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Construction of Instanton Solutions in Gauge Theory on Eight Dimensional Noncommutative Space

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D-branes are solitons in string theory and have played an important role in the study of nonperturbative aspects of string theory. One way to study properties of D-brane systems is to investigate a low energy effective theory of open strings which have their end points on D-branes. It is well-known that such a low energy effective theory is written as a supersymmetric Yang-Mills theory.

From the viewpoint of the supersymmetric Yang-Mills theory, BPS bound states of D0-branes and D4-branes can be seen as instanton solutions of the four dimensional Yang-Mills theory which is the effective field theory of D4-branes. It is of interest to generalize the above systems of D0-branes and D4-branes to other systems of D-branes and to study them by means of the low energy effective theory.

Then I studied a system of D0-branes and D8-branes under the background of a constant NS-NS B field. The reason why I introduced the constant B field is that the low energy effective field theory of D-branes under such background field is known to be written as the Yang-Mills theory on noncommutative space and D0-branes on D8-branes with the B-field is expected to be described as instanton solutions of the Yang-Mills theory on eight dimensional noncommmutative space.

It is known that there exists a system of D0-branes and D8-branes which keeps some supersymmetries under the background of the constant B-field. Therefore it is expected that there exists a description of D0-branes and D8-branes as instanton solutions of the Yang-Mills theory on eight dimensional noncommmutative space.

These studies reduce to finding solutions of an eight dimensional analogue of the self-duality equations which are the first order linear relations amongst the components of the field strength. It has been shown that there are many kinds of the self-duality equations on eight dimensional space. In this paper I focused on the case that the ADHM construction on four dimensional space can be straightforwardly extended to eight dimensional space since in this case the extended ADHM construction on eight dimensional space can be easily extended to eight dimensional noncommutative space.

Then in this thesis, I have tried to construct the instanton solutions on eight dimensional noncommutative space which correspond to the BPS states of D0-branes and D8-branes with the B-field by using the noncommutative version of the extend ADHM construction. There are a few studies on this noncommutative version of the extend ADHM construction, but the interpretation of the extended ADHM construction as in the case of four dimensional space has not yet made clear. Therefore it is worth explicitly constructing instanton solutions of the Yang-Mills theory on eight dimensional noncommutative space by using the noncommutative version of the extend ADHM construction and it is also worth studying properties of the instanton solutions. From these studies, properties of the system of D0-branes and D8-branes are expected to be understood from the viewpoint of the low energy effective theory.

A topological charge $Q^{(4)}$ and a rank of the gauge group $U(N)$ of the four dimensional instantons respectively correspond to parameters k and N which appear in the ADHM construction on four dimensional space. And the four dimensional instantons have the D-brane interpretation of the bound states of k D0-branes and N D4-branes. By naively generalizing this D-brane interpretation to the instantons on eight dimensional noncommutative space, it is expected that the parameters N and k which appear in the extended ADHM construction respectively correspond to the rank of the gauge group of the Yang-Mills theory on eight dimensional space and the topological charge $Q^{(8)}$ of the instantons. Furthermore, the instanton solutions on eight dimensional noncommmutative space is expected to have the D-brane interpretation of the bound states of k D0-branes and N D8-branes with the B-field.

However I made it clear that the above expected interpretation is not always correct by explicitly constructing the noncommutative instanton solutions. In the case that the gauge group is $U(1)$ and the extended ADHM equations are deformed, I have pointed out that the integer k which appears in the extended ADHM construction should be interpreted as the charge of D4-branes rather than the charge of D0-branes by explicitly calculating topological charges in a wide subspace of the extended ADHM data.

In the case that the extended ADHM equations are not deformed, I have shown that the instanton solutions correspond to the system of k D0-branes and N D8-branes with the B-field by explicitly constructing the noncommutative instanton solutions. And I have made it clear that these noncommutative instanton solutions reproduce the solutions constructed by using a solution generating technique which is apparently different from the extended ADHM construction.