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Dynamic Analysis of Parallel Manipulators under the Singularity-Consistent Parameterization

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SUMMARY

Based on the singularity-consistent parameterization framework, we analyze motion at direct kinematics singularities for a broad class of parallel manipulators. It will be shown that taking into account the instantaneous motion direction of the output link, additional insight can be gained for the possibility to move through such singularities. We argue that direct kinematics singularities should be analyzed over the dual space which, in turn, involves the state of the passive joints. We perform such an analysis based on the conditioning of the equation of motion. It is shown that, depending on the instantaneous motion direction, at certain direct kinematics singularities it is possible to obtain a consistent solution in terms of torque. This implies that in combination with the continuity of the singularity-consistent inverse kinematic solution, motion through such direct kinematics singularities is feasible.

KEYWORDS: Parallel manipulators; Dynamic analysis; Parametrization framework.

1. INTRODUCTION

The singularity analysis of parallel manipulators has drawn considerable attention in literature.[1–9] Pierrot et al.[2] mentioned two types of singularities, called *overmobility* and *undermobility,* which correspond to the indeterminacy of the direct and inverse kinematics, respectively. Gosselin and Angeles[4] added a third type, when both the direct and the inverse kinematics become simultaneously indeterminate. To identify any of these three types of singularities, it is sufficient to know the position of the active joints only. Recently, some other classifications appeared which take into account the position of the passive joints as well.[6–8] The work of Zlatanov et al.[7,8] is helpful in obtaining additional insights into the problem. Some authors note, however, that their classification has an inherent redundancy since the various types of singularities occur always in pairs.[9]

Notably, all the singularity analysis in literature is based on the static state of the manipulator. On the other hand, a number of recent studies show that it is possible to move through or to initialize a motion in a certain direction at a singularity.[10–15] In our opinion, there is a clear implication for future singularity analysis from those studies: *the analysis should be done over the phase space rather than over the configuration space.* Although such an analysis is not the ultimate goal of our present work, we shall provide some examples to justify the above assumption. Especially, with regard to singularities of parallel manipulators, it can be shown that some of the "redundant" classification groups in the work of Zlatanov et al. have then a distinct meaning.

The aim of this paper is to present a motion feasibility study at and around direct kinematics singularities which are relevant to a broad class of parallel manipulators. Our study will be based upon our recent work on a new, general method for solving the inverse kinematic problem of nonredundant serial-link manipulators. We called the method[16,17] *singularity-consistent* (*SC*).* Accordingly, the kinematic function is parameterized and a *nonlinear* inverse kinematics solution is obtained in terms of an autonomous differential equation. Under the parameterization, some of the kinematic singularities can be regarded as regular points. Thus, a stable motion can be guaranteed, even in the case when the manipulator follows a path passing close to, or through such singularities. Since the SC approach is general, it can be applied in a straightforward manner to parallel manipulators as well.[18] More specifically, we have shown that by means of a closed-loop command generator type controller, it is possible to generate feasible paths that would reconfigure the mechanism, moving thereby through a so-called *instantaneous self-motion* singularity. This has been also experimentally verified at our lab[19] with a six degree-of-freedom parallel robot HEXA.[2,20] Other types of singularities, such as *dual self-motion* and *bifurcation,* have been also discussed, but the motion feasibility remains to be studied.

The paper is organized as follows: In section 2 we present some background on the singularity-consistent parameterization for a parallel manipulator. Section 3 provides some new insight regarding singularities of the direct kinematics. In section 4, analysis based on the equation of motion is performed. Illustrative examples are presented in section 5. Finally, the conclusions can be found in section 6.

2. PRELIMINARIES: THE SINGULARITY-CONSISTENT PARAMETERIZATION OF THE KINEMATICS

In this paper, we consider a n dof nonredundant parallel manipulator which consists of two rigid bodies, one fixed

* "SC" will be used below as an acronym for "singularity-consistent."

another mobile, connected in parallel by a number of serial sub-chains. The kinematics of the parallel manipulator can be represented by the following nonlinear homogeneous equation:

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta}_a) = \mathbf{0} \tag{1}$$

which reflects the physical phenomenon of a closed kinematic chain.[4,21] In the above notation $\boldsymbol{\theta}_a \in \Re^n$ denotes the position of the active joints (the actuated joints), $\boldsymbol{x} \in \Re^n$ stands for the output-link coordinates i.e. the coordinates of the mobile body, and $\boldsymbol{f}: \Re^n \rightarrow \Re^n$ is a smooth vector-valued function. The subscript *a,* as in the above notation, will be used throughout the paper to denote an active joint variable. This is to be distinguished from passive joint variables which will be denoted by the subscript *p*.

Velocity-based control of a parallel manipulator utilizes the following equation, obtained after differentiation of equation (1):

$$\mathscr{D}_x \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta}_a)\dot{\boldsymbol{x}} + \mathscr{D}_\theta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta}_a)\dot{\boldsymbol{\theta}}_a = \mathbf{0} \tag{2}$$

where $\mathscr{D}_{(\circ)}$ denotes the differentiation with respect to $(\circ)$. In coordinate form, both differential mappings $\mathscr{D}_x\boldsymbol{f}$ and $\mathscr{D}_\theta\boldsymbol{f}$ are represented by $n \times n$ matrices.

Note that the last equation is valid for any type of parallel manipulator. In our further discussion, we shall make several assumptions regarding the parallel manipulator's structure. These assumptions will gradually restrict the scope but mainly from a theoretical viewpoint. Many parallel manipulators existing in practice will be covered by our framework. In such a sense, the first and weakest assumption we make is that the inverse kinematics of the parallel manipulator is solvable in closed form. Then, the following proposition establishes an important property of one of the above mappings:

Proposition 1: The mapping $\mathscr{D}_\theta\boldsymbol{f}$ is block-diagonal.

Proof: Since the inverse kinematics of the parallel manipulator is solvable in closed form, we can express equation (1) in more explicit separable form as follows:

$$\boldsymbol{e}_i(\boldsymbol{\theta}_{ai}) = \boldsymbol{\eta}_i(\boldsymbol{x}) \tag{3}$$

where $\boldsymbol{\theta}_{ai}$ is the vector of the active joint angles in the ith serial sub-chain; $\boldsymbol{e}_i$ and $\boldsymbol{\eta}_i$ are nonlinear vector-valued functions of appropriate dimensions. Note that at this stage we make no assumption about the number of active joints per serial sub-chain. The above form of equation (1) clearly indicates that it is possible to express equation (2) in such a way that there is no term in $\mathscr{D}_\theta\boldsymbol{f}$ cross-coupling the active joint angles of different sub-chains, which in turn implies $D_\theta\boldsymbol{f}$ is block diagonal.

Q.E.D.

The second assumption regarding the parallel manipulator's structure is that it comprises one active joint per serial sub-chain. A large class of parallel-link manipulators including spatial, such as the HEXA robot,[2,20] as well as planar ones, such as the five bar mechanism and the 3 dof manipulator used as illustrative examples below,* is covered by this assumption. For this class, the mapping $\mathscr{D}_\theta\boldsymbol{f}$ can be written in a diagonal form:

$$\mathscr{D}_\theta\boldsymbol{f} = \text{diag}\,[j_{aa1} \quad j_{aa2} \quad \cdots \quad j_{aan}].$$

Here, a double a subscript has been used to express the fact that the active joint space is mapped onto itself.

2.1 *The nonlinear inverse kinematic solution*

Recall that closed-loop velocity-based control of mechanical manipulators usually relies upon both the inverse kinematics and the direct kinematic solutions. In the case of a parallel manipulator, these solutions are derived in a straightforward manner from equation (2). A problem arises, however, when any of the mappings $\mathscr{D}_x\boldsymbol{f}$, $\mathscr{D}_\theta\boldsymbol{f}$, or both become ill-conditioned. This is the well-known singularity problem. In our further discussion we shall refer to the singularities of $\mathscr{D}_x\boldsymbol{f}$ and $\mathscr{D}_\theta\boldsymbol{f}$ as the singularities of the direct and the inverse kinematics, respectively. In the case when both of these mappings are singular, we shall speak about simultaneous singularities of the inverse and direct kinematics. This notion is in accordance with the three types of singularities defined by Gosselin and Angeles.[4]

To alleviate the above mentioned singularity problem, we proposed a reformulation of the kinematics under the assumption that the output-link path can be parameterized.[18] Suppose that $\boldsymbol{x} = \boldsymbol{g}(s)$ is the parameterization, where $\boldsymbol{g}: \Re \rightarrow \Re^n$ is a smooth function and the parameter s is not time. We will assume that the parameterizations does not induce singularities and hence, $\mathscr{D}_s\boldsymbol{g} \neq \mathbf{0}$ along the path. Then, the kinematic function is rewritten as:

$$\boldsymbol{f}(\boldsymbol{g}(s), \boldsymbol{\theta}_a) = \mathbf{0}. \tag{4}$$

After differentiation, we obtain

$$\mathscr{D}_s\boldsymbol{f}(\boldsymbol{g}(s), \boldsymbol{\theta}_a)\dot{s} + \mathscr{D}_\theta\boldsymbol{f}(\boldsymbol{g}(s), \boldsymbol{\theta}_a)\dot{\boldsymbol{\theta}}_a = \mathbf{0} \tag{5}$$

where the mapping $\mathscr{D}_s\boldsymbol{f} = [j_{s1} \quad j_{s2} \quad \cdots \quad j_{sn}]^T$ is an n-dimensional vector-valued function. It is apparent that with this representation, the system's dimension is decreased, as compared to the dimension of the "conventional" equation (2).

Besides decreased dimension, the parameterization yields another significant advantage: at some kinematic singularities a continuous inverse kinematic solution can be obtained. This can be explained as follows. First, we augment the active-joint space by the path parameter s:

$$\boldsymbol{q} = [s, \boldsymbol{\theta}_a^T]^T. \tag{6}$$

Equation (4) can be rewritten as

$$\boldsymbol{h}(\boldsymbol{q}) = \mathbf{0} \tag{7}$$

where $\boldsymbol{h}: \mathscr{R}^{n+1} \rightarrow \mathscr{R}^n$ is smooth because it is composed of

* See reference 4 for other examples.

two smooth mappings. Next, we introduce a linear local model at $\boldsymbol{q}$:

$$\mathscr{D}_q \boldsymbol{h}(\boldsymbol{q})\dot{\boldsymbol{q}} = \boldsymbol{0} \tag{8}$$

where the tangential space mapping $\mathscr{D}_q \boldsymbol{h}$ is composed of $\mathscr{D}_s \boldsymbol{f}$ and $\mathscr{D}_\theta \boldsymbol{f}$, and can be represented as

$$\mathscr{D}_q \boldsymbol{h}(\boldsymbol{q}) = \begin{bmatrix} j_{s1} & j_{aa1} & & & \boldsymbol{0} \\ j_{s2} & & j_{aa2} & & \\ \cdots & & & \ddots & \\ j_{sn} & \boldsymbol{0} & & & j_{aan} \end{bmatrix}.$$

Equation (8) denotes a homogeneous $n \times (n+1)$-dimensional system. A set of solutions exists that can be written as follows:

$$\dot{\boldsymbol{q}} = b\boldsymbol{n}(\boldsymbol{q}) \tag{9}$$

where b is an arbitrary scalar and $\boldsymbol{n} \in \Re^{n+1}$ is the so-called *null space function.* This function can be written as

$$\boldsymbol{n}(\boldsymbol{q}) = [v_0 \quad v_1 \quad \cdots \quad v_n]^T,$$
$$v_0 = \prod_{k=1}^{n} j_{aak},\; v_i = -j_{si} \prod_{\substack{k=1 \\ k \neq i}}^{n} j_{aak} \quad (i = 1, \ldots, n). \tag{10}$$

The system (9) represents an autonomous dynamical system.[22] This formulation is easily implemented for path planning and control purposes, as shown in our previous work.[16–18] Since b is arbitrary, at regular points of the kinematic function, it can be determined from the desired motion velocity as a function of time. Around and at kinematic singularities, b is modified to yield a feasible joint velocity. Note that thereby, the direction of motion is preserved continuously, as long as the null space function does not vanish.

Equation (9) will be used below in the analysis of kinematic singularities. We emphasize that this equation incorporates the instantaneous motion of the output link. This will help us in gaining important additional information regarding the behavior at kinematic singularities.

2.2 The kinematic singularities under the SC parameterization

In our previous work we described a classification of kinematic singularities within the framework of the SC parameterization.[18] The classification is based on the analysis of the elements of the null space function $\boldsymbol{n}$. These elements represent the relation between the output-link and the active-joint velocities, for the given instantaneous motion direction of the output-link. We will show below that at some specific kinematic singularities there is always a velocity relation that guarantees the continuity of the inverse kinematic solution. This, in turn, is related to the controllability of the parallel manipulator at kinematic singularities and is therefore motivated from a practical viewpoint.

There are generally two large classes of velocity relations at a kinematic singularity which we called *Type A* and *Type B* velocity relation, respectively.[16] Type A relation yields a "self-motion." This term is familiar from studies on serial kinematically redundant manipulators. It is used to describe the state when the output link is motionless while one or more of the other links are moving continuously. Below we shall apply this term in a broader sense, to cover also the state of motionless output-link and *instantaneously* moving intermittent links. The type of self-motion just described will be refered to as a *trivial self-motion.* Next, we note that in the case of parallel manipulators, there is another, "dual" type self-motion. It is characterized by a motion of the output link (either instantaneous, or continuous) while all the active joints are immobilized.[18] This state will be called *dual self-motion.*

The two types of self-motion can be distinguished by analyzing the elements of the null space function. In the case of trivial self-motion, from the definition (10) it is apparent that the first element of $\boldsymbol{n}$ vanishes, and there is exactly one nonzero element among the rest. This state occurs due to the vanishing of one of the diagonal elements j_{aai}. The inverse kinematics becomes indeterminate, with codimension one. We have an "undermobility" at hand. On the other hand, in the case of a dual self-motion, the analysis of the null space function reveals that the first element (i.e. the determinant of the diagonal matrix) is nonzero. Hence, the inverse kinematics is solvable. The rest of the elements of the null space function vanishes, which is due to the vanishing of all of the path-related elements j_{si}. This indicates the indeterminateness of the direct kinematics, and hence, shows the state of "overmobility." It is interesting to note that the two types of self-motions can be defined as Boolean functions over the elements of $\mathscr{D}_s \boldsymbol{f}$ and $\mathscr{D}_\theta \boldsymbol{f}$, when their nullity is expressed in binary form. Trivial self-motion is a logical "or" over the elements of $\mathscr{D}_\theta \boldsymbol{f}$, while dual self-motion is a logical "and" over the elements of $\mathscr{D}_s \boldsymbol{f}$.

In the case when Type B velocity relation is established at a kinematic singularity, the null space function vanishes entirely. This is a stationary point of the autonomous system (9). It is possible to distinguish further between two types of bifurcations: one of them occurring at specific simultaneous singularities of both the inverse and the direct kinematics, and the other – due to inverse kinematic singularities with codimension over one.[18] Such analysis goes, however, beyond the scope of the present work.

The important result is that, under any of the two types of self-motion, a smooth path leading through the kinematic singularity can be obtained via the inverse kinematics solution (9).

3. THE SINGULARITIES OF THE DIRECT KINEMATICS

From the discussion in the previous section it is clear that dual self-motion occurs at specific singularities of the direct kinematics. Because of the continuous inverse kinematic solution obtained under the SC parameterization, we are motivated to analyze the motion also at other types of direct kinematic singularities. We shall exclude, however, the case of simultaneous inverse and direct kinematics singularities yielding a bifurcation-type velocity relation, since motion fades out.

The following proposition shows that dual self-motion occurs at a specific subset of direct kinematics singularities.

Proposition 2: Dual self-motion occurs only at those singularities of the direct kinematics, at which the output-link velocity $\dot{\boldsymbol{x}} \neq \mathbf{0}$ belongs to the kernel of $\mathscr{D}_x\boldsymbol{f}$.

Proof: The direct kinematics singularities are defined as $\det \mathscr{D}_x\boldsymbol{f} = 0$. On the other hand, the state of dual self-motion is characterized with $\mathscr{D}_s\boldsymbol{f}\dot{s} = \mathbf{0},\ \dot{s} \neq 0$. From the two kinematic equations (2) and (5) it follows then that $\mathscr{D}_x\boldsymbol{f}\dot{\boldsymbol{x}} = \mathbf{0}$ must hold. Q.E.D.

It is apparent that the SC parameterization "masks" a large class of direct kinematics singularities, i.e. all those direct kinematics singularities at which the instantaneous motion of the output link is out of Ker $\mathscr{D}_x\boldsymbol{f}$. At such singularities, the velocity relation implies neither a self-motion nor an equilibrium. In other words, the first element of the null space function is nonzero, and in addition, there is at least one other nonzero element. This means that, similarly to the case of self-motion, the solution (9) will yield a smooth path through the singularity at hand.

4. ANALYSIS BASED ON THE EQUATION OF MOTION

Inspite of the continuity of the inverse kinematics solution at the direct kinematics singularities discussed in the previous section, it would be appropriate to analyze for possible problems with regard to the torque requirement.

First, recall that under dual self-motion the output-link moves due to the motion in the passive joints only. This means that the analysis should include the state of the passive joints and a relation of this state to the already known quantities such as the mappings $\mathscr{D}_x\boldsymbol{f}$ and $\mathscr{D}_\theta\boldsymbol{f}$.

4.1 Passive joint kinematics

At this point we make the third and final assumption regarding the parallel manipulator's structure. Namely, we shall assume that each of the n serial subchains has k dof, $k-1$ among them being passive, and such that the system is statically determined at a non-singular configuration. Let $\boldsymbol{\phi}_i$ be the coordinates of all active and passive joints of the ith serial sub-chain. Similarly to the kinematic relations shown at the beginning in section 2, we can write

$$\boldsymbol{\varphi}_i(\boldsymbol{x}, \boldsymbol{\phi}_i) = \mathbf{0}, \tag{11}$$

where $\varphi_i \in \Re^k$ is a vector-valued nonlinear function. Differentiating equation (11) with respect to time, we obtain:

$$\mathscr{D}_x\varphi_i\dot{x} + \mathscr{D}_{\phi_i}\boldsymbol{\varphi}_i\dot{\boldsymbol{\phi}}_i = \mathbf{0}. \tag{12}$$

Without any loss of generality, we can assume that θ_{ai} is the first element of $\boldsymbol{\phi}_i$ i.e., $\boldsymbol{\phi}_i = [\theta_{ai}, \boldsymbol{\theta}_{pi}^T]^T$. Then, the two mappings $\mathscr{D}_x\boldsymbol{\varphi}_i \in \Re^{k\times n}$ and $\mathscr{D}_{\phi_i}\boldsymbol{\varphi}_i \in \mathscr{R}^{k\times k}$ can be written in a partitioned form, by employing equation (3):*

$$\mathscr{D}_x\boldsymbol{\varphi}_i = \begin{bmatrix} \boldsymbol{j}_{xai} \\ \boldsymbol{J}_{xpi} \end{bmatrix} \tag{13}$$

and

$$\mathscr{D}_{\phi i}\boldsymbol{\varphi}_i = \begin{bmatrix} j_{aai} & \mathbf{0} \\ \boldsymbol{j}_{api} & \boldsymbol{J}_{ppi} \end{bmatrix} \tag{14}$$

where $\boldsymbol{j}_{xai}$ denotes the ith row of $\mathscr{D}_x\boldsymbol{f}$, $\boldsymbol{J}_{xpi} \in \mathscr{R}^{(k-1)\times n}$, $\boldsymbol{j}_{api} \in \mathscr{R}^{k-1}$ and $\boldsymbol{J}_{ppi} \in \mathscr{R}^{(k-1)\times(k-1)}$.

4.2 The equation of motion

The *unconstraint* equation of motion of each serial sub-chain can be written as:

$$\begin{bmatrix} \tau_{ai} \\ \mathbf{0} \end{bmatrix} + (\mathscr{D}_{\phi i}\boldsymbol{\varphi}_i)^T\boldsymbol{w}_i = \begin{bmatrix} \boldsymbol{M}_{ai}(\phi_i) \\ \boldsymbol{M}_{pi}(\phi_i) \end{bmatrix}\ddot{\phi}_i + \begin{bmatrix} c_{ai}(\phi_i, \dot{\phi}_i) \\ \boldsymbol{c}_{pi}(\phi_i, \dot{\phi}_i) \end{bmatrix} + \begin{bmatrix} g_{ai}(\phi_i) \\ g_{pi}(\phi_i) \end{bmatrix} \tag{15}$$

where $\boldsymbol{M}$, $\boldsymbol{c}$ and $\boldsymbol{g}$ denote parts of the inertia matrix, the vector of Coriolis and centrifugal forces, and the gravity forces at the respective serial sub-chain. Vector $\boldsymbol{w}_i$ denotes the Lagrange multiplier vector. τ_{ai} is the joint torque which is to be determined.

Next, assuming that the mass of the output link is distributed and attached to each of the serial sub-chains, the constraint equation imposed over their motion can be written as:

$$\sum_{i=1}^{n} (\mathscr{D}_x\varphi_i)^T\boldsymbol{w}_i = \mathbf{0}. \tag{16}$$

Note that the above form of the equation of motion is typical for *under-actuated* manipulators. The constraint (16) renders the system "regularly" actuated. Our goal, however, is to represent the above equation of motion in such a form which helps explicitly displaying the potential problems arising at the direct kinematics singularities under consideration.

Vector $\boldsymbol{w}_i$ can be partitioned similarly to the partitioning of ϕ_i: $\boldsymbol{w}_i = [w_{ai}\boldsymbol{w}_{pi}^T]^T$. Then, we solve for $\boldsymbol{w}_{pi}$ from the last $k-1$ equations of equation (15), as follows:

$$\boldsymbol{w}_{pi} = \boldsymbol{J}_{ppi}^{-T}\boldsymbol{d}_{pi} \tag{17}$$

where

$$\boldsymbol{d}_{pi} = \boldsymbol{M}_{pi}\ddot{\phi}_i + \boldsymbol{c}_{pi} + \boldsymbol{g}_{pi}. \tag{18}$$

Substitute $\boldsymbol{w}_{pi}$ from equation (17) into the equation of motion (15) and combine all n equations to obtain

$$\begin{gathered} \tau + \mathscr{D}_\theta\boldsymbol{f}\boldsymbol{w}_a = \boldsymbol{d}_a, \\ \boldsymbol{\tau} = \{\tau_{ai}\},\ \boldsymbol{w}_a = \{w_{ai}\},\ \boldsymbol{d}_a = \{d_{ai}\}, \\ d_{ai} = \boldsymbol{M}_{ai}\ddot{\phi}_i + c_{ai} + g_{ai} - \boldsymbol{j}_{api}^T\boldsymbol{J}_{ppi}^{-T}\boldsymbol{d}_{pi} \\ (i = 1, \ldots, n). \end{gathered} \tag{19}$$

The form of the equation of motion, as in equation (19), is suitable for the analysis. First of all it is seen that the

* This equation becomes scalar under the assumption one active joint per sub-chain.

inverse kinematics cannot deteriorate the dynamics, since the mapping $\mathscr{D}_\theta \boldsymbol{f}$ does not appear in an inverse form. In order to conclude about the role of the direct kinematics, consider the vector $\boldsymbol{w}_a$. From the constraint equation (16) and equation (17), we obtain:

$$(\mathscr{D}_x \boldsymbol{f})^T \boldsymbol{w}_a + \sum_{i=1}^{n} \boldsymbol{J}_{xpi}^T \boldsymbol{J}_{ppi}^{-T} \boldsymbol{d}_{pi} = \mathbf{0}. \quad (20)$$

It is apparent that $\boldsymbol{w}_a$ will be feasible as long as the last equation is solvable. In any other case, an infeasible torque will be produced through the equation of motion (19).

The general conclusion from this analysis is that for parallel manipulators, it is insufficient to use just the SC formulation, which in fact provides a singularity-consistent solution only with regard to the inverse kinematics. A formulation which includes also the singularity-consistency of the direct kinematics is desirable. It must be noted, however, that the consistency of the direct kinematics has to be analyzed over the dual space, which involves the state of the passive joints. It is impossible to get the necessary information just from the mapping $\mathscr{D}_x \boldsymbol{f}$ that appears in the "conventional" velocity equation (2). This conclusion confirms our initial assumption that the singularity analysis of parallel manipulators should be based on the full state space.

5. ILLUSTRATIVE EXAMPLES

Two planar structures, a five-bar mechanism, and a planar three dof manipulator with revolute actuators will be used to illustrate the above theories. In the simulations, the desired motion of both the active joints and the output link on the path is ensured at velocity level, by employing the command-generator type SC closed-loop controller.[18] Positions are derived through numerical integration by a fourth-order Runge–Kutta method with 0.01 s time step. Accelerations are obtained by numerical differentiation. The passive joint positions are calculated from the geometrical relationship and thereafter differentiated numerically to obtain the respective velocities and accelerations. The whole set of joint variables is then substituted into the equations of motion (equations (19) and (20)) to finally obtain the joint torque.

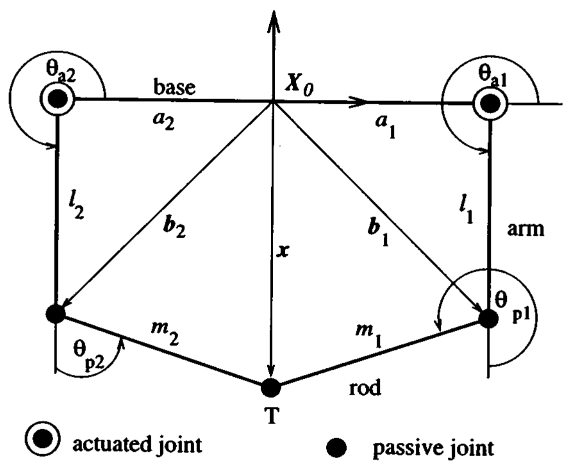

Fig. 1. The five bar mechanism.

Kinematic singularities will be identified by means of det $\mathscr{D}_x \boldsymbol{f}$ and det $\mathscr{D}_\theta \boldsymbol{f}$ which will be denoted as DK and IK, respectively.

5.1 The five-bar mechanism

The five bar mechanism is shown in Figure 1. Point T is the output-point, a_i, l_i and m_i denote the distance from the origin to the active joint, the arm length, and the rod length, respectively. We shall present the result from computer simulated motion through the various types of kinematic singularities. The output-point path is always linear. No specific end-point on the path will be specified. The reason is that, under the SC parameterization, the motion is always cyclic. Note that whenever the output-point reaches the workspace boundary, its motion is "reflected" back on the path and thus stays always within the workspace. The velocity on the path is defined under the *natural motion* constraint,[23] with $b = \text{const} = 1$. The derivation of the kinematic relations can be found in reference 18.

The dynamic parameters are the same in all simulations. We assume that the links do not have any moments of inertia, and three mass points are assigned to the passive joints as: one at the output-point (1 kg), and two at the passive joints (0.01 kg). We assume also that the linkage is in a horizontal plane, and thus gravity terms are eliminated.

With the first example, the geometry of the mechanism is: $l_i = a_i = m_i = 1$ m. The output-point moves on a horizontal linear path at a distance of 1 m from the origin. Figure 2 shows the data. The graph of the determinants shows that several trivial self-motions occur (at the points where the IK graph is zero), and three direct kinematics singularities are passed (at the points where the DK graph is zero). Representative configurations of the mechanism are attached to the graph. At one of the direct kinematics singularities (at about 9 s) a dual self-motion occurs. This is apparent from the null space vector graph which displays the output-point (OP) velocity, and the two active joint velocities (#2 and #3 standing for the first and second active joint, respectively). It is seen that the two active joint velocities are exactly zero, while the output point velocity is nonzero. At the other two direct kinematics singularities no dual self-motion occurs. From the torque graph it is seen that with the dual self-motion, the torque requirement is very high. This is due to the ill-conditioning of equation (20), and consequently, to the ill-conditioning of the equation of motion. Note also that the dual self-motion continues for a relatively long period of time during which a chattering-type behavior is observed. This means that, in practice, motion through this singularity might be impossible. To see the torque requirement at the other two direct kinematic singularities, we have to zoom in. The result shown in Figure 3 suggests that again, the equation of motion becomes ill-conditioned — there are high peak torques. Notably, no chattering occurs, however.

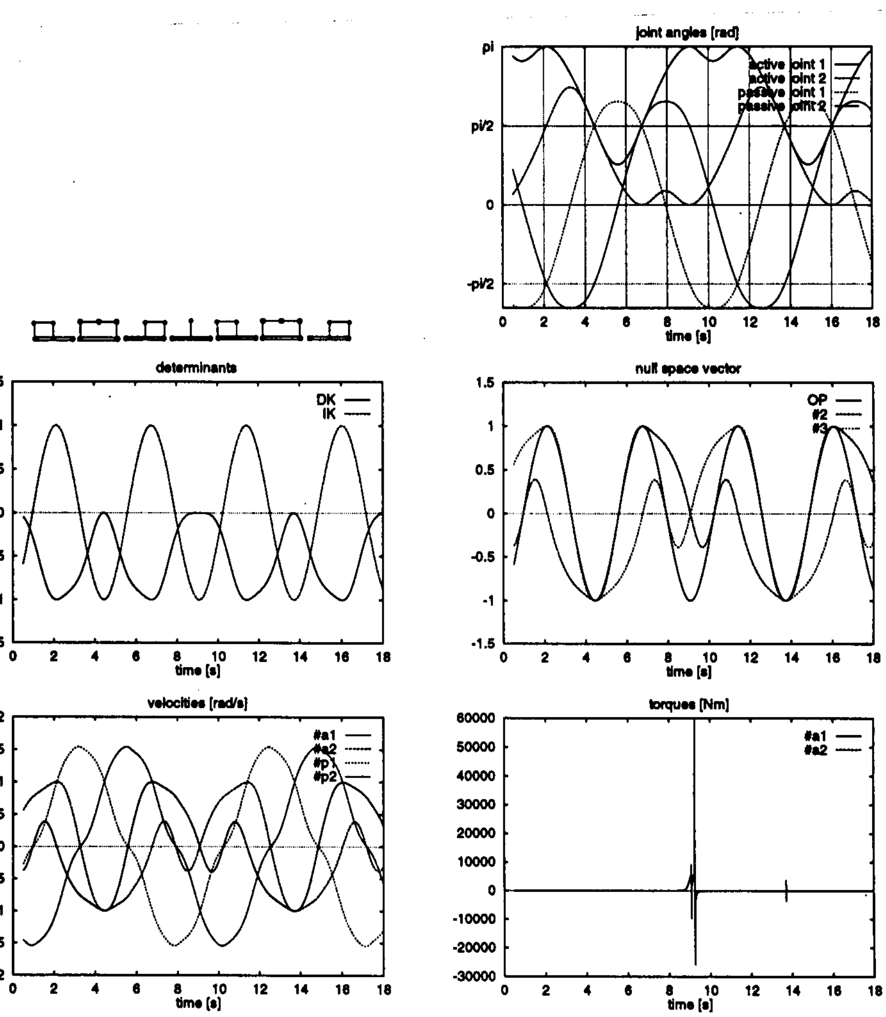

Fig. 2. Motion along a horizontal linear path.

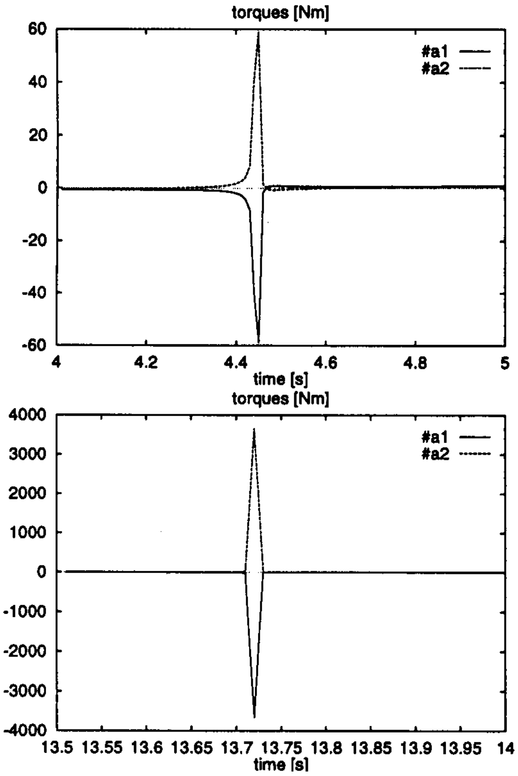

Fig. 3. Zoom in at the two of the DK singularities.

With the second example, the geometry of the five-bar mechanism is changed slightly with longer rods: $m_1 = m_2 = 1.12$ m. The motion is on a vertical path going through the origin. The two sub-chains are moving symmetrically. Figure 4 shows several consecutive configurations of the mechanism. The motion starts at the configuration with negative y coordinate of the output-point. Very soon a dual self-motion is performed. This is seen from the data in Figure 5. In fact, in order to avoid the necessity of zoom-in for the torque, the graphs

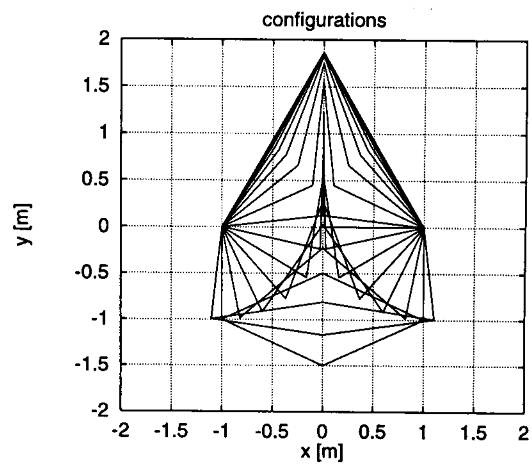

Fig. 4. Motion of the five-bar mechanism on a vertical path.

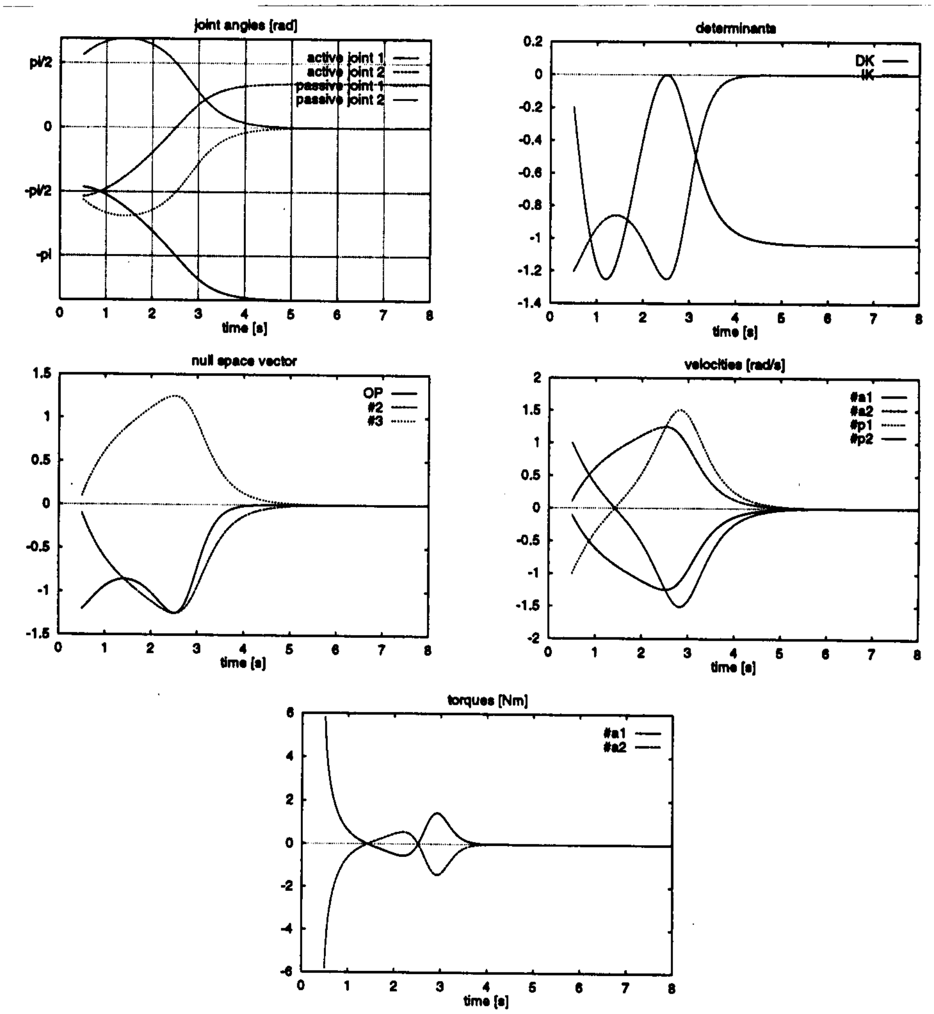

Fig. 5. Motion along a vertical linear path.

show the data just after the dual self motion. It is seen that besides the dual self-motion, another direct kinematics singularity is encountered, when the two rods are aligned (the active joint angles are $-\pi$ and 0, respectively). At that singularity, however, equation (20) and hence, the equation of motion is well-conditioned. This can be implied from the smooth torque graph. Thus, there will be no problem to move through this direct kinematic singularity, in practice. Note that finally, the motion ends up at a stationary point.

The third example demonstrates a motion through a simultaneous singularity of the inverse and direct kinematics. We note that this singularity does not imply a bifurcation, and hence, the inverse kinematic solution is expected not to vanish. The link geometry is $a_i = m_i = 1$ m, $l_1 = \sqrt{5}$ m, $l_2 = 1$ m. The simulation data is presented in Figure 6. It is seen that the SC inverse kinematics delivers a smooth solution, however, at the direct kinematic singularities, the system is ill-conditioned, and the torque requirement is infeasible.

5.2 The planar three dof manipulator with revolute actuators

Figure 7 shows a planar three dof parallel manipulator which contains a mobile plate, a fixed plate and three serial sub-chains. The position and orientation of the frame $\boldsymbol{X}_m$, attached to the center of gravity of the mobile plate, with respect to the frame $\boldsymbol{X}_o$, attached to the center of gravity of the fixed plate, are considered as the output. The detailed description of the manipulator along with the major kinematic relations can be found in reference 4.

We assume the fixed and the mobile plates of the planar manipulator to be two isoscales triangles with sides equal to 2.0 m and 1.0 m, respectively. The mass and inertia of the mobile plate are assumed to be 10.0 kg and 10.0 kg m^2, respectively. The geometrical parameters are set as $l_i = m_i = 0.7$ m $(i = 1, 2, 3)$. The mass corresponding to m_i or l_i is equal to 10.0 kg. The SC parameterization of the path is linear, and the dynamic modeling of the manipulator is done considering each serial sub-chain as lumped mass linkages.

The path chosen for the simulation contains the rotation of the mobile plate about its own axis while $\boldsymbol{X}_m$ always stays at the origin of $\boldsymbol{X}_o$. The velocity on the path is defined again under the natural motion constraint, with $b = \text{const} = 1$. The initial configuration is symmetric, as is the resultant path (in terms of motions in the three serial sub-chains). We present therefore the results for a single sub-chain only (see Figure 8). It is clear from the plot of

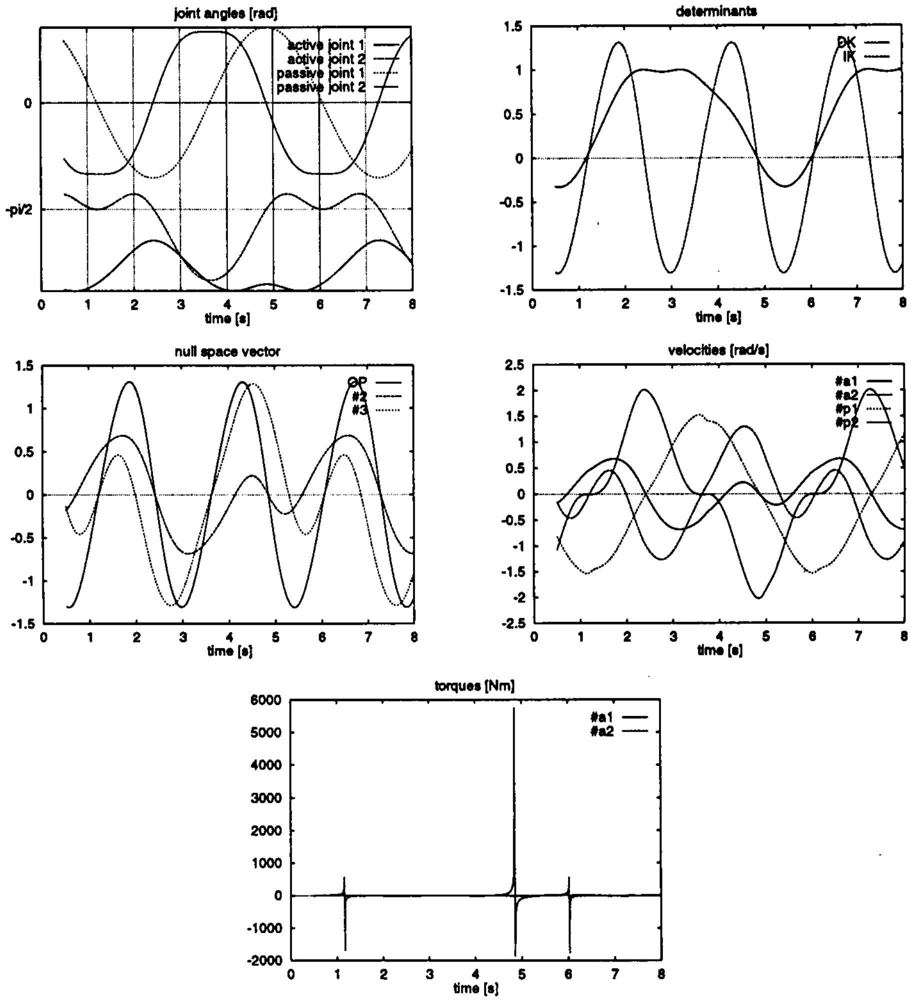

Fig. 6. Motion through simultaneous singularities of the inverse and direct kinematics.

the determinants that the manipulator crossed a direct kinematics singularity on its path. Note that under natural motion, the mobile plate's angular velocity is proportional to det $\mathscr{D}_\theta \boldsymbol{f}$. The corresponding velocity plot implies then that the velocity relation is that of a dual self-motion (the active-joint velocity #a3 is zero at the singularity). The plot of the torque at the active joint shows that the equation of motion is ill-conditioned at this specific singularity. When zoomed in, a quick change in the sign of the torque becomes evident across the singular point, however, no chattering is observed.

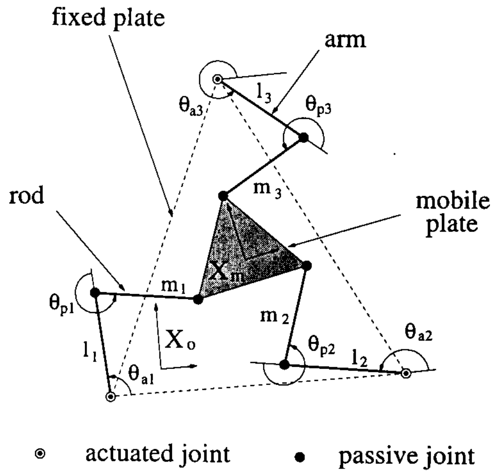

Fig. 7. A three dof planar parallel manipulator.

6. CONCLUSIONS

Based on the singularity-consistent parameterization framework, we analyzed the motion at direct kinematics singularities for a broad class of parallel manipulators. The result obtained shows that taking into account the instantaneous motion direction of the output link, additional insight can be gained for the possibility to move through the singularity at hand. This complements a similar result obtained through the analysis of the inverse kinematics.

We have shown that the singularities of the direct kinematics affect the conditioning of the equation of motion. This means that such singularities are to be analyzed over the dual space, which in turn involves the state of the passive joints. It is impossible to conclude

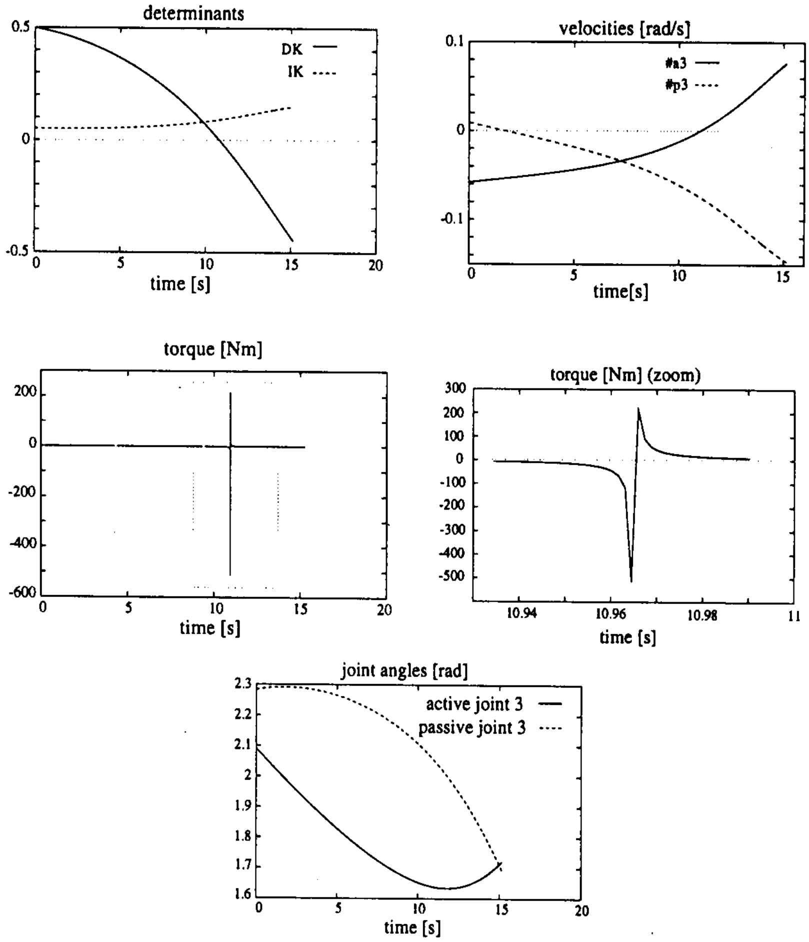

Fig. 8. Motion of the three dof planar parallel manipulator through a DK singularity.

about the consistency of the direct kinematics just from the velocities of the active joints and at the output link. More specifically, it was shown that, depending on the instantaneous motion direction, at certain direct kinematics singularities it is possible to obtain a consistent solution in terms of torque. This implies that in combination with the continuity of the SC inverse kinematic solution, motion through such direct kinematics singularities is feasible. The result obtained is useful for path planners to ensure paths that would cross the singularity, and hence, to avoid paths that yield the ill-conditioning of the equation of motion.

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