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On the Solutions of the Haissinski Equation with Some Simple Wake Functions

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Electrons in an accelerator interact with their environment because they are enclosed in metals (vacuum pipe, the RF cavities, etc.). An Electron feels a force produced by the other electrons. This force is called the wake force. The force acting on an electron by another electron is represented by the wake function. The wake force is an integration of this wake function. The motion of an electron in a bunch is described by the Fokker-Plank equation with the effective (one particle) potential term which represents the wake force and the stochastic effect coming from the synchrotron radiation.

A stationary solution of the Fokker-Planck equation is described by the Haissinski equation, which is a nonlinear integral equation. For most of the cases, there exists a threshold value for the total number of particles in the bunch, beyond which a solution of the Haissinski equation is unstable. Usually, the accelerator is designed and operated below the threshold. Beyond the threshold, the bunches tend to become long and show uncontrollable turbulent behavior.

Recently, the need for shorter bunches becomes serious for the better performance of e^+e^- colliding rings, high brightness light sources, the free electron lasers, etc.. To respond to these needs, a better understanding of the above instability becomes more important.

At present, on the other hand, the evaluation of the threshold is done fully numerically: wake functions for different vacuum chamber elements are calculated by computer programs. The threshold is calculated using a computer program. Sometimes, multiparticle tracking is used to see what happens beyond the threshold. There is very few analytical approaches. And, most of analytical approaches are done with a lot of approximations. To understand the nature of the beam and proceed to shorter bunches, more rigorous analytic approaches are needed.

For example, the existence and uniqueness of a solution for the Haissinski equation is not clear. There is not any successful approach to prove its existence and uniqueness. In this paper, we will prove it for two examples: for a purely capacitive wake function, we prove the existence and uniqueness rigorously. This wake function is a very special one but it is the first step towards the rigorous proof for the general cases.

Previous independent numerical studies seem to indicate that the Haissinski equation always has a unique solution, except for a single well-known case: a purely inductive wake function. Wake functions for the modern storage and damping rings tend to be purely inductive, because their vacuum chambers tend to be smooth. They contain small discontinuities only, such as shallow steps, transitions masks and bellows etc.. When the bunch length is sufficiently larger than these objects, these objects produce the inductive wake functions. A purely inductive wake function, however, is known to have a peculiar property: it is the only known example of the case where a solution of the Haissinski equation does not exist beyond a certain threshold.

In the main part of this paper, we numerically show that such strange property come from the ill-defined treatment of a purely inductive wake function. After introducing a physical regularization of its singularity, as we will show numerically, there always exists the solution of the Haissinski equation. Some authors claim that the threshold of the existence of the Haissinski equation has physical meaning. That is, beyond the threshold, the bunch is unstable. This argument is apparently wrong. The existence of a stationary solution and its stability are not the same. Actually, we will show that the instability exists but the threshold is different.