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Quantum Group Symmetries in Models of Elementary Particle Physics

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Abstract

Introduction

A serious conceptual problem when one tries to apply the techniques usually used in quantum field theory to the theory of gravity can be understood as follows: On the one hand when we want to use quantum field theoretical techniques the metric has to be specified a prioi. On the other hand in quantum gravity the metric itself becomes a dynamical variable and we meet the problem of having to specify the dynamical variables before the spacetime points have any physical identity. Due to this circumstance the investigations in quantum gravity were concentrated on either using perturbative methods around some fixed background metric or applying the method of the canonical quantization of gravity. However neither of them gave a complete description of the quantum gravity. These investigations indicate that the classical concepts of spacetime have to be modified very drastically in the transition to the quantum domain of gravity. Recently the existence of the quantum group is discovered. Since the quantum group is defined as a class of non-commutative non-cocommutative Hopf algebra, requiring the covariance under the action of the quantum group we get the simple model to investigate a geometry where we can consider to discard the commutativity of the algebra of coordinate functions, and such a step may be interpreted as a kind of quantization of the geometry. It is the subject of this thesis to investigate whether this new structure offers the possibility to describe the behaviour of the spacetime near the Planck scale.

Part I

First we give a brief overview of the mathematical structure of the quantum group. Then we investigate, as a first step, the q-deformation of the Lorentz group and the algebra of the q-deformed Mikowski space.

We give here an improved description of our original construction of the quantum Lorentz group by defining the commutation relation including the $*$-cojugated elements, which define the algebra called real form of the quantum double. This construction also shows the new feature peculiar to the quantum group that there are two possibilities to construct $Fun_q(SO(4,C))$, which reduce to either the $Fun_q(SO(4))$

or the $Fun_q(SO(3,1))$ under the reality condition. We also define a q-deformed non-commutative, 4-dimensional space which corresponds to the Minkowski space of the non-deformed case. The relation between this q-deformed Minkowski space coordinate algebra and the algebra of the quantum 2-sphere is given in the appendix A.1. For completeness we also show the construction of the q-deformed Clifford algebra related with the quantum Lorentz group [CSSW2].

Part II

We investigate the generalization of the formalism of differential geometry to the case with quantum group symmetry. The bicovariant bimodule has been first defined by Woronowicz, however the differential calculus was constructed only for the case of $Fun_q(SU(2))$. We have developed the $\hat{R}$-matrix representation of the formalism, which makes it possible to generalize the differential calculus to the cases of $Fun_q(SU(N))$ and $Fun_q(SO(N))$. Although most of the proofs were given in our original paper we largely straightened the logic here and give an improved description.
Furthermore we give a proof of the invariance of our differential calculus under the $*$-operation, which is not contained in our original paper. The differential calculus is insofar instructive for our aim since it teaches us the behaviour of the ghost fields, and thus gives us an insight into the possibilities of constructing q-deformed gauge theories by using the BRST formalism.

Part III

We investigate the connection between the differential calculus on quantum group and the quantum space algebra which gives us quite instructive relations between the ordinary differential calculus and its q-analogue:
We derive from the q-deformed algebra of functions over $SU(N)$ the commutation relations of the dual algebra by using the q-deformed Maurer-Cartan equation. Explicit results are given for the case of $Fun_q(SU(2))$. Then, considering the action of the dual algebra of $Fun_q(G)$ on the quantum plane we obtain a natural generalization of the concept of the infinitesimal transformation to the q-deformed case. As an example we define the infinitesimal quantum Lorentz group transformation.
To accomplish our considerations it is necessary to understand not only the kinmatics but also the dynamics on the non-commutative spacetime. For this aim we investigate the Schrödinger equation on the q-deformed Euclidian space. This line is still in a very early stage of its development and only a few results are known. Here we only examine the structure of the 'q-wave function ' which corresponds to the ground state of the harmonic oscillator.

Discussion and Conclusion

In this part we discuss the new possibilities of a q-deformed theory as a regularization. Since in such a theory the q-deformation is systematically incorporated into the basic structure, we may expect that the q-deformation will give us some profound conceptional changes.