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Scaling Analysis of Domain-Wall Free Energy in the Edwards-Anderson Ising Spin Glass in a Magnetic Field

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The stability of the spin-glass phase against a magnetic field is studied in the three- and four-dimensional Edwards-Anderson Ising spin glasses. Effective couplings $J_{\rm eff}$ and effective fields $H_{\rm eff}$ associated with length scale L are measured by a numerical domain-wall renormalization-group method. The results obtained by scaling analysis of the data strongly indicate the existence of a crossover length beyond which the spin-glass order is destroyed by field H. The crossover length well obeys a power law of H which diverges as $H \to 0$ but remains finite for any nonzero H, implying that the spin-glass phase is absent even in an infinitesimal field. These results are well consistent with the droplet theory for short-range spin glasses.

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In spite of extensive studies for more than two decades, a basic problem on the field-temperature phase diagram of the short-range Ising spin glass (SG) is still controversial. The mean-field theory predicts the existence of the SG phase in a magnetic field up to a certain strength at a temperature below T_c, the critical temperature in a zero field [1]. On the other hand, the droplet theory [2–4], a phenomenological theory for short-range SGs, predicts the absence of the SG phase even in an infinitesimal field.

In experiments, this issue has been addressed by the study of Ising SG $Fe_xMn_{1-x}TiO_3$. Although the presence of the SG phase in field was first reported [5], the subsequent work concluded its absence by careful analyses of its relaxation time [6]. The same conclusion was recently drawn by Jönsson *et al.* [7]. From a theoretical point of view, numerical studies of the Edwards-Anderson (EA) short-range SG model [8] have yielded rather conflicting results: some data support the presence of the SG phase in field [9,10], and others support its absence [11–13]. However, a recent numerical work on the correlation length has shown the absence of the SG phase in three dimensions at a low field $H = 0.05J$ [13], where J is the standard deviation of couplings $\{J_{ij}\}$. Since this field is much below a critical field $H_c \approx 0.65J$ suggested by a previous study [10], the result strongly indicates the absence of the SG phase. However, there still remains the possibility that the SG phase is stable at lower fields. Furthermore, the physical mechanism which destroys the SG state remains to be clarified.

In the present work, we study the EA SG in field in both three and four dimensions by the numerical domain-wall renormalization-group (DWRG) method [14–17]. We also show some corresponding results of the Migdal-Kadanoff (MK) SG model, in which the absence of the SG phase in field has already been shown [15]. For this model we can easily access huge sizes such as $L \approx 10^{10}$. This compensates the Monte Carlo (MC) results on the EA model within a limited range of length scales. Quite interestingly, both the models exhibit the same scaling behavior. All the observables are scaled as a function of $L/\ell_{\rm cr}(H)$, where L is the system size and $\ell_{\rm cr}(H)$ a crossover length beyond which the SG order is destroyed by field H. The existence of such a length scale is indeed predicted by the droplet theory. The observed scaling behavior indicates the absence of the SG phase even in an infinitesimal field.

The droplet theory.—Droplets are spin clusters which can be flipped with a low excitation energy [2–4]. The typical excitation energy of droplets with length scale ℓ is assumed to scale as ℓ^θ, where θ is the so-called stiffness exponent. Now let us consider applying field H. Although we consider a uniform field for simplicity, the following argument is also valid for random fields. In Ising SGs, the spins in a droplet are either $+1$ or -1 with equal probability, implying that the order of Zeeman energy of droplets with size ℓ is $H\ell^{d/2}$. Since the droplet theory provides some arguments which support the inequality $\frac{d-1}{2} \geq \theta$ [2], the theory claims that there exists a characteristic length, given as $\ell_{\rm cr}(H) \approx H^{-1/\zeta}$ with $\zeta \equiv d/2 - \theta$, that droplets larger than $\ell_{\rm cr}(H)$ are forced to flip by the field. As a result, the SG state at $H = 0$ becomes unstable beyond $\ell_{\rm cr}(H)$. We call $\ell_{\rm cr}$ and ζ the (field) crossover length and the crossover exponent, respectively. Furthermore the droplet theory claims that the system is paramagnetic beyond $\ell_{\rm cr}(H)$ as it happens in random field Ising model. Because $\ell_{\rm cr}(H)$ diverges as $H \to 0$ but remains finite for any nonzero H, the droplet theory asserts the absence of the SG phase in any field.

DWRG method and observables.—Let us first explain models for the DWRG method. Figure 1(a) shows the way to construct the hierarchical lattice for the MK SG. The

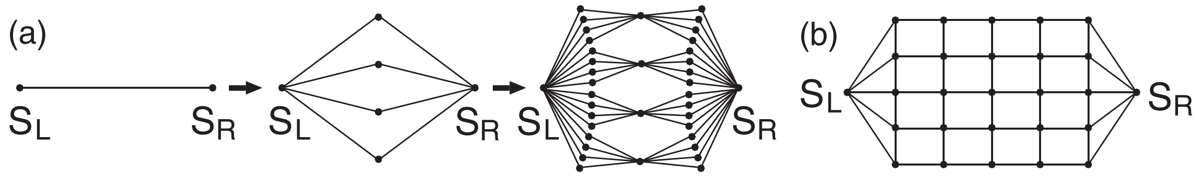

FIG. 1. (a) The construction of the hierarchical lattice ($b = 2$, $d = 3$). (b) The lattice for the DWRG method in the EA SG.

lattice is made iteratively by replacing each bond with b^{d-1} paths, where d is the dimension of the lattice. Each path consists of b bonds, and new $(b-1)$ spins (full circles) are inserted in between. We hereafter call the two outermost spins (S_L and S_R) *boundary spins*. The size L of the lattice is multiplied by b as the replacement is done once. Figure 1(b) is the lattice for the EA SG. The lattice is the same as that in Ref. [17]. It consists of two boundary spins and L^d spins on a d-dimensional hyper-cubic lattice. The boundary condition is open in the direction along which the boundary spins lie, and is periodic in other directions. The Hamiltonian is $\mathcal{H} = -\sum_{\langle ij\rangle} J_{ij} S_i S_j - \sum_i H_i S_i$, where the first term is exchange energies between two nearest neighboring spins and the second term is Zeeman energies by field H_i.

In the DWRG method, we measure the effective coupling $J_{\rm eff}$ and the effective fields $H_{\rm eff}$ defined by

$$Z(S_L, S_R) \equiv \mathrm{Tr}' e^{-H\{S\}/T} \propto e^{-\mathcal{H}_{\rm eff}(S_L,S_R)/T}, \quad (1)$$

$$\mathcal{H}_{\rm eff}(S_L, S_R) = -J_{\rm eff} S_L S_R - H_{\rm eff}^{(L)} S_L - H_{\rm eff}^{(R)} S_R, \quad (2)$$

where the trace in Eq. (1) is for all the spins except S_L and S_R. In the MK SG, $Z(S_L, S_R)$ is estimated exactly by taking the trace sequentially from the later generated spins to the earlier generated ones [15]. In the EA SG, on the other hand, probability $P(S_L, S_R)$ is measured by MC simulations [17]. Since $J_{\rm eff}$ and the free-energy difference δF caused by twisting the two boundary spins are related by $J_{\rm eff} = -\delta F/2$ in zero field, we consider that $J_{\rm eff}$ represents the strength of the SG order. Because $J_{\rm eff}$ is either positive or negative, we calculate the standard deviation of sample-to-sample fluctuations of effective couplings, $\sigma_J(L, H)$. We also measure that of effective fields $\sigma_H(L, H)$. The correlation between effective couplings in zero field and those in field H is also estimated by the correlation coefficient

$$C(L, H) = \frac{\overline{J_{\rm eff}(L, H) J_{\rm eff}(L, 0)}}{\sigma_J(L, H)\sigma_J(L, 0)}, \quad (3)$$

where the overbar denotes the sample average. Here $J_{\rm eff}(L, H)$ and $J_{\rm eff}(L, 0)$ are calculated for the same realization of $\{J_{ij}\}$.

Results in the MK SG.—Figure 2 shows the size dependences of σ_J and σ_H in the four-dimensional MK SG. The values of couplings $\{J_{ij}\}$ are taken from a Gaussian distribution of mean 0 and width 1. We apply random fields H_i of strength H by following the method from Ref. [15]. The pool method [18] is used to access huge sizes such as $L \approx 10^{10}$. The Zeeman energy ($\sigma_H \propto HL^{d/2}$) overwhelms the effective coupling ($\sigma_J \propto L^\theta$) around $L \approx H^{-1/\zeta}$, i.e., around the crossover length $\ell_{\rm cr}(H)$ in the droplet theory. After the crossing, σ_J exhibits roughly exponential decay and the exponent of σ_H changes from $d/2$ to $(d-1)/2$. These observations naturally lead us to the idea that $H^{\theta/\zeta}\sigma_J$, $H^{\theta/\zeta}\sigma_H$ and C are scaled as functions of $X \equiv LH^{1/\zeta}$ which we call the scaling variable. The idea is tested in Fig. 3. The data with different H and L nicely collapse into scaling curves. These results clearly show the existence of the crossover length $\ell_{\rm cr}(H)$.

Results in the EA SG.—Let us first explain some details of our simulation. The values of J_{ij} are either $+J$ or $-J$ with equal weights. Uniform field H is applied to all the spins except S_L and S_R. The number of samples is more than 1500 for all the sets of (L, H). We use the exchange MC method [19] to accelerate the equilibration, and the method in Ref. [16] to overcome a hard-relaxing problem of the boundary spins which is originated from their high connectivities. The temperature ranges used for the exchange MC are $0.5J \le T \le 4.0J$ ($T_c \approx 1.1J$ [20]) for $d = 3$ and $1.0J \le T \le 4.5J$ ($T_c \approx 2.0J$ [21]) for $d = 4$. We

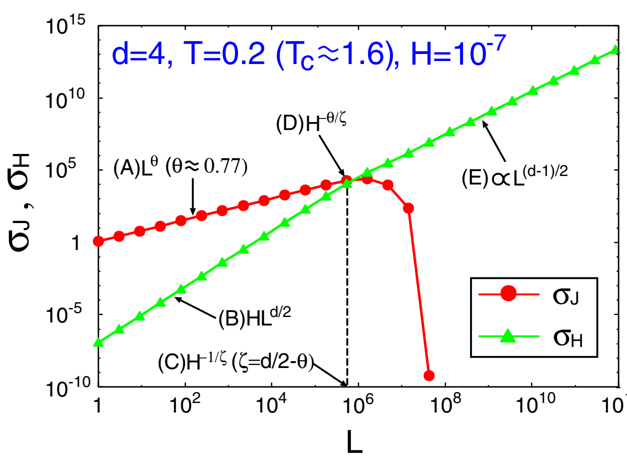

FIG. 2 (color online). Size dependences of σ_J and σ_H in the four-dimensional MK SG at $T \approx 0.125T_c$, where T_c is the critical temperature in zero field. Strength of random fields is 10^{-7}. (a) Size dependence of σ_J for smaller L. (b) Size dependence of σ_H for smaller L. (c) The size where the crossover between σ_J and σ_H occurs. (d) The value of σ_J and σ_H at the crossing point. (e) Size dependence of σ_H for larger L.

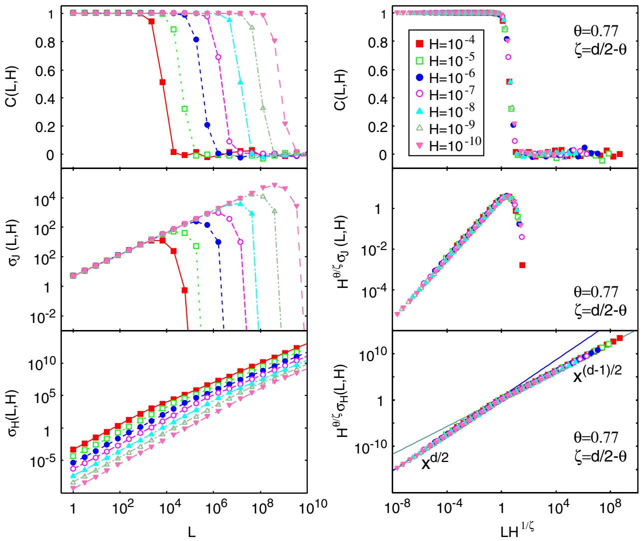

FIG. 3 (color online). Raw data (left panel) and their scaling plots (right panel) in the MK SG. The dimensions and the temperature are the same as those in Fig. 2. The value $\theta = 0.77$ obtained in Ref. [25] is used for the scaling. The two straight lines in the right-bottom panel are proportional to $x^{d/2}$ and $x^{(d-1)/2}$, respectively.

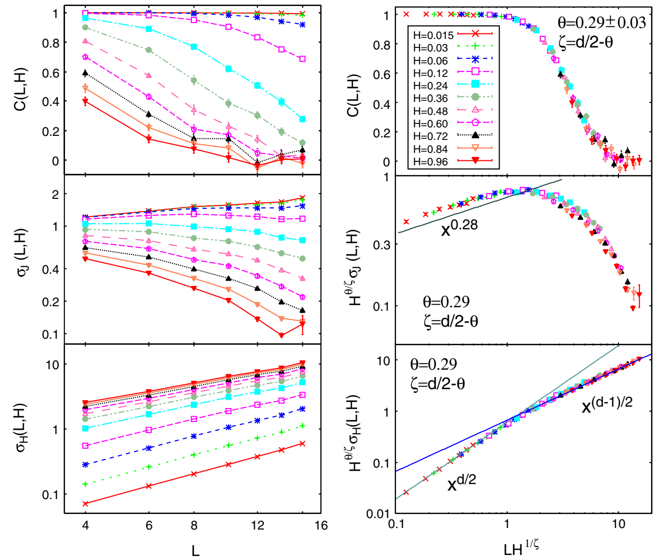

FIG. 4 (color online). Raw data (left panel) and their scaling plots (right panel) in the EA SG at $T = 0.5J$ for $d = 3$. The value of θ is estimated by fitting the data of C, and the value is used for the scaling plot of σ_J and that of σ_H. The slope of the line in the right-middle panel is obtained by linear least-square fits of $\ln[\sigma_J(L, H = 0)]$ against $\ln(L)$.

hereafter focus on the data at the lowest temperature which is well below T_c. We set the MC step for thermalization and that for measurement to be equal. They are sufficiently (at least 5 times) larger than the ergodic time to ensure the equilibration, where the ergodic time is the average MC step for a specific replica to move from the lowest to the highest temperature and return to the lowest one. As in Ref. [16], we have also checked that MC runs starting from parallel and antiparallel boundary spins yield identical results within error bars.

Figures 4 and 5 show the results for $d = 3$ and those for $d = 4$, respectively. Since C is a dimensionless quantity, it is a function of only $X = LH^{1/\zeta}$. We therefore estimate ζ by fitting the data of C. By assuming the scaling relation $\zeta = d/2 - \theta$ predicted by the droplet theory, θ is estimated to be 0.29(3) for $d = 3$ and 0.60(2) for $d = 4$. Since they are close to both recent estimations by Boettcher [22] [0.24(1) for $d = 3$ and 0.61(2) for $d = 4$] and direct estimations by linear least-square fits of $\ln[\sigma_J(L, H = 0)]$ against $\ln(L)$ (0.28(3) for $d = 3$ and 0.69(3) for $d = 4$), our data support the scaling relation.

We next examine scaling properties of σ_J and σ_H by using the values of θ determined above. The scaling reasonably works except σ_J for $d = 4$. The deviation suggests that the fields investigated are too large and/or the sizes are too small so that corrections to the scaling is not negligible. In fact, if we closely observe the scaling plot of σ_J for $d = 3$, we notice that the data with large H, say $H \geq 0.48$, systematically deviate from the master curve, while the data with small field are scaled quite well. If we estimate θ by forcing all the data to be scaled *approximately*, we get apparently larger values of θ in both σ_J and σ_H. For example, θ estimated from σ_J in such way is 0.36 ± 0.02 for $d = 3$ and 0.84 ± 0.04 for $d = 4$.

The crossover behavior in the MK SG shown in Fig. 3 look very sharp in comparison with that in the EA SG. This is simply because the ranges of H and L, and so X, are quite wide. The enlargement of the crossover region such as to $0.1 \lesssim X \lesssim 10$ is realized by choosing the parameter ranges, for example, $0.015J \leq H \leq 0.95J$ and $L = 3^n$

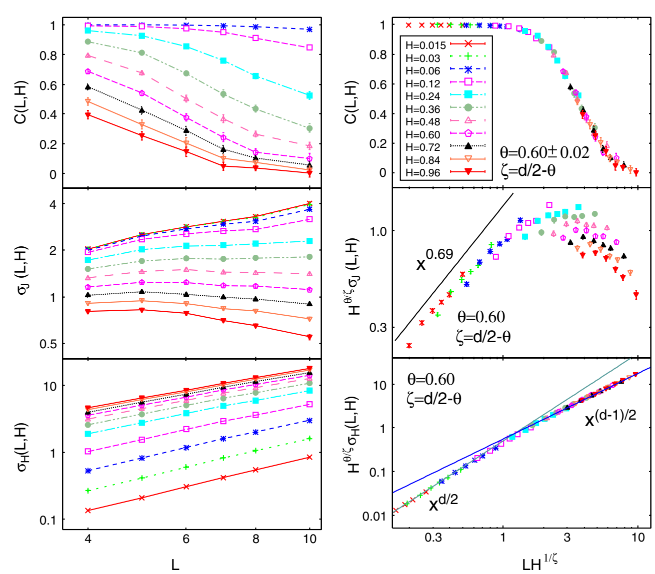

FIG. 5 (color online). The same plot as that in Fig. 4 for the four-dimensional EA SG at $T = 1.0J$.

with $1 \leq n \leq 4$. These ranges are close to those we are forced to choose for the EA SG. We then obtain quite similar results (not shown) as those obtained in the EA SG (Fig. 4). Within these ranges σ_J increases monotonically for smaller H and it decreases monotonically for larger H. Still, such data are reasonably scaled with the same value of θ as indicated in Fig. 3. These observations imply that the observed scaling behaviors in both the MK and EA SGs are essentially the same.

Interpretation of results.—We first consider the crossover behavior in σ_H. Since the field is not applied to the boundary spins, the effective fields $H_{\rm eff}^{(L)}$ and $H_{\rm eff}^{(R)}$ originate only through the influence of the field applied to the bulk spins $\{S_i\}$. For example, if the correlation between S_i and S_L is positive, an upward field to S_i tends to direct S_L upwards. Since the correlation can be either positive or negative depending on the site, the contribution to the effective fields can also be either positive or negative. When the effective coupling $J_{\rm eff}$ exists, S_L and S_R receive such random contributions, whose amplitude is proportional to H, from all the bulk spins. This yields σ_H which is proportional to $HL^{d/2}$. When $J_{\rm eff}$ vanishes, on the other hand, $\sigma_H \propto L^{(d-1)/2}$ because the boundary spins, which interact with L^{d-1} spins, receive contributions only from spins around their surfaces.

Now the meaning of our results are clear. As shown in Fig. 2, the Zeeman energy (σ_H) begins to overwhelm the effective coupling (σ_J) around $L \approx \ell_{\rm cr}(H)$ since σ_H increases faster than σ_J. After that, σ_J rapidly drops to zero. This means that the SG order whose length scale is longer than $\ell_{\rm cr}(H)$ is destroyed by the field. As described above, σ_H is kept increasing but with the change of its exponent from $d/2$ to $(d-1)/2$. The decay of $C(L, H)$ to zero for larger scaling variable X indicates the vanishing of the correlation between the state in zero field and that in field. The observed scaling behavior consistently implies the absence of the SG phase in any nonzero field.

Discussion and conclusions.—Let us comment on the fragility of the SG phase to other perturbations. According to the droplet theory, a change in the temperature (or bonds) by the amount Δ gives rise to a new SG state which is decorrelated from the original one beyond the length scale $\ell_{\rm ch}(\Delta)$. Here $\ell_{\rm ch}(\Delta)$ is proportional to Δ^{-1/ζ^*} with $\zeta^* \equiv d_s/2 - \theta$, d_s being the fractal dimension of droplets. This type of the fragility of the SG state is called temperature or bond chaos [23]. Our recent DWRG study [17] in the EA SG has indeed revealed the existence of $\ell_{\rm ch}(\Delta)$. A key observable in such studies is the correlation coefficient C of Eq. (3) with H replaced by Δ. As shown in Figs. 3–5, the system exhibits similar fragility against the field perturbation. However, it must be noted that vanishing of σ_J for larger X strongly indicates that the magnetic field destroys the SG order itself in sharp contrast to the cases of temperature and bond perturbations.

To conclude, thermodynamic observables of the EA SG are confirmed to follow the scaling in terms of the crossover length $\ell_{\rm cr}(H)$ as predicted by the droplet theory, the consequence of which is the absence of the SG phase in field. It should be noted that above the upper critical dimension $d_u = 6$ different scenario may hold [24]. We consider that all of our results concerning the temperature, bond and field perturbations provide strong evidence for the appropriateness of the droplet theory for the description of SGs in low dimensions.

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