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Construction function for three-dimensional sinograms of the time-of-flight positron emission tomograph

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A new method of image reconstruction for the time-of-flight positron emission tomograph (TOF-PET) has been developed. A construction function was found to reconstruct a positron image by directly convoluting three-dimensional sinograms. This construction function consists of the product of the well-known function for a conventional PET and the response function of time-of-flight measurement system for annihilation γ rays.

INTRODUCTION

Positron emission tomography (PET)[1–3] enables us to obtain a quantitative image of the metabolism of viable cells. By using tracers of short-lived positron-emitter nuclides produced by cyclotron, PET is widely applied to cancer diagnosis, and to studies of physiology, and of cardiac and pulmonary functions. Now, a number of dedicated models of PET are commercially available, and their spatial resolution is reaching to an ultimated value. On the other hand, a new type of PET[4–8] in which the position of positron annihilation is localized by the time-of-flight (TOF) technique and expected to be much superior to a conventional type PET, is still under development partly due to the fact that the TOF-PET requires an extremely reliable software system to reconstruct in a limited time a high-quality image from the multidimensional sinograms.

In a TOF-PET, the position (t) of positron–electron annihilation is determined from

$$t = C(T_2 - T_1)/2, \qquad (1)$$

where T_1 and T_2 are the arrival times at detectors for a pair of γ rays, respectively, and C is the light velocity (see Fig. 1). Accordingly, the accuracy of position (Δt) of positron–electron annihilation is limited by the time resolution (ΔT) of the TOF system

$$\Delta t = \Delta T \times C/2. \qquad (2)$$

With a time resolution of several tens of picoseconds,[9] we can obtain directly the distribution of positron emitter with an accuracy of several mm without any back projection technique. Unfortunately, the currently obtainable time resolution is limited to several hundred picoseconds. Therefore, an image reconstruction by back projection is still needed, even for a TOF-PET, in order to obtain a position resolution of several mm.[10] However, TOF-PET has many advantages,[11,12] for example, the improvement of contrast in image, which enables us to find a cancer in its early stage.

As seen in Fig. 2, the coincidence line of a pair of γ rays is located at a distance (s) from the center of a region of interest (ROI) with an angle (θ) to the fundamental axis of ROI. In the case of TOF-PET, moreover, the position of positron–electron annihilation (t) is determined on the coincidence line. Therefore, a three-dimensional sinogram $S(t, s, \theta)$ can be formed from coincident events of γ rays.

As three-dimensional sinograms generally require a big memory size in a computer system and also the construction function for three-dimensional sinograms has not been found, a method of image reconstruction which convolutes directly three-dimensional sinograms has not yet been developed. Snyder *et al.*[13] have introduced a reconstruction method without using three-dimensional sinograms directly. Most of current TOF-PET systems adopt this method for the image reconstruction. In this method one makes a data image, called "preimage" by simply back projecting three-dimensional sinograms, then this preimage is transferred into a frequency space with a fast Fourier transformation (FFT), divided by a Fourier component of point-spread function for TOF-PET. Finally, this is returned to the original space with an inverse FFT to reconstruct a position image (hereafter we refer to this method as "preimage method"). Thus the preimage is an image convoluted with a distribution function of positron emitter and a point-spread function. In the preimage method, therefore, a positron image defocused by the spread of TOF resolution is obtained at

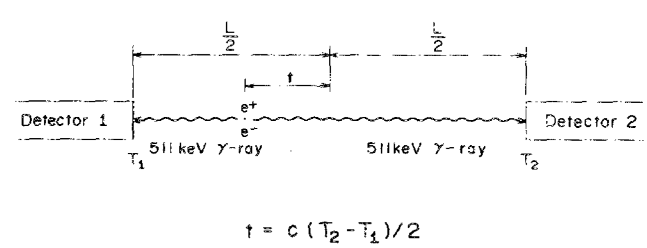

FIG. 1. TOF measurement for annihilation γ rays. The arriving times of γ rays at detectors (T_1 and T_2) are measured by referring to a standard time. The position of positron annihilation can be estimated from the difference in the time of flights of γ rays by Eq. (1). Here, L is the distance between the detectors.

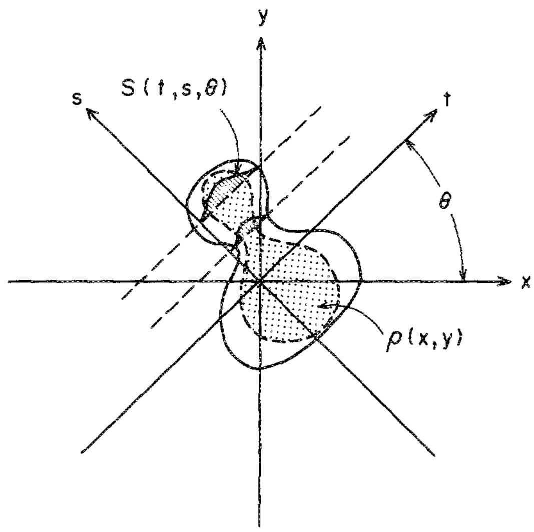

FIG. 2. Three-dimensional sinogram. The intensity of coincident events with TOF information, namely, three-dimensional sinogram $S(t, s, \theta)$ is obtained in a rotatory coordinate, while positron-emitter-radionuclides are distributed with a function $\rho(X, Y)$ in the fixed coordinate.

the first stage. Because of lack of fine information on the angle (θ) in making the preimage, the spatial resolution obtained by this method is inferior to that of a conventional PET, but the signal-to-noise ratio of a reconstructed image is improved due to the TOF technique.

On the other hand, the reconstruction method using directly three-dimensional sinograms can be considered as an extension of the convolution method of conventional PET. As compared with the abovementioned preimage method in Fig. 3, this method is very simple and, therefore, is expected to provide positron images with both good spatial resolution and high quality. The construction function for convolution in this method, however, has not been known up to the present. In this paper, we derive the construction function for three-dimensional sinograms, and introduce an image-reconstruction method which directly convolutes three-dimensional sinograms.

I. THEORY

When a positron is emitted from a radionuclide and encounters an electron, a positron and electron pair is annihilated and two γ rays are emitted in the direction of 180° to each other. The position of positron–electron annihilation vertex is obtained by measuring the time of flight of γ rays along the coincidence line. Thus, we can get the distribution of positron-emitter radionuclide $\rho(X, Y)$ from coincident events of the annihilation γ rays $S(t, s, \theta)$ (see Fig. 2). The intensity of $S(t, s, \theta)$, however, does not directly reflect the density $\rho(X, Y)$ because the current accuracy of TOF technique is not that good as to determine the position of annihilation within the error of several mm. The intensity $S(t, s, \theta)$ is given by the convoluted integral between the density $\rho(X, Y)$ and the response function $h(t)$ of TOF measurement system

$$S(t, s, \theta) = \int_{-\infty}^{+\infty} dt' \, h(t - t')\rho(X', Y') \qquad (3)$$

with

$$X' = t' \cos\theta - s \sin\theta \quad \text{and} \quad Y' = t' \sin\theta + s\cos\theta.$$

Hereafter, we call the intensity $S(t, s, \theta)$ a three-dimensional sinogram.

In the case of conventional PET,[14] a positron image can be reconstructed by convoluting the sinograms $S(s, \theta)$ without TOF information and is given by[15]

$$\rho(X, Y) = \frac{1}{2\pi} \int_0^{\pi} d\theta \int_{-\infty}^{+\infty} ds' \, S(s', \theta) g_0(s - s') \qquad (4)$$

with $s = X\cos\theta + Y\sin\theta$, and

$$g_0(s) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk \, |k| e^{iks}. \qquad (5)$$

We predict a formula similar to Eq. (4) for the case of the TOF-PET[16]

$$\rho(X, Y) = \frac{1}{2\pi} \int_0^{\pi} d\theta \int_{-\infty}^{+\infty} ds' \, S(t, s', \theta) g_{\mathrm{TOF}}(s - s') \qquad (6)$$

with

$$s = X\cos\theta + Y\sin\theta \quad \text{and} \quad t = -X\sin\theta + Y\cos\theta. \qquad (6')$$

Now we derive the convolution function $g_{\mathrm{TOF}}(s)$ as follows.

The density $\rho(X, Y)$ can be considered to be a set of points which have a strength of ρ_i at the position of (X_i, Y_i)

$$\rho(X, Y) = \sum_i \rho_i \delta(X_i - X)\delta(Y_i - Y),$$

where $\delta(X)$ is the Dirac delta function. In the present calculation, therefore, the generality of theory is kept by replacing $\rho(X, Y)$ with the Dirac function $\delta(X)\delta(Y)$ in Eqs. (3), (4), and (6). We obtain

$$\delta(X)\delta(Y) = \frac{1}{2\pi} \int_0^{\pi} d\theta \, g_0(s) \qquad (7)$$

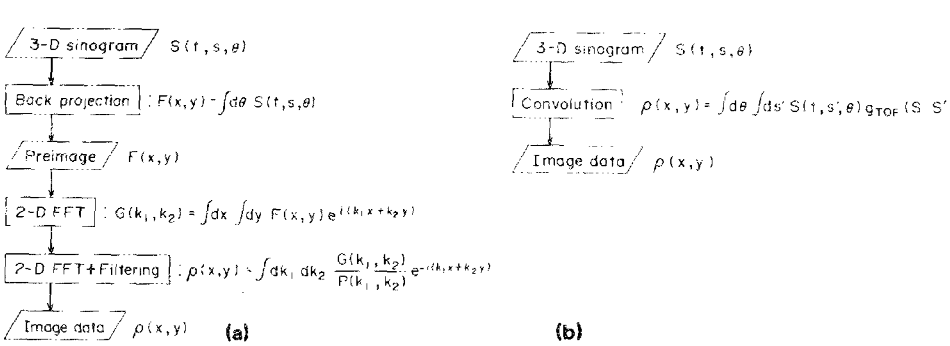

FIG. 3. Image reconstruction methods for TOF-PET; $P(k_1,k_2)$ is the point-spread function of TOF-PET in the frequency space. (a) Preimage method. (b) Convolution method.

and

$$\delta(X)\delta(Y)=\frac{1}{2\pi}\int_0^{\pi}d\theta\, g_{\rm TOF}(s)h(t). \quad (8)$$

On the other hand, the response function $h(t)$ of TOF-measurement system obeys a normal distribution and is expressed by

$$h(t)=\frac{1}{\Delta t}\sqrt{\frac{\ln 2}{\pi}}\, e^{-(t/\Delta t)^2\ln 2}, \quad (9)$$

where Δt is given by Eq. (2) with the FWHM of TOF measurement. Now, we assume that Δt is independent of s, θ, and t and the function $g_{\rm TOF}(s)$ can be expressed by

$$g_{\rm TOF}(s)=g_1(s)\times h(s), \quad (10)$$

where the function $g_1(s)$ will be determined later. Substituting Eq. (10) into Eq. (8) and using the relation of Eq. (6'), Eq. (8) becomes

$$\delta(X)\delta(Y)=\frac{\ln 2}{(\Delta t)^2\pi}e^{-[(X^2+Y^2)/(\Delta t)^2]\ln 2}\frac{1}{2\pi}\int_0^{\pi}d\theta\, g_1(s)$$

$$=\frac{\ln 2}{(\Delta t)^2\pi}\frac{1}{2\pi}\int_0^{\pi}d\theta\, g_1(s).$$

This equation coincides with Eq. (7) except for the factor $\ln 2/(\Delta t)^2\pi$. Thus we obtain

$$g_1(s)=\frac{\pi(\Delta t)^2}{\ln 2}g_0(s). \quad (11)$$

As a result, the convolution function of TOF-PET $g_{\rm TOF}(s)$ is found to be a simple product of the response function of TOF $h(t)$ and the convolution function[17,18] of a conventional PET $g_0(s)$

$$g_{\rm TOF}(s)=\frac{\pi(\Delta t)^2}{\ln 2}g_0(s)h(s). \quad (12)$$

We have previously measured[19] the response function $h(t)$ of TOF measurement using BaF_2 scintillators and found that $h(t)$ has the form of the Gauss function, the FWHM ΔT is constant for the axis along the coincidence line (t axis) and also that ΔT is independent of the geometrical arrangement of BaF_2 crystals. However, ΔT is a function of s and θ as each detector has different responses to the time resolution, so that Eq. (12) cannot be directly applied to Eq. (6).

Now, $\overline{\Delta t}$ denotes the value of Δt averaged over S and θ, and $S(t,s,\theta;\Delta t)$ means the sinogram with the time resolution Δt. Then from Eqs. (3) and (9), the three-dimensional sinogram $S(t,s,\theta;\overline{\Delta t})$ can be expressed by

$$S(t,s,\theta;\overline{\Delta t})=\frac{1}{\overline{\Delta t}}\sqrt{\frac{\ln 2}{\pi}}\times\int_{-\infty}^{+\infty}dt'\, e^{-[(t-t')/\overline{\Delta t}]^2\ln 2}\rho(X',Y').$$

By expanding the exponential term of the right-hand side into a series of $(\Delta t-\overline{\Delta t})$, this equation is approximated by

$$S(t,s,\theta;\overline{\Delta t})=S(t,s,\theta;\Delta t)-\frac{(\Delta t)^2}{2\ln 2}\times\frac{\partial^2 S(t,s,\theta;\Delta t)}{\partial t^2}\frac{\Delta t-\overline{\Delta t}}{\overline{\Delta t}}. \quad (13)$$

Thus, we can construct the three-dimensional sinogram with an average $(\overline{\Delta t})$ of TOF-time resolution from the sinogram $S(t,s,\theta;\Delta t)$ and the second-order differential coefficient $\partial^2 S(t,s,\theta;\Delta t)/\partial t^2$ and we can use the convolution method of Eqs. (6) and (12) for sinograms with the average value of Δt.

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