A New Structure for Feedforward Active Noise Control Systems With Improved Online Secondary Path Modeling

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***Abstract*—This paper proposes a new structure for feedforward active noise control (ANC) systems with online secondary path modeling. The proposed method: 1) uses the same error signal for updating the noise control process as used for the secondary path modeling process and 2) incorporates an adaptive filtering with averaging based filtered-reference algorithm in the noise control process. The computer simulations have been conducted with both narrowband and broadband noise signals. It is shown that in the proposed ANC system the residual noise signal and the secondary-path-modeling error can be reduced at a faster convergence rate than the existing methods. This improved performance is achieved at the expense of a slightly increased computational complexity.**

***Index Terms*—Active noise control, averaging, FxLMS algorithm, online secondary path modeling.**

I. Introduction

A feedforward active noise control (ANC) [1]–[3] system using the FxLMS algorithm comprises two filters; a noise control filter (hereafter called the control filter), and a secondary path modeling filter (hereafter called the modeling filter). As shown in Fig. 1, the control filter $W(z)$ is adaptive and generates the secondary canceling signal $y(n)$. The objective of the modeling filter $\hat{S}(z)$ is to compensate for the secondary path $S(z)$, which is present between the output of the control filter and that of the error microphone.

The FxLMS algorithm appears to be very tolerant of errors made in the modeling of $S(z)$ by the filter $\hat{S}(z)$. As shown in [4] and [5], with in the limit of slow adaptation, the algorithm will converge with nearly 90° of phase error between $\hat{S}(z)$ and $S(z)$. Therefore, offline modeling can be used to estimate $S(z)$ during an initial training stage for ANC applications [2]. For some applications, however, the secondary path may be time varying, and it is desirable to estimate the secondary path online when the ANC is in operation [6].

The basic additive random noise technique for online secondary path modeling in ANC systems is proposed by Eriksson *et al.* [7]. As shown in Fig. 2, this ANC system comprises two

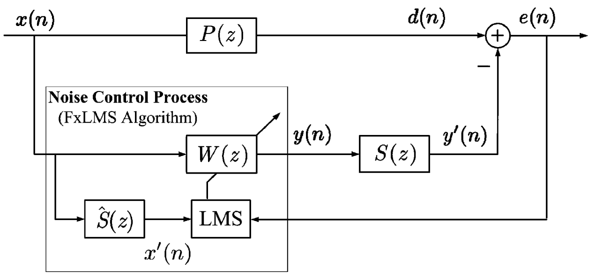

Fig. 1. Block diagram of FxLMS based feedforward ANC system.

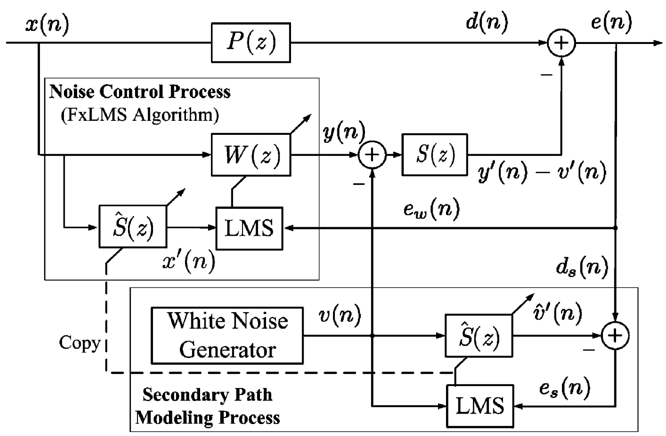

Fig. 2. ANC system of Fig. 1 with online secondary path modeling (Eriksson's method).

processes: a noise control process (hereafter called the control process), and a secondary path modeling process (hereafter called the modeling process). The main problem with this system is that the white random noise, $v(n)$, injected into the ANC system for the modeling process, appears in the residual error signal $e(n)$. Thus $e(n)$ comprises two parts: a part required for the control process and a part required for the modeling process. Since $e(n)$ is used in both the control process and modeling process, the part required for one acts as a disturbance for the other. Due to this intrusion between the control process and modeling process, the overall performance of the ANC system is further degraded.

Improvements in the Eriksson's method have been proposed in [8]–[10]. These improved methods introduce another adaptive filter into the ANC system of Fig. 2. In [8], [9], the third filter removes the interference from the modeling process, and the modeling process therefore converges fast. Here no effort

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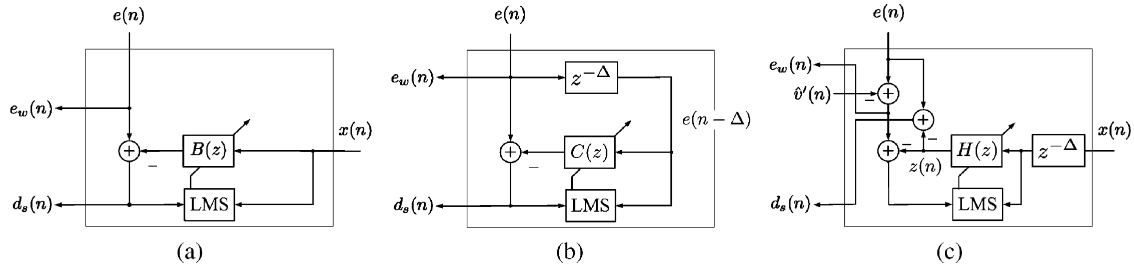

Fig. 3. Third adaptive filter for improved online secondary path modeling in ANC system of Fig. 2. (a) Adaptive noise cancelation (ADNC) filter in Bao's method. (b) Prediction error filter in Kuo's method. (c) ADNC filter with cross-updating in Zhang's method.

is made to improve the control process [see Fig. 3(a) and (b)]. In [10], the control filter, the modeling filter, and the third filter are *cross-updated* to reduce the mutual interference between the control process and modeling process [see Fig. 3(c)]. Simulation results presented in [10] show that this cross-updated ANC system gives the best performance for ANC systems with online secondary path modeling.

The main idea in this paper is to develop an ANC system that can achieve improved online secondary path modeling without introducing any extra adaptive filter into the ANC system of Fig. 2. Since the control process is perturbed by the interference which is random in nature [note that $v(n)$ is a white random noise process], we suggest using an adaptive filtering with averaging (AFA) based filtered-x (FxAFA) algorithm in the control process. With a fast convergent control process, the interference in the modeling process is removed quickly and hence the modeling process can converge fast. Simulations show that the proposed ANC system can achieve better performance than the existing methods.

The organization of this paper is as follows. Section II explains the proposed method in connection to the operation of Eriksson's method. Section III discusses the computational complexity issue, Section IV details the simulation results and Section V presents concluding remarks.

II. Proposed Method for ANC Systems With Online Secondary Path Modeling

A. Proposed Method

Consider Eriksson's method for ANC systems with online secondary path modeling, shown in Fig. 2. Assuming that the control filter $W(z)$ is an FIR filter of tap-weight length L, the secondary signal $y(n)$ is expressed as

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_L(n) \tag{1}$$

where $\boldsymbol{w}(n) = [w_0(n)w_1(n)\cdots w_{L-1}(n)]^T$ is the tap-weight vector, $\boldsymbol{x}_L(n) = [x(n)x(n-1)\cdots x(n-L+1)]^T$ is the L-sample reference signal vector, and $x(n)$ is the reference signal obtained by the reference microphone. An internally generated zero-mean white Gaussian noise signal, $v(n)$, uncorrelated with the reference noise $x(n)$, is injected at the output $y(n)$ of the control filter. The residual noise signal $e(n)$ is given as

$$e(n) = d(n) - y'(n) + v'(n) \tag{2}$$

where $d(n) = p(n) * x(n)$ is the primary disturbance signal at the error microphone, $y'(n) = s(n) * y(n)$ is the secondary canceling signal, $v'(n) = s(n) * v(n)$ is the modeling signal, $*$ denotes the convolution operation, and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. The residual noise signal $e(n)$ is used as an error signal for the control process, i.e.,

$$e_w(n) = e(n) = d(n) - y'(n) + v'(n) = u(n) + v'(n) \tag{3}$$

where

$$u(n) = d(n) - y'(n) \tag{4}$$

is a component of the error signal due to the canceling noise only.

The coefficients of the control filter $W(z)$ are updated by the FxLMS algorithm

$$\begin{aligned}\boldsymbol{w}(n+1) &= \boldsymbol{w}(n) + \mu_w e_w(n)\boldsymbol{x}'(n)\\ &= \boldsymbol{w}(n) + \mu_w \boldsymbol{x}'(n)u(n) + \mu_w \boldsymbol{x}'(n)v'(n)\end{aligned} \tag{5}$$

where μ_w is the step size for the control process, $\boldsymbol{x}'(n) = [x'(n)x'(n-1)\cdots x'(n-L+1)]^T$, and $x'(n)$ is the reference signal $x(n)$ filtered through the modeling filter $\hat{S}(z)$. We see that the control process is perturbed by an undesired term $\mu_w \boldsymbol{x}'(n)v'(n)$.

Assuming that $\hat{S}(z)$ is represented by an FIR filter of tap-weight length M, the filtered-reference signal $x'(n)$ is obtained as

$$x'(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}_M(n) \tag{6}$$

where $\hat{\boldsymbol{s}}(n) = [\hat{s}_0(n)\hat{s}_1(n)\hat{s}_2(n)\ \cdots \hat{s}_{M-1}(n)]^T$ is the impulse response of the modeling filter $\hat{S}(z)$ and $\boldsymbol{x}_M(n) = [x(n)x(n-1)\cdots x(n-M+1)]^T$ is the M-sample reference signal vector.

The residual noise signal $e(n)$ is used as a desired response in the modeling process, i.e., $d_s(n) = e(n)$, and hence the error signal for the modeling process is generated as

$$e_s(n) = d_s(n) - \hat{v}'(n) = u(n) + [v'(n) - \hat{v}'(n)]. \tag{7}$$

The LMS update equation for $\hat{S}(z)$ is given as

$$\begin{aligned}\hat{\boldsymbol{s}}(n+1) &= \hat{\boldsymbol{s}}(n) + \mu_s e_s(n)\boldsymbol{v}(n)\\ &= \hat{\boldsymbol{s}}(n) + \mu_s \boldsymbol{v}(n)[v'(n) - \hat{v}'(n)] + \mu_s \boldsymbol{v}(n)u(n)\end{aligned} \tag{8}$$

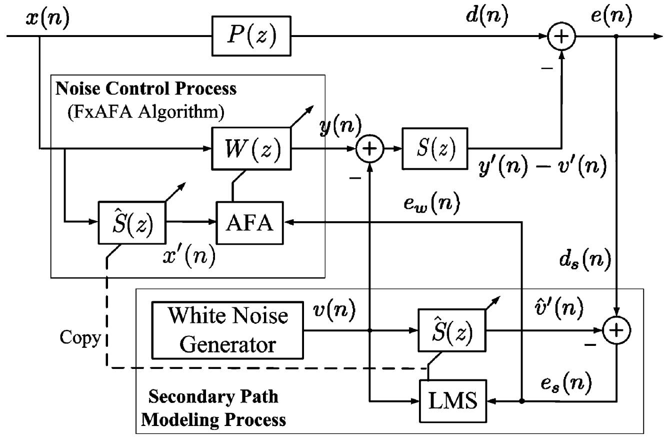

Fig. 4. Proposed method for ANC systems with online secondary path modeling.

where μ_s is the step size of the modeling process, $\hat{v}'(n) = \hat{s}(n) * v(n)$ is an estimate of $v'(n)$ obtained from the modeling filter, and $\boldsymbol{v}(n) = [v(n)v(n-1)\cdots v(n-M+1)]^T$. Equation (8) shows that the performance of the modeling process is degraded by an undesired term $\mu_s \boldsymbol{v}(n)u(n)$ and in worst case the modeling process may diverge.

The error signals for the control process and the modeling process are given in (3) and (7), respectively. From these expressions, we observe the following.

- Both $e_w(n)$ and $e_s(n)$ contain $u(n)$, which is the error signal required for the control process. In $e_w(n)$, $u(n)$ is corrupted by the component $v'(n)$, and in $e_s(n)$, $u(n)$ is corrupted by a term $[v'(n) - \hat{v}'(n)]$.
- As compared with $e(n)$, which is equal to $e_w(n)$ in the conventional formulation, $e_s(n)$ appears to be a better error signal for the control process, because $|v'(n) - \hat{v}'(n)| < |v'(n)|$ and when $\hat{S}(z)$ converges then (ideally) $v'(n) \approx \hat{v}'(n) \Rightarrow [v'(n) - \hat{v}'(n)] \rightarrow 0$.
- Since $v(n)$ is white Gaussian noise of zero mean, both $v'(n)$ and $[v'(n) - \hat{v}'(n)]$ are random in nature and can be averaged out.

On the basis of the above analysis, two modifications are suggested to Eriksson's method. The first is using $e_s(n)$ as the error signal for both the control process and the modeling process, i.e., $e_w(n) = e_s(n) = e(n) - \hat{v}'(n)$. The second modification is replacing the FxLMS algorithm with an (AFA [11] based FxAFA algorithm. The proposed ANC system is shown in Fig. 4.

B. FxAFA Algorithm

Replacing $e_w(n)$ by $e_s(n)$ in (5), we get

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w u(n)\boldsymbol{x}'(n) + \mu_w [v'(n) - \hat{v}'(n)] \boldsymbol{x}'(n). \quad (9)$$

We see that the control process is perturbed by an undesired term $\mu_w [v'(n) - \hat{v}'(n)] \boldsymbol{x}'(n)$. Taking the expectation of (9) and noting that $v(n)$ and $x(n)$ are uncorrelated with each other, we obtain

$$E[\boldsymbol{w}(n+1)] = E[\boldsymbol{w}(n)] + \mu_w E[u(n)\boldsymbol{x}'(n)] + \mu_w E[\hat{v}'(n) - v'(n)] E[\boldsymbol{x}'(n)]. \quad (10)$$

TABLE I
SUMMARY OF THE PROPOSED ANC SYSTEM

Parameters:	$L =$ tap-weight length of $W(z)$
	$M =$ tap-weight length of $\hat{S}(z)$
	$\mu_w =$ step size for $W(z)$
	$\mu_s =$ step size for $\hat{S}(z)$
	$\gamma \in (0.5, 1)$
Computation:	For $n = 0, 1, 2, \cdots$ compute
	$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_L(n)$
	$\hat{v}'(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{v}(n)$
	$e_w(n) = e_s(n) = e(n) - \hat{v}'(n)$
	$\hat{\boldsymbol{s}}(n+1) = \hat{\boldsymbol{s}}(n) + \mu_s e_s(n)\boldsymbol{v}(n)$
	$x'(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}_M(n)$
	$\boldsymbol{g}(n) = \mu_w e_w(n)\boldsymbol{x}'(n)$
	$\overline{\boldsymbol{g}}(n) = (1 - 1/n)^\gamma \overline{\boldsymbol{g}}(n-1) + (1/n^\gamma)\boldsymbol{g}(n)$
	$\overline{\boldsymbol{w}}(n) = (1 - 1/n)\overline{\boldsymbol{w}}(n-1) + (1/n)\boldsymbol{w}(n)$
	$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}}(n) + \overline{\boldsymbol{g}}(n)$

Since $\hat{v}'(n)$ is an estimate of $v'(n)$ and both are generated by a zero-mean random process; hence $E[\hat{v}'(n) - v'(n)] \rightarrow 0$. From this analysis, we conclude that averaging can be used to remove the effect of the perturbation term. To realize this, we incorporate the concept of AFA [11] to the FxLMS algorithm. In [11], two averaging based adaptive filtering algorithms are proposed. The first algorithm uses averaging in iterates only and in the second algorithm averaging is incorporated with both iterates and observations. It is shown that the second approach results in better performance than the first approach [11]. Motivated by the second approach, we incorporate averaging with both the iteration vector [the tap-weight vector $\boldsymbol{w}(n)$], and the observation vector [the gradient vector $\mu_w e_w(n)\boldsymbol{x}'(n)$)] of the FxLMS algorithm. This results in the FxAFA algorithm [12], which is described as

$$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}}(n) + \overline{\boldsymbol{g}}(n) \quad (11)$$

where

$$\overline{\boldsymbol{w}}(n) = \frac{1}{n}\sum_{k=1}^{n} \boldsymbol{w}(k) \quad (12)$$

$$\overline{\boldsymbol{g}}(n) = \frac{1}{n^\gamma}\sum_{k=1}^{n} \mu_w e_w(k)\boldsymbol{x}'(k);\ \frac{1}{2} < \gamma < 1. \quad (13)$$

The operation of the proposed ANC system is summarized in Table I. It is important to note that the averages introduced in (12) and (13) are computed recursively.

Equation (13) shows that the effective step-size for the FxAFA algorithm is $\mu_w/n^\gamma = \alpha(n)$ (say). This is a time varying gain parameter with the property $\lim_{n\rightarrow\infty} \alpha(n) \rightarrow 0$. It is seen that $\gamma = 1$ will rapidly decrease the gain parameter, and hence the adaptation process may be very slow. Therefore one may wish to choose $\gamma < 1$. On the contrary, if γ is selected close to zero then $\alpha(n)$ is very slowly decreasing. This is also not desirable for large mismatch. Hence $1/2 < \gamma < 1$ is the recommended range for the values of γ [13].

Let's give some reasons why "averaging" can improve the performance. We know that the method of steepest descent computes a tap-weight vector that moves down the ensemble-av-

TABLE II
COMPUTATIONAL COMPLEXITY (NUMBER OF MULTIPLICATIONS/ITERATIONS) COMPARISON OF THE PROPOSED METHOD WITH THE EXISTING METHODS
(HERE, $(\cdot)$ IS THE RATIO OF COMPUTATIONAL COMPLEXITY TO THAT OF ERIKSSON'S METHOD.)

	Analytical Expression	$M = L/3; N = L/2$	$M = L; N = L/2$	$M = N = L$
Eriksson s Method	$2L + 3M + 2$	$\approx 3L$	$\approx 5L$	$\approx 5L$
Improved Methods	$2L + 3M + 2N + 3$	$\approx 4L(1.33)$	$\approx 6L(1.2)$	$\approx 7L(1.4)$
Proposed Method	$6L + 3M + 2$	$\approx 7L(2.33)$	$\approx 9L(1.8)$	$\approx 9L(1.8)$

erage error-performance surface along a deterministic trajectory that terminates on the Wiener solution (although it takes infinite number of iterations, n, to do so). The LMS algorithm, on the other hand behaves differently because of the presence of the gradient noise: rather than terminating on the Wiener solution, the tap-weight vector computed by the LMS algorithm executes a random motion around the minimum point of the error performance surface [14, p. 234]. Furthermore, by assigning a small value to the step size parameter, the adaptation is made to progress slowly, and the effects of the gradient noise on the tap weights are largely filtered out [14, p. 235].

In the proposed algorithm, the aim is to have the iterations move to the Wiener solution reasonably fast. For the averaging approach of (13), with $\gamma < 1$ the estimates from (11) are allowed to approach the vicinity of the true value faster. At the same time, averaging removes the random fluctuations in the gradient vector and ensures that the iterations move toward the optimal (Wiener) solution. Now better noise reduction performance is expected and ANC will reduce the residual noise component $u(n)$ at a fast convergence rate. This means that the modeling process is now expected to converge fast.

C. Effect of Using Same Error Signal

As said earlier, in the proposed method, both the modeling filter and the control filter are updated using the same error signal, i.e., $e_w(n) = e_s(n) = e(n) - \hat{v}'(n) = e'(n)$(say). Taking the z-transform and making necessary substitutions, we get following expression for this error signal:

$$E'(z) = [P(z) - S(z)W(z)]X(z) + [S(z) - \hat{S}(z)]V(z). \quad (14)$$

By convergence of $W(z)$, we mean that the error signal is minimized to (ideally) zero. This requires $W(z)$ to adapt to the following optimal solution:

$$W^{\circ}(z) = \frac{P(z)}{S(z)} + \left(\frac{S(z) - \hat{S}(z)}{S(z)}\right)\frac{V(z)}{X(z)}. \quad (15)$$

This equation shows that $W(z)$ will converge to the optimal solution $P(z)/S(z)$, if and only if, modeling error reduces to zero, i.e., $\hat{S}(z) \rightarrow S(z)$. Converse is also true, that the modeling error reduces to zero, if and only if, $W(z)$ converges to the optimal solution $P(z)/S(z)$. Thus in the proposed method the convergence of the control filter and the modeling filter is mutually dependent.

III. COMPUTATIONAL COMPLEXITY

Table II presents a computational complexity (multiplications per iteration) comparison of the proposed method with the existing methods. It is assumed that three adaptive filters, $B(z)$, $C(z)$, and $H(z)$ in Bao's method, Kuo's method, and Zhang's method, respectively, are selected of tap-weight length N. Hence, these methods (called improved methods in Table II) have the same computational complexity.

The data represented in Table II shows that the computational complexity of the proposed method is greater than the existing schemes. The source of this increased computational burden is the recursive computation of the averages introduced in (11).

IV. COMPUTER SIMULATIONS

In this section, we compare the performance of the proposed method with that of Eriksson's method and Zhang's method. The performance comparison is done on the basis of two performance measures. The first is the residual error signal $e(n)$. The second is the relative modeling error being defined as

$$\Delta S(\text{dB}) = 10\log_{10}\left[\frac{\sum_{i=0}^{M-1}\{s_i(n) - \hat{s}_i(n)\}^2}{\sum_{i=0}^{M-1}\{s_i(n)\}^2}\right]. \quad (16)$$

For the primary acoustical path $P(z)$ and the secondary path $S(z)$, the experimental data provided by [1] is used, where both are modeled by IIR filters of order 25. The frequency response of the acoustic paths is shown in Fig. 5. The modeling filter $\hat{S}(z)$ and control filter $W(z)$ are FIR filters of tap-weight length L and M, respectively. The adaptive noise cancelation (ADNC) filter $H(z)$ in Zhang's method is selected as an FIR filter of tap-weight length N. The control filter $W(z)$ is initialized by the null vector $\boldsymbol{w}(0) = \mathbf{0}$, and ADNC filter $H(z)$ is initialized by the null vector $\boldsymbol{h}(0) = \mathbf{0}$. To initialize the modeling filer, offline modeling is performed[1] which is stopped when the modeling error [as defined in (16)] has been reduced to -5 dB. The resulting weights are used for $\hat{\boldsymbol{s}}(0)$ when the ANC system is started. A sampling frequency of 4 kHz is used and the simulations are carried out with the signals having frequency content below 500 Hz. The parameters are adjusted for fast and stable convergence and are summarized in Table III. All the results

[1]As stated in Section II-C, the adaptation of $W(z)$ and $\hat{S}(z)$ in the proposed method is mutually dependent. If $\hat{S}(z)$ is initialized by a null vector, then ANC system may be unstable. To avoid this situation, we have used offline modeling to initialize the modeling filter $\hat{S}(z)$. It is worth mentioning that practically the first stage in ANC system design is offline measurements. It is better to initialize the ANC system by offline measurements [15].

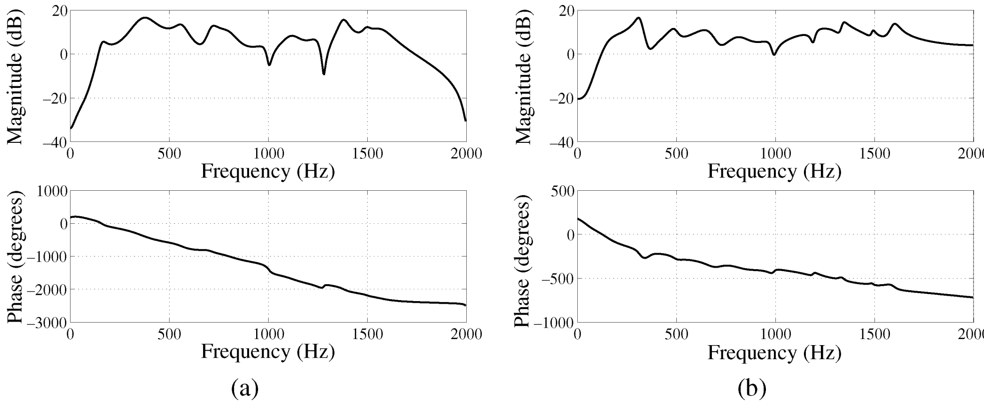

Fig. 5. Frequency response of acoustic paths used in computer simulations. (a) Primary path $P(z)$. (b) Secondary path $S(z)$.

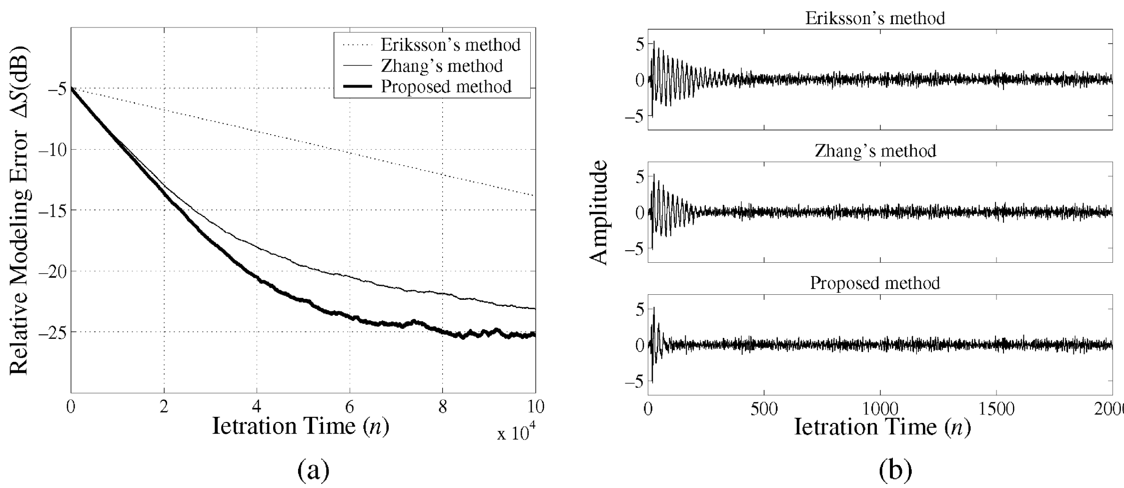

Fig. 6. Performance comparison between existing methods and proposed method for Case 1. (a) Relative modeling error ΔS (dB) versus iteration time n. (b) Residual error signal $e(n)$ versus iteration time n.

TABLE III
SIMULATIONS PARAMETERS FOR COMPUTER EXPERIMENTS

Case	Eriksson's Method (μ_w, μ_s)	Zhang's Method (μ_w, μ_s, μ_h)	Proposed Method (μ_w, μ_s, γ)
1, 2	$1 \times 10^{-5}, 1 \times 10^{-3}$	$2 \times 10^{-5}, 5 \times 10^{-3}, 1 \times 10^{-3}$	$2 \times 10^{-3}, 5 \times 10^{-3}, 0.6$
3	$1 \times 10^{-5}, 1 \times 10^{-3}$	$2 \times 10^{-5}, 3 \times 10^{-3}, 1 \times 10^{-3}$	$1 \times 10^{-3}, 3 \times 10^{-3}, 0.5$

presented below are averaged over ten realizations of the noise process.

1) Case 1: First we consider a sinusoidal signal of 200 Hz as a reference noise signal. The variance of this signal is 2, and a zero-mean white Gaussian noise is added to it with SNR of 20 dB. In order to maintain low residual noise in steady state, zero-mean white Gaussian noise of variance 0.01 is used in the modeling process. The tap-weight lengths for the adaptive filters are chosen as $L = 128$, $M = 64$, and $N = 64$. The delay Δ in Zhang's method is 30. Fig. 6(a) shows the curves of the relative modeling error, ΔS, as defined in (16). We see that the proposed method achieves best performance among the existing methods. The corresponding curves for the residual error signal are shown in Fig. 6(b). We see that proposed method can reduce the residual noise at a much faster rate than Zhang's method. This is because in the proposed method a large value for the step size μ_w for $W(z)$ can be selected (see Table III). This confirms the discussion presented earlier, that due to an efficient control process, the residual noise is reduced efficiently, which in turn improves the performance of the modeling process.

2) Case 2: In this case, the reference noise is a broadband signal comprising sinusoids of frequencies 200, 250, 425, and 500 Hz. The variance of the reference noise signal is adjusted to 2 and a zero-mean white Gaussian noise is added to it with SNR of 20 dB. As in Case 1, the modeling process is excited by a zero-mean white Gaussian noise of variance 0.01. The tap-weight lengths for the adaptive filters are chosen to be the same as in Case 1, i.e., $L = 128$, $M = 64$, and $N = 64$. The delay Δ in Zhang's method is 30. In Fig. 7(a), curves of the relative modeling error, ΔS, are shown for the proposed method in comparison to the existing methods. Fig. 7(b) shows the curves for residual error signal $e(n)$. As in Case 1, the proposed method gives better performance than Zhang's method, both in noise reduction and secondary path modeling. It is interesting to note that here the convergence of $e(n)$ is slower as compared with the curves shown in Fig. 6(b). The reason is the reference signal, which comprises multiple frequency components as compared with the single tone in Case 1.

3) Case 3: Here the reference noise signal is generated by filtering a zero mean white Gaussian noise of unit variance through a bandpass filter with the passand 100–400 Hz. The

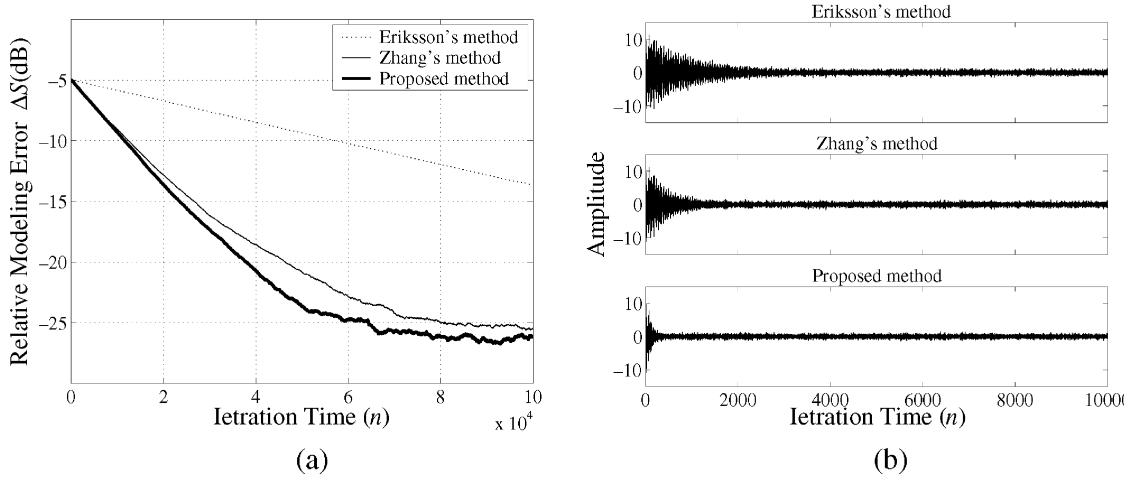

Fig. 7. Performance comparison between existing methods and proposed method for Case 2. (a) Relative modeling error ΔS (dB) versus iteration time n. (b) Residual error signal $e(n)$ versus iteration time n.

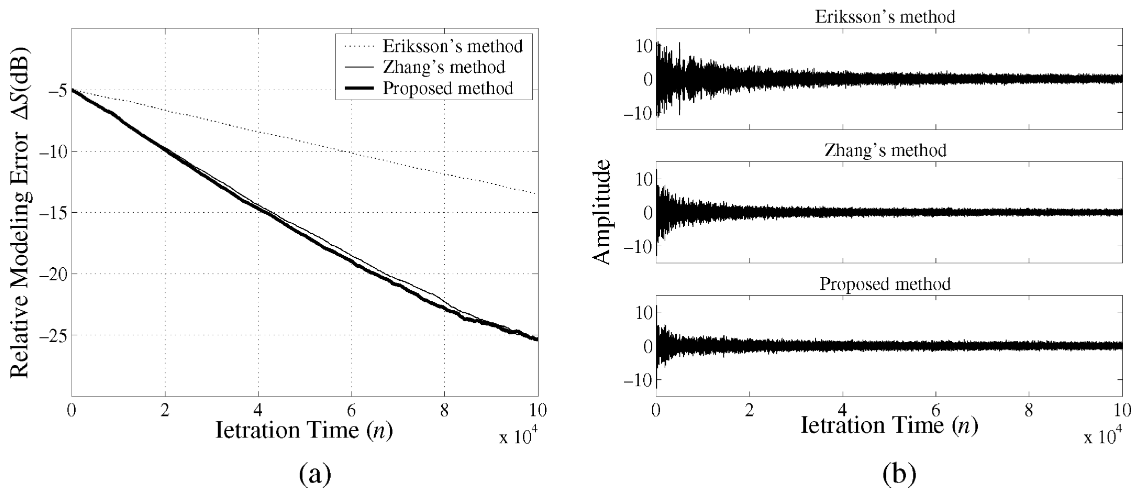

Fig. 8. Performance comparison between existing methods and proposed method for Case 3. (a) Relative modeling error ΔS (dB) versus iteration time n. (b) Residual error signal $e(n)$ versus iteration time n.

variance of the filtered signal is adjusted to 2 and a zero-mean white Gaussian noise is added to it with SNR of 20 dB. The step size parameters are adjusted for fast and stable performance and are given in Table III. As in the previous cases, zero-mean white Gaussian noise of variance 0.01 is used in the modeling process. The simulations results are presented in Fig. 8. We see that the convergence of the three methods is very slow as compared with the results presented in Figs. 6 and 7. Nevertheless, proposed method achieves better performance than the existing methods, by using two adaptive filters only.

V. Concluding Remarks

In this paper, we have proposed a new method for feedforward ANC systems with improved online secondary path modeling. The main features of the proposed ANC system are summarized below.

- Since the primary noise is much stronger than the online secondary path modeling excitation signal, the influence from the primary noise to the secondary path modeling process is more significant, especially at the beginning of the ANC process while the system has no idea of the secondary path model. An effort, therefore, is made to improve the control process, so that better noise reduction is achieved and hence the modeling process converges fast.
- The proposed structure uses two adaptive filters, $W(z)$ and $\hat{S}(z)$, to perform the noise control and secondary path modeling simultaneously. This is in contrast to the existing improved methods, which use three adaptive filters.
- The proposed ANC system can reduce the residual noise signal and the secondary-path-modeling error at a faster convergence rate than the existing methods.

In the noise-control process, two rounds of averaging are incorporated with the FxLMS algorithm. The first round of averaging removes random fluctuations from the gradient vector and the second round of averaging ensures that tap weights converge to the optimal solution. Due to this averaging, the proposed FxAFA algorithm has long memory, and hence poor tracking properties. This problem can be overcome, for example, by using weighted averaging with exponential forgetting factor [16], or moving averaging with sliding window can be used. Another idea may be to re-initialize the averaging process at regular intervals. Improving the tracking properties of the proposed ANC system is a task of future work.

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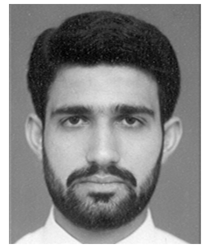

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