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Magnetohydrodynamical Winds with Finite Electrical Conductivities

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We discuss a numerical treatment of the *resistive* MHD winds which are driven by strongly magnetized, rotating stars. For simplicity, the axis of rotation is assumed to coincide with that of magnetization.

It is shown that rotation-driven wind solutions surely exist, to the set of resistive MHD equations even when the stellar magnetic fields remain of closed type.

Keywords: Magnetohydrodynamics(MHD), resistive MHD, stellar winds, centrifugal force.

§1. Introduction

Magnetohydrodynamical (MHD) stellar wind theory has been developed by many authors[1)] since Weber and Davis[2)] first found the split-monopole solution for a rotating magnetized star, and Michel[3)] generalized their work to a relativistic treatment, almost all of these authors have made the assumption that the electrical conductivity is infinite everywhere (i.e. the ideal MHD approximation). As far as adopting this approximation, we have to assume that the stellar magnetic fields are deformed to have open structures in the wind zone.

One of the present authors (Kaburaki[4)]) has proposed that a more realistic structure of the stellar magnetic fields in the wind zone is an elongated dipole-type structure, when a finite resistivity of the plasma is taken into account. He has derived analytically a self-consistent approximate solutions to the resistive MHD equations which are fairly simplified under the assumption that the magnetospheric plasma forms an equatorial disk owing to the action of centrifugal force on it. In this paper, we solve these equations numerically in order to improve the accuracy of the solutions.

§2. Equations for Disk-like Winds

The plasma in a rapidly rotating stellar magnetosphere tends to form a disk-like structure. This is due to the centrifugal force, the components of which along the magnetic lines of force lead the plasma to gather around the equatorial plane. In this paper, we restrict our attention only to the cases of non-relativistic centrifugal wind. Such situations are realized when the magnetospheric plasma density is so dense that the centrifugal force can deform the stellar magnetic field well within the light cylinder.

The set of non-relativistic resistive MHD equations are first simplified under the assumption that the disk is thin (i.e. $\Delta \ll 1$ where Δ is the angular thickness of the disk and is assumed to be a constant). Then, the separation of variables is approximetely performed in the axisymmetric and stationary configurations, so that we have a set of ordinary differential equations to describe disk-like stellar winds. In this paper, we discuss only a dense and non-relativistic centrifugal wind. The method is identical to that has been developed in the discussion of magnetized accretion disks (Kaburaki[5]).
The non-dimensional forms of the equations become as follows:
equation of motion (r, ϕ, and θ components, respectively);

$$\hat{v}_r \frac{d\hat{v}_r}{dx} - \frac{\hat{v}_\phi^{\,2}}{x} = -\frac{1}{\zeta^3 x^2} - \frac{\hat{b}_r}{\hat{\rho} x^4} \quad , \qquad (1)$$

$$\frac{d}{dx}(x\hat{v}_\phi) = \frac{\hat{b}_\phi}{\hat{\rho}\hat{v}_r}\frac{1}{x^3} \quad , \qquad (2)$$

$$\hat{P} = \hat{b}_\phi^{\,2} \quad , \qquad (3)$$

Ohm's low (r and ϕ components);

$$\hat{b}_r = R_A \frac{\hat{v}_r}{x^2} \quad , \qquad (4)$$

$$\hat{b}_\phi = R_A\left(1 - \frac{\hat{v}_\phi}{x}\right)\frac{1}{x} \quad , \qquad (5)$$

equation of states;

$$\hat{P} = K_A \hat{\rho}^\gamma \quad , \quad (\gamma \text{ is a constant}) \qquad (6)$$

where the non-dimensional variables are;
radial coordinate

$$x = \frac{r}{r_A} \quad (r_A\text{: inner boundary}),$$

velocity

$$\hat{v}(x) = \frac{\tilde{v}(r)}{V_A} \quad (V_A = r_A\Omega,\ \Omega\text{:angular velocity of the central star}),$$

magnetic field

$$\hat{b}(x) = \frac{\tilde{b}(r)}{B_A} \quad (B_A = \frac{\mu}{r_A^3}, \ \mu\text{:magnetic dipole moment of the central star}),$$

pressure

$$\hat{P}(x) = \frac{\tilde{P}(r)}{P_A} \quad (P_A = \frac{B_A^2}{8\pi}) ,$$

and density

$$\hat{\rho}(x) = \frac{\tilde{\rho}(r)}{\rho_A} \quad (\rho_A = \frac{B_A^2}{4\pi\Delta V_A^2}) .$$

Other parameters are

$$\zeta = \frac{r_A}{r_K} \ (r_K = (\frac{GM}{\Omega^2})^{1/3}) ,$$

$$R_A = \frac{4\pi\sigma\Delta r_A V_A}{c^2} \quad \text{(matnetic Reynolds number)},$$

$$K_A = \frac{K\rho_A^{\gamma}}{P_A} ,$$

where G is the gravitational constant, M is the mass of the central star, c is the light velocity and σ is the electrical conductivity (also assumed to be a constant, for simplicity). These six equations and six variables $\hat{v}_r$, $\hat{v}_\phi$, $\hat{b}_r$, $\hat{b}_\phi$, $\hat{P}$, $\hat{\rho}$ construct a closed set of equations. Other quantities, such as electric currents, the θ-component of the electric field, and the θ-component of the velocity are calculated from the remaining set of equations after the above six quantities are determined.

§3. Boundary Conditions

We set the inner boundary conditions according to the following consideration.

The acceleration of a wind works only outside the radius where centrifugal force is just balanced by gravity and the magnetic restoring force. Solving (1) for the density under the condition $d\hat{v}_r/dx = 0$, we have

$$\hat{\rho}_b = R_A \frac{\hat{v}_r}{x^5 \hat{v}_\phi^2 - x^4 \zeta^{-3}} . \tag{7}$$

We require that this value gives not only the boundary value for the density,

$$\hat{\rho} = \hat{\rho}_b \quad \text{at } x = 1 , \tag{8}$$

but also its gradient,

$$\frac{d\hat{\rho}}{dx} = \frac{d\hat{\rho}_b}{dx} \quad \text{at } x = 1 , \tag{9}$$

where $\hat{\rho} = (\hat{b}_{\phi}^2/K_A)^{1/\gamma}$ which is determined from (3) and (6). Then solving (8) and (9), we have the boundary values, $\hat{v}_r(x=1)$ and $\hat{v}_{\phi}(x=1)$. With these four boundary conditions, we can solve the equations (1) ∿ (6) numerically.

§4. An Example of the Solutions

We show in Fig. 1 ∿ Fig. 3 only one example of the numerical solutions to the disk equations.

The important features of this solution are summarized below.

1. Resistive MHD wind solutions which are consistent with closed-type magnetic field lines surely exist for certain regions of parameters and proper boundary conditions.
2. In this example, angular momentum and energy transfer from the central star takes place only within a narrow region in r.
3. The terminal velocity for v_r is of the order of the corotational velocity at the inner boundary ($r = r_A$).

The above property 2 is qualitatively different from that of the analytical solution discussed by Kaburaki[6]. However, this fact is due to the difference in γ adopted in the above example. We have obtained also solutions for a different value of γ whose behaviours are analogous to that of the analytic solution. However, a full explanation of the whole types of solution will be published elsewhere later.

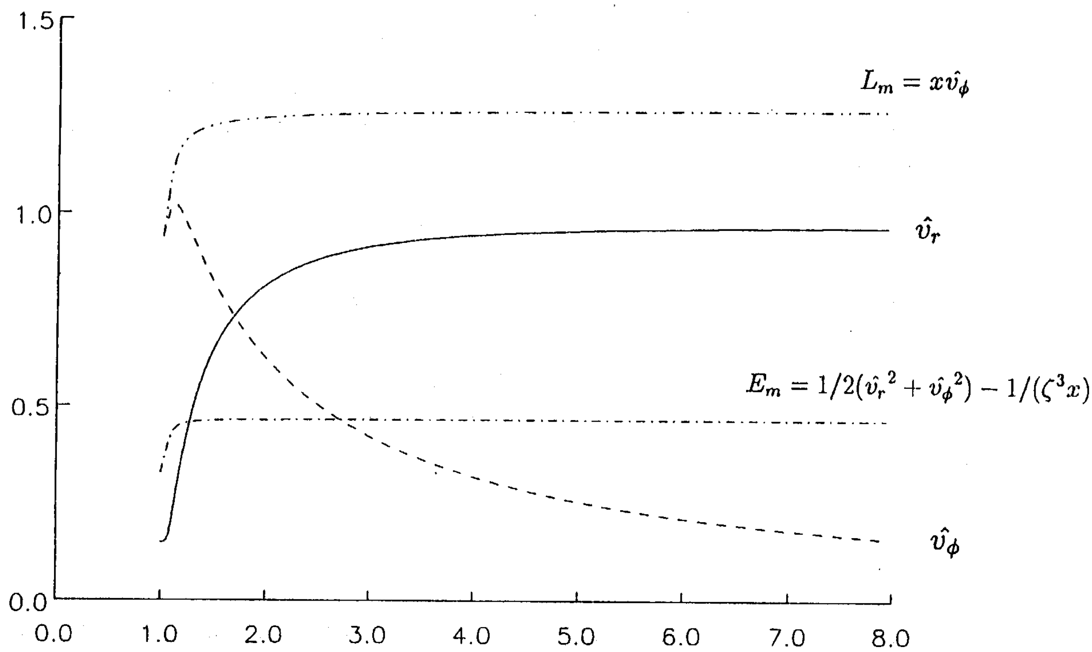

Fig. 1 Variations as a function of $x = r/r_A$ are plotted for the non-dimensional radial velocity $\hat{v}_r$, the toroidal velocity $\hat{v}_{\phi}$, the specific angular momentum of fluid $\hat{L}$, and the specific energy of fluid $\hat{E}$. The adopted parameters are $R_A = 5.0$, $\gamma = 4/3$, $K_A = 0.1$, $\zeta = 2.0$.

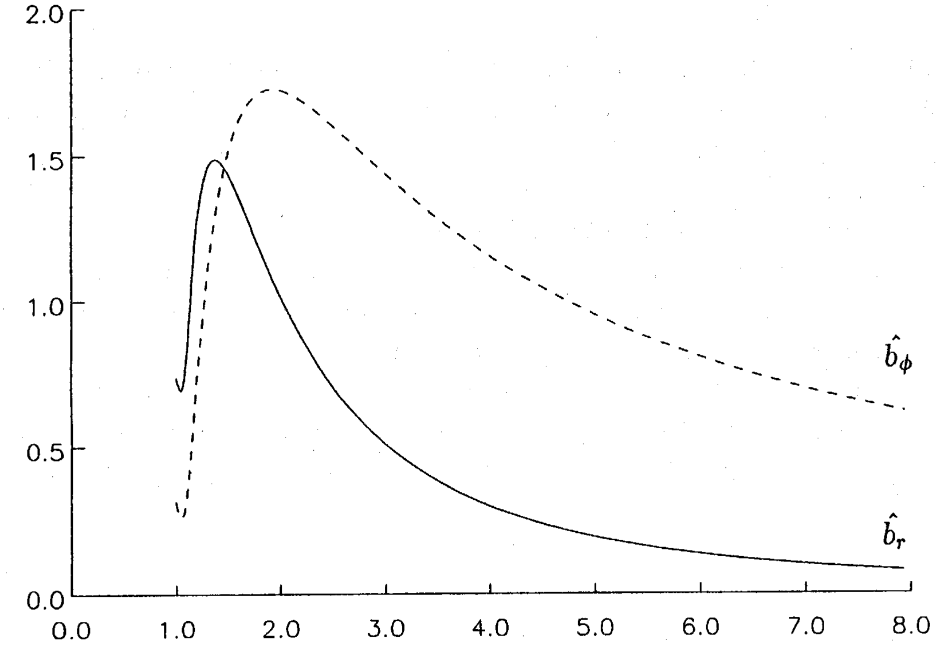

Fig. 2 The same are shown for the non-dimensional radial magnetic field $\hat{b}_r$ and the toroidal magnetic field $\hat{b}_\phi$.

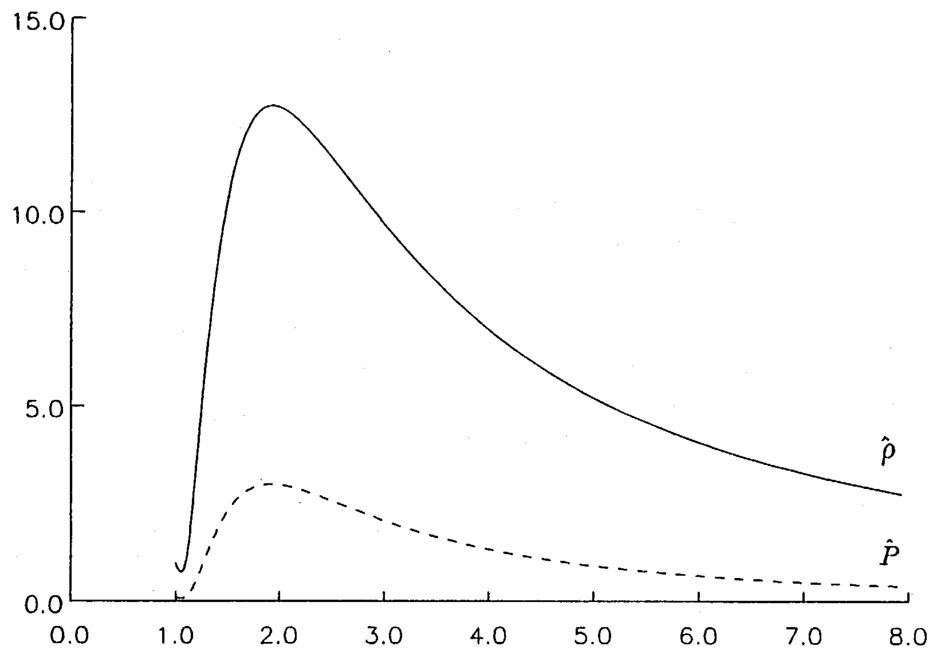

Fig. 3 The same are shown for the non-dimensional density $\hat{\rho}$ and the pressure $\hat{P}$.

§5. Discussion

We have established within the framework of resistive MHD that there exist stellar wind solutions which are consistent with the closed configuration of the magnetospheric lines of force. It should be emphasized here that, in the resistive MHD solution, the electric field has a component even along the wind path. Although this field does not play an essential role in the acceleration of the centrifugal winds, as far as the plasma density is large enough, the electric force can dominate over the centrifugal force when the plasma is extremely tenuous as in the pulsar magnetospheres. Indeed, in the actual pulsar magnetospheres, the electric acceleration of the wind within the light cylinder seems to be quite essential[6]. In this sense, the extension of the present work to a fully relativistic regime would be an important future task.

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