emitted for various metals.

Metals and type of dislocation	m	$\eta=\dot{\tau}^{\frac{(m+1)}{(m+2)}}t$	N for smaller N	N for larger N
Al (edge)	1	$\dot{\tau}^{2/3}t$	1385η ($N\leqq 5$)	$3.293\times10^5\eta^2$ ($N>5$)
Fe (screw)	2.6	$\dot{\tau}^{3.6/4.6}t$	$0.5081\eta^{2.6}$ ($5\leqq N\leqq 20$)	$0.1182\eta^{3.6}$ ($N>20$)
Fe (edge)	3	$\dot{\tau}^{4/5}t$	$6.458\eta^3$ ($5\leqq N\leqq 20$)	$3.956\eta^4$ ($N>20$)
W238-2 (edge)	4.0	$\dot{\tau}^{5/6}t$	$0.001965\eta^{4.0}$ ($N\leqq 30$)	$0.0001539\eta^{5.0}$
W (edge)	4.8	$\dot{\tau}^{5.8/6.8}t$	$0.0006938\eta^{4.8}$ ($N\leqq 30$)	$0.00006423\eta^{5.8}$ ($N>30$)
Mo (edge)	6.4	$\dot{\tau}^{7.4/8.4}t$	$0.02055\eta^{6.4}$ ($N\leqq 30$)	$0.005171\eta^{7.4}$ ($N>30$)

TABLE VII. Values of $\gamma_0(m)$ and $\gamma(m)$ with parameter of m.

m	1	2.6	3	4.0	4.8	6.4
$\gamma_0(m)$	2.637	1.445	0.9432	0.5458	0.3687	0.2090
$\gamma(m)$	1.191	0.5023	0.3043	0.1745	0.1263	0.07557

For example, in Fig. 1 the calculated data for N edge dislocations in pure aluminum ($m=1$) are plotted, and line ① is the theoretical one obtained from Eq. (11) and line ② is obtained from Eq. (10).

The good approximation formula for the calculated relations between N and η for various metals is shown in Table VI. The calculated values of $\gamma(m)$ and $\gamma_0(m)$ are shown in Table VII and Fig. 2. From these data $\gamma(m)$ and $\gamma_0(m)$ are in good approximation expressed by the following formulas:

$$\gamma(m)=1.396m^{-1.45} \tag{12}$$

$$\gamma_0(m)=3.380m^{-1.34}. \tag{13}$$

On the other hand, the following relation is well established in $v=M\tau^m$:

$$M=v_0\left(\frac{1}{\tau_0^*}\right)^m. \tag{14}$$

Now let us denote by t_a the time required for the applied stress to reach τ_a, that is,

$$t_a=\tau_a/\dot{\tau}. \tag{15}$$

Substituting Eqs. (14) and (15) into Eqs. (10) and (11), and analyzing dimensionally, we obtain for large N:

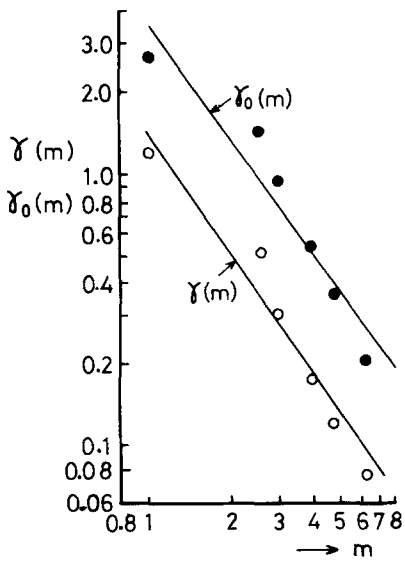

FIG. 2. The value of $\gamma_0(m)$ and $\gamma(m)$.

$$N=\gamma(m)\left(\frac{b}{t_a v_0}\right)^{-(m+1)/(m+2)}\left(\frac{\tau_0^*}{G}\right)^{-m(m+1)/(m+2)}\left(\frac{\tau_a}{G}\right)^{(m+1)^2/(m+2)} \tag{16}$$

and for small N

$$N=\gamma_0(m)\left(\frac{b}{t_a v_0}\right)^{-m^2/(m+2)}\left(\frac{\tau_0^*}{G}\right)^{-m^2/(m+2)}\left(\frac{\tau_a}{G}\right)^{m(m+1)/(m+2)}. \tag{17}$$

D. The case in which v is given by a thermally activated process

In the model described above Eq. (1) is used as the empirical formula. That is, the model does not require that the velocity of an isolated dislocation is given by a thermally activated process. In this section, let us consider the case in which the velocity v of an isolated dislocation is given[2] by a thermally activated process as follows:

$$v=A_1\exp(-H/kT), \tag{18}$$

where A_1 is a constant, the product of the area swept out per successful thermal agitation, the number of sites per unit length of the dislocation at which thermal activation can occur, and the frequency of vibration of the dislocation segment concerned. H is the activation energy of the rate process and is expressed[3] as

$$H=H_k\left(1+\tfrac{1}{4}\ln\frac{16\tau_p^0}{\pi\tau^*}\right) \tag{19}$$

in which H_k is the kink energy, τ_a is the applied shear stress, τ_μ is the long-range internal stress, τ^* is the effective stress $=\tau_a-\tau_\mu$, and τ_p^0 is the Peierls stress at 0 °K. Here in the case of high-purity low in-grown dislocation density crystals, τ_μ is assumed very small compared with τ_a. T is absolute temperature.

Thus, substituting Eq. (19) into Eq. (18) and rearranging, we get

$$v=M\tau_a^m, \tag{20}$$

where

$$m=H_k/4kT, \tag{21}$$

$$M=v_0(1/\tau_0^*)^m, \tag{22}$$

$$\tau_0^*=\tau_{00}(A_1/v_0)^{-1/m},\qquad \tau_{00}\equiv e^4(16/\pi)\tau_p^0, \tag{23}$$

and $v_0=1$ cm/sec.

Substituting Eq. (23) into Eq. (16), using Eq. (21) and rearranging, the number N of dislocations emitted up to time t_a is given by

$$N=A_*t_a^{(m+1)/(m+2)}\left(\frac{\tau_a}{G}\right)^{(m+1)/(m+2)}\times\exp\left[-\frac{m+1}{m+2}H_k\ln\left(\frac{\tau_{00}}{\tau_a}\right)(4kT)^{-1}\right], \tag{24}$$

where

$$A_*=\gamma(m)(b/A_1)^{-(m+1)/(m+2)}.$$

For usual materials such as pure iron ($m=3$ at room temperature) in the range between -100 and 0 °C, the value of $(m+1)/(m+2)$ is from 0.85 to 0.80, and for pure aluminum ($m=1$ at room temperature) in the range between 0 and 300 °C, $(m+1)/(m+2)$ is from 0.67 to 0.60, which is not very dependent on temperature. On the other hand, the change of the value of the logarithm of $\gamma(m)$ with temperature change is neglected as compared with the change of the logarithm of $A_*t_a^{(m+1)/(m+2)}(\tau_a/G)^{(m+1)/(m+2)}$ in Eq. (24) in the range $573\geq T\geq 273$ °K for pure iron. Thus Eq. (24) shows an equation of Arrehenius type. It is to be noted that the number N of dislocations emitted from the source up to time t_a from the instant of application of stress under constant stress rate is expressed in terms of an apparent single rate process, and the apparent activation energy is $(m+1)/(m+2)$ times the activation energy for the motion of an isolated dislocation, although the dislocation groups movement is not a single rate process, but consists of a composite rate process.

From small number of N, using Eq. (17) we can obtain

$$N=A_{**}t_a^{m/(m+2)}\left(\frac{\tau_a}{G}\right)^{m/(m+2)}\times\exp\left[-\frac{m}{m+2}H_k\ln\left(\frac{\tau_{00}}{\tau_a}\right)(4kT)^{-1}\right], \tag{25}$$

where

$$A_{**}=\gamma_0(m)(b/A_1)^{-m/(m+2)}.$$

It can be seen that the argument made on Eq. (24) holds on Eq. (25) also.

E. Mean mobile dislocation density along one slip plane

The mean mobile dislocation density f_{md} along one slip plane is represented by x_1, the distance moved by the lead dislocation as follows:

$$f_{md}=N/x_1. \tag{26}$$

On the other hand, x_1 can be written as follows[1]:

$$x_1=a_1x_{iso}=a_1\frac{M}{m+1}\dot{\tau}^m t^{m+1}, \tag{27}$$

where a_1 is approximately estimated as constant[1] over the range of the larger N. Substituting Eqs. (10) and (27) into Eq. (26), f_{md} is expressed as follows:

$$f_{md}=\frac{k^1(m)}{b}\left(\frac{b\dot{\tau}}{G^{m+1}M}\right)^{1/(m+2)}, \tag{28}$$

where $k^1(m)=\gamma(m)(m+1)/a_1$. From Eqs. (10), (11), and (28), it is conjectured that when the resistant stress τ_0^* is small, that is, M is large, the mean mobile dislocation density on slip plane is small, although a dislocation is emitted easily from the source. Let us assume that the mean mobile dislocation density D_{md} per unit area is expressed as follows:

$$D_{md}=f_{md}^2. \tag{29}$$

Substituting Eq. (28) into Eq. (29) and rearranging, we obtain

$$D_{md}=\frac{[k^1(m)]^2}{b^2}\left(\frac{b\dot{\tau}}{G^{m+1}M}\right)^{2/(m+2)}. \tag{30}$$

From experimental data,[4] τ_0^* may be expressed as

$$\tau_0^*=\tau_{00}\exp[-n_1k(\ln b_1)T] \tag{31}$$

which is equivalent to the theoretical equation (23). n_1 and b_1 are positive constants. In the case of germanium, however, m in Eq. (21) is experimentally practically independent of temperature.[4] For germanium, using experimental data,[4] τ_0^* is expressed as

$$\tau_0^* = 1.93\times10^8 \exp(-0.01680T). \tag{32}$$

Let us assume applied stress rate $\dot{\tau}$ is proportional to tensile strain rate $\dot{\epsilon}$, that is

$$\dot{\tau} = \tfrac{1}{2}\dot{\sigma} = \tfrac{1}{2}E\dot{\epsilon}. \tag{33}$$

Substituting Eqs. (14), (31), and (33) into Eq. (30), we obtain

$$\log_{10} b^2 D_{md} = 2\log_{10}\left[k^1(m)\left(\frac{b(1+\nu)}{v_0}\right)^{1/(m+2)}\left(\frac{\tau_{00}}{G}\right)^{m/(m+2)}\right] + \frac{2}{m+2}\log_{10}\dot{\epsilon} - \frac{2}{2.303}\frac{m}{(m+2)} n_1 k(\ln b_1) T. \tag{34}$$

Using $b = 4\times10^{-7}$ mm, $\nu = 0.3$, $G \approx 5\times10^3$ kg/mm^2 and experimental data, $\tau_{00} = 1.93\times10^8$ kg/mm^2, and $n_1 k \ln b_1 = 0.01680$, Eq. (34) becomes

$$\log_{10} b^2 D_{md} = 0.2286 + \frac{2}{m+2}\log_{10}\dot{\epsilon} - \frac{2}{2.303}\frac{m}{m+2} n_1 k(\ln b_1) T. \tag{35}$$

For constant strain rate $\dot{\epsilon}$ of 1.1×10^{-4} sec, Eq. (35) becomes

$$\log_{10} b^2 D_{md} = -2.03 - 0.006253T. \tag{36}$$

For constant temperature T of 873 °K, Eq. (35) becomes

$$\log_{10} b^2 D_{md} = -5.23 + 0.571\log_{10}\dot{\epsilon}. \tag{37}$$

It is interesting to note that if we assume D_{md} essentially equal to ρ, Eqs. (36) and (37) are quantatively in accordance with the semiexperimental data,[5] which were obtained by assuming the relation $\dot{\epsilon} = \rho v b$ in usual tensile testing under several constant strain rates $\dot{\epsilon}$ and temperatures, respectively.

IV. DISCUSSION

In the treatment the source activation stress τ_s is assumed to be zero. Let us study the effect of τ_s on the dynamic factor for the case of τ_s having some positive value. Let us denote by t_s the time when the first dislocation is emitted from the source, and define T^* as

$$T^* = t - t_s, \tag{38}$$

where t is the time measured from the instant of load application. Then by the similar treatment as in a previous paper,[1] for the case of $\tau_s > 0$ we obtain the dynamic factor as follows:

$$\eta^c = \dot{\tau}^{(m+1)/(m+2)} T^* \left[\left(1+\frac{t_s}{T^*}\right)^{m+1} - \left(\frac{t_s}{T^*}\right)^{m+1}\right]^{1/(m+2)} \tag{39}$$

which is equivalent to η for the case $\tau_s = 0$. For the case $T^* \gg t_s$, η^c will be

$$\eta^c = \dot{\tau}^{(m+1)/(m+2)} T^*. \tag{40}$$

If some positive value is taken for τ_s, the number of dislocations emitted will be smaller than for the case $\tau_s = 0$. The effect, however, may be expected to be small, by inference from the result obtained by Kanninen and Rosenfield[6] on the influence of the source activation stress τ_s for the case of pile-up formation under constant stress.

V. CONCLUSIONS

From computer simulation of dynamics of linear arrays of edge dislocations emitted from a source under constant rate of stress application in the case of a power relation between stress and dislocation velocity, the following conclusions were obtained.

(i) The dynamic behavior such as the ratio of the position x_i of, the ratio of the effective stress $\tau_{eff,i}$ on, and the ratio of the velocity v_i of an individual dislocation in the array to those of an isolated dislocation and the number N of dislocations emitted up to time t are determined only by the dynamic factor Θ $[\equiv (\dot{\tau}/\tau_0)^{(m+1)/(m+2)} t/t_0]$, as far as $\dot{\tau}$ and t are concerned.

(ii) For various metals in the range $m = 1$–6.4 the effective stress on the dislocation groups calculated is nearly constant and almost equal to the applied stress, and the velocity of the lead dislocation is lower than two times the velocity of an isolated dislocation.

(iii) The number N of dislocations emitted up to time t is obtained from the following simple and good approximation formulas:

$$N = \gamma(m)(M/Gb)^{(m+1)/(m+2)}\eta^{m+1}, \quad \text{for larger } N$$

$$N = \gamma_0(m)(M/Gb)^{m/(m+2)}\eta^m, \quad \text{for smaller } N,$$

where $\eta \equiv \dot{\tau}^{(m+1)/(m+2)} t$ (the dynamic factor in dimensional form).

(iv) For metals in which the velocity of an isolated dislocation is given by a thermally activated process, the number of dislocations emitted from the source up to time t from the instant of stress application is still expressed in terms of an apparent single rate process, and the apparent activation energy is a constant multiple [such as $(m+1)/(m+2)$ or $m/(m+2)$] of the activation energy for the motion of an isolated dislocation, although the dislocation groups movement is not a single rate process but consists of a composite rate process.

(v) The mean mobile dislocation density along one slip plane is represented by the following equation:

$$f_{md} = \frac{k^1(m)}{b}\left(\frac{\dot{\tau} b}{G^{m+1} M}\right)^{1/(m+2)},$$

which is in good agreement with the semiexperimental data in the literature.

ACKNOWLEDGMENTS

The computations were carried out by NEAC 2200-MODEL 700 computer, Computer Centre, Tohku University. The authors wish to express hearty thanks to Mrs. N. Endo and M. Takahashi for assistance in carrying out the study.

[1] T. Yokobori, A.T. Yokobori, Jr. and A. Kamei, Philos. Mag. **30**, 367 (1974).
[2] H.L. Prekel and H. Conrad, Acta Metall. **15**, 955 (1967).
[3] A. Seeger, Philos. Mag. **1**, 651 (1956).
[4] A.R. Chaudhuri, J.R. Patel, and L.G. Rubin, J. Appl. Phys. **33**, 2736 (1962).
[5] K. Sumino and K. Kojima, Cryst. Lattice Defects **2**, 159 (1971).
[6] M.F. Kanninen and A.R. Rosenfield, Philos. Mag. **20**, 569 (1969).