# The study on exact dynamics of one-dimensional electron system(Abstracts of Doctoral Dissertations,Annual Report(from April 1999 to March 2000))

# The study on exact dynamics of one-dimensional electron system

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Strong correlation between one-dimensional electrons has drastic consequences to their physical properties. The most spectacular is the spin-charge separation; the elementary excitations are spinons and holons both of which obey the fractional statistics. This is in contrast with the fermionic Landau quasi-particles realized in higher dimensions. Recently the spin-charge separation has been observed in the quasi one-dimensional compounds such as $SrCuO_2$ and $Sr_2CuO_3$ by the angle-resolved photoemission spectroscopy.

In the supersymmetric $t$-$J$ model with $1/r^2$ interaction, these fractional particles appear in the simplest manner. Exact thermodynamics for the model can be interpreted in terms of free spinons and holons. For the supersymmetric $t$-$J$ model, Ha and Haldane analyzed numerical results for finite-size systems, and found that only a few number of elementary excitations contribute to spectral functions, namely dynamical structure factors. They proposed the region where the spectral weight is nonzero for each spectral function in the thermodynamic limit, but did not obtain the spectral functions themselves. Analytical knowledge should obviously provide deeper insight into strong correlation effects on dynamics. In this thesis, I derive the analytical expressions for dynamical charge structure factor $N(Q,\omega)$ and one electron addition spectral function $A^+(k,\omega)$ of the supersymmetric $t$-$J$ model in the small momentum region.

I consider the supersymmetric $t$-$J$ model given by

$$\mathcal{H}_{tJ} = \mathcal{P}\sum_{i<j}\left[-t_{ij}\sum_{\sigma=\uparrow,\downarrow}\left(c_{i\sigma}^{\dagger}c_{j\sigma}+h.c.\right)+J_{ij}\left(\boldsymbol{S}_i\cdot\boldsymbol{S}_j-\frac{1}{4}n_in_j\right)\right]\mathcal{P}, \qquad (1)$$

where $c_{i\sigma}$ is the annihilation operator of an electron at site $i$ with spin $\sigma$, $n_i$ is the number operator and $\boldsymbol{S}_i$ is the spin operator. The projection operator $\mathcal{P}$ excludes double occupation at each site. The transfer energy $t_{ij}$ and exchange one $J_{ij}$ are given by $t_{ij} = J_{ij}/2 = tD_{ij}^{-2}$ where $D_{ij} = (N/\pi)\sin\left(\pi\left(i-j\right)/N\right)$ with $N$ being the number of lattice sites.

In the supersymmetric $t$-$J$ model, strong spin-charge separation is found in the dynamical charge structure factor $N(Q,\omega)$ with $Q$ smaller than Fermi wave number. $N(Q,\omega)$ with a given electron density does not depend on the spin polarization for arbitrary size of the system. In the thermodynamic limit, only two holons and one antiholon contribute to $N(Q,\omega)$. These results are generalized to the SU($K$,1) supersymmetric $t$-$J$ model (in the case of $K=2$ the SU($K$,1) model is reduced to eq. (1) ).

In the SU($K$,1) $t$-$J$ model, a holon has the spectrum $\epsilon_{\mathrm{h}}(q)=Ktq[2\pi(1-\bar{n})/K+q]/2$ with $\bar{n}$ being the average electron number per site, and another charge excitation called the antiholon has the spectrum $\epsilon_{\mathrm{a}}(q)=tq[2\pi(1-\bar{n})-q]/2$. The system has $N_\sigma$ electrons with spin $\sigma$. In the small momentum region $0<Q\leq k_F^{\mathrm{Min}}$, with $k_F^{\mathrm{Min}}$ being the smallest Fermi number of electrons, *i.e.*, $k_F^{\mathrm{Min}}=\pi\mathrm{Min}(N_1,\cdots N_K)/N$, I derive $N(Q,\omega)$ in the form:

$$\begin{aligned} N(Q,\omega) \;\propto\; & Q^2\int_0^{2\pi(1-\bar{n})}dq'\prod_{i=1}^{K}\int_0^{2\pi/K}dq_i \\ & \times\delta\left(Q-q'-\sum_{i=1}^{K}q_i\right)\,\delta\left(\omega-\epsilon_{\mathrm{a}}(q')-\sum_{i=1}^{K}\epsilon_{\mathrm{h}}(q_i)\right) \\ & \times\frac{\prod_{i<j}|q_i-q_j|^{2/K}}{\prod_{i=1}^{K}(Kq_i+q')^2}\,\epsilon_{\mathrm{a}}(q')^{K-1}\prod_{i=1}^{K}\epsilon_{\mathrm{h}}(q_i)^{1/K-1}, \end{aligned} \tag{2}$$

where $q_i$ ($i=1,\cdots,K$) and $q'$ represent the momenta of $K$ holons and one antiholon. In deriving the result I use the solution of the SU($K$,1) Sutherland model in the strong coupling limit. I have checked the validity of finite size results in the case of $K=2$ by comparison with numerical results up to $N=16$.

In the small momentum region $k\leq 2k_{\mathrm{F}}$ ($k_{\mathrm{F}}$ is Fermi wave number, *i.e.*, $k_{\mathrm{F}}=\pi\bar{n}/2$), the excitations which contribute to $A^+(k,\omega)$ consist of one spinon, one holon and one antiholon. I obtain the analytical expression for $A^+(k,\omega)$ in the form:

$$\begin{aligned} A^+(k,\omega) \;\propto\; & \int_0^{k_{\mathrm{F}}}dq_{\mathrm{s}}\int_0^{k_{\mathrm{F}}}dq_{\mathrm{h}}\int_0^{2\pi-4k_{\mathrm{F}}}dq_{\mathrm{a}} \\ & \times\delta\left(k-k_{\mathrm{F}}-(q_{\mathrm{s}}+q_{\mathrm{h}}+q_{\mathrm{a}})\right)\,\delta\left(\omega-(\epsilon_{\mathrm{s}}(q_{\mathrm{s}})+\epsilon_{\mathrm{h}}(q_{\mathrm{h}})+\epsilon_{\mathrm{a}}(q_{\mathrm{a}}))-\mu\right) \\ & \times\frac{\epsilon_{\mathrm{a}}(q_{\mathrm{a}})}{\sqrt{\epsilon_{\mathrm{s}}(q_{\mathrm{s}})\epsilon_{\mathrm{h}}(q_{\mathrm{h}})}(q_{\mathrm{h}}+1/2q_{\mathrm{a}})^2}, \end{aligned} \tag{3}$$

where $\mu$ denotes the chemical potential, *i.e.*, $\mu/t=-\pi^2/12(3\bar{n}^2-6\bar{n}+4)$. The $q_{\mathrm{s}}$, $q_{\mathrm{h}}$ and $q_{\mathrm{a}}$ represent the momenta of spinon, holon and antiholon. The dispersion relations of the excitations are given by; $\epsilon_{\mathrm{s}}(q)=tq(v_{\mathrm{s}}-q)$, $\epsilon_{\mathrm{h}}(q)=tq(v_{\mathrm{c}}+q)$ and $\epsilon_{\mathrm{a}}(q)=tq(2v_{\mathrm{c}}-q)/2$, where $v_{\mathrm{s}}=\pi$ and $v_{\mathrm{c}}=\pi(1-\bar{n})$. In deriving the result I use the solution of SU(1,1) Sutherland model. Analytical expressions for the matrix elements of the spectral function are conjectured by investigation of the finite systems. I also have checked the validity by comparison with numerical results up to $N=16$. Asymptotic behavior is fully consistent with the conformal field theory.

The both analytical results for $N(Q,\omega)$ and $A^+(k,\omega)$ can be interpreted in terms of the fractional statistics.