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Revision of Pulsation Properties of Yellow Bright Stars

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Linear adiabatic periods, which have been mostly used to evaluate the masses of pulsating stars, shouldn't be used for low surface-gravity variable stars, because those differ considerably with linear nonadiabatic periods and nonlinear theoretical periods in some cases. The use of nonadiabatic periods sometimes derive drastically low stellar masses. Taking into account the small occurrence of variable stars at the bluer side of cepheid instability strip on the HR diagram, it may be supposed that most of these variables will be less massive evolved stars.

§1. Introduction

A lot of variable supergiant stars is classified as cepheid variables which are located on the cepheid instability strip in the HR diagram. The cepheid variables have been intensively investigated for more than a half a century. Their periods are estimated from radial pulsation theory of stars. The pulsational properties have been solved by numerical calculations in both linear and nonlinear approximations. On the other hand, other variable stars found among supergiant stars, which are not situated in the cepheid instability strip, were not investigated so precisely. Stars found at the red side of the instability strip have still problems to investigate them, caused from convection developed in their outer layers. Dynamical effects may be important to construct model envelopes for these stars. Even static structures of atmospheres have not been solved for the Mira variables. We shall discuss here, therefore, only the stars which are found at the bluer side of the cepheid strip.

In the study of these bright stars, pulsation masses, which are derived from the comparison of theoretical periods with observed periods, are a useful tool. Takeuti's theoretical periods for the upper domain of the HR diagram have been sometimes used to evaluate their pulsation masses 1). Unfortunately, nonadiabatic theoretical periods differ considerably from the adiabatic ones in

some cases. Periods determined from hydrodynamic models are also different from the linear adiabatic periods. In the present paper, we summarize this important properties on the basis of various theoretical calculations. Consequence of the use of nonadiabatic periods are also discussed.

§2. Disagreement between Linear Adiabatic and Nonadiabatic Pulsation Periods

In general, it was believed that linear adiabatic pulsation periods and linear nonadiabatic pulsation periods are approximately identical for radially pulsating stars. The adiabatic periods are almost equal to the nonadiabatic ones at least for classical cepheids. Aikawa however, showed that the linear nonadiabatic period differs from the adiabatic one considerably for a model stellar envelope. Aikawa halo argued on the disagreements between the linear adiabatic and nonadiabatic periods for stellar models of which surface gravities are very small. General trends in pulsation periods of yellow bright stars are that their nonadiabatic periods are reduced compared with adiabatic ones when their surface gravities become smaller. Takeuti found the same characteristics for the models of an F supergiant in ϵ Aur system. The periods deduced from nonlinear hydrodynamic models for supergiants constructed by Fadeyev $^{6,7)}$ are also considerably different from their linear adiabatic periods.

This reduction of theoretical periods is important because the pulsation masses of yellow bright stars are reduced considerably when the nonadiabatic periods are applied to the investigation of supergiant stars. Aikawa found the pulsation mass of a variable star HDl61796 is approximately one solar mass, even though Fernie $^{8)}$ and Takeuti $^{9)}$ had concluded it may be a massive star based on the comparison between adiabatic periods and the observed period. Zalewski $^{10)}$, who studied the star, found a less massive model can explain the variability. The F star of ϵ Aur should be about one solar mass when the nonadiabatic periods are compared with its quasi-period. The cause of this disagreement is analysed by Aikawa $^{3)}$.

Pulsation periods calculated in the linear nonadiabatic approximation are tabulated in Table I. The trends discussed above are clearly indicated. Strange modes often yielded on the way to search the solution of pulsation functions of the ordinary mode are ignored because they are pulsationally stable. Since the code includes only radiative transport of energy, the calculations are confined to models near the blue edge of the instability strip.

Figures la-d show the pulsation constants in the adiabatic and non-adiabatic pulsations of pulsationally unstable models as a function of the mass-radius ratio, $\log(M/R)$. The tendency of the difference between the adiabatic and non-adiabatic pulsation constants is clearly demonstrated. And the constants are smooth functions of the mass-radius ratio both for the adiabatic and non-adiabatic pulsations except for extremely high luminosity and high temperature models (points at the upper-left corner in Fig. 1d). In the latter cases, there

Table I. Linear adiabatic and nonadiabatic periods for low surface-gravity stars. (The mass M, the luminosity L, and the radius R are expressed in solar units. The period P is given in days. η is the growth rate a period.)

			a) log l	5 = 3.5.					
Te	М	log(M/R)	$P_{\mathbf{L}\mathbf{A}}$	$P_{\mathtt{LNA}}$	$\log Q_{_{\mathbf{L}\mathbf{A}}}$	$\log Q_{ extsf{LNA}}$	η		
5000	1.0	-1.876	57.5	44.3	-1.055	-1.165	+0.69		
5500 5500 5500 5500	1.0 1.5 2.0 3.0	-1.793 -1.617 -1.492 -1.316	39.3 26.2 20.0 14.0	31.4 23.5 18.9 13.7	-1.095 -1.183 -1.239 -1.306	-1.192 -1.23 -1.264 -1.315	-0.02 +0.26 +0.23 +0.14		
6000 6000 6000 6000	1.0 1.5 2.0 3.0	-1.718 -1.541 -1.417 -1.240	27.4 18.6 14.4 10.2	23.2 16.9 13.6 9.9	-1.139 -1.217 -1.268 -1.330	-1.211 -1.25 -1.29 -1.341	-0.72 -0.19 -0.05 +0.00		
6500 6500	2.0 3.0	-1.347 -1.171	10.5 7.6	10.1 7.4	-1.300 -1.355	-1.317 -1.364	-0.37 -0.14		
b) $\log L = 3.75$									
Te	М	$\log(M/R)$	$P_{\mathtt{LA}}$	$P_{\mathtt{LNA}}$	$\log \mathit{Q}_{\mathtt{LA}}$	$\log Q_{ extsf{LNA}}$	η		
5000 5000	1.0 1.5	-2.001 -1.825	111. 70.6	70.8 55.6	-0.956 -1.064	-1.151 -1.168	+0.34 +0.69		
5500 5500 5500 5500 5500	1.0 1.5 2.0 3.0 4.0	-1.918 -1.742 -1.617 -1.441 -1.316	75.6 48.2 36.0 24.5 19.0	53.2 39.3 31.9 23.3 18.5	-0.999 -1.106 -1.170 -1.249 -1.297	-1.151 -1.195 -1.223 -1.272 -1.309	-0.35 +0.02 +0.20 +0.18 +0.12		
6000 6000 6000 6000	1.5 2.0 3.0 4.0	-1.666 -1.542 -1.365 -1.240	33.8 25.5 17.6 13.8	28.9 23.0 16.8 13.4	-1.147 -1.207 -1.309 -1.323	-1.215 -1.251 -1.325 -1.336	-0.65 -0.29 -0.08 -0.03		
			c) log	L = 4.0					
Te	М	log(M/R)	$P_{\mathbf{L}\mathbf{A}}$	PLNA	$\log Q_{_{{ m LA}}}$	$\log Q_{ ext{LNA}}$	η		
5000 5000 5000	1.0 1.5 2.0	-2.126 -1.950 -1.825	225. 138. 99.2	114. 90.1 75.5	-0.838 -0.961 -1.042	-1.133 -1.146 -1.160	+0.36 +0.39 +0.63		
5500 5500 5500 5500 5500 5500	1.0 1.5 2.0 3.0 4.0 6.0 8.0	-2.043 -1.867 -1.742 -1.566 -1.441 -1.265 -1.140	154. 93.9 67.8 44.2 33.6 23.4 18.3	88.6 67.2 54.0 39.7 31.6 22.8 18.1	-0.877 -1.004 -1.083 -1.180 -1.237 -1.307	-1.117 -1.149 -1.083 -1.227 -1.264 -1.318 -1.356	-0.20 -0.39 -0.07 +0.18 +0.15 +0.08 +0.05		
6000 6000 6000	3.0 4.0 6.0 8.0	-1.490 -1.365 -1.189 -1.064	31.4 24.0 17.0 13.4	28.7 22.8 16.6 13.2	-1.216 -1.271 -1.333 -1.372	-1.255 -1.293 -1.343 -1.379	-0.28 -0.14 -0.05 -0.02		

d) $\log L = 4.25$										
Тe	М	log(M/R)	$P_{\mathtt{LA}}$	PLNA	$\log Q_{ extbf{LA}}$	$\log Q_{ extsf{LNA}}$	η			
5000 5000	1.5 2.0	-2.075 -1.950	290. 200.	145. 122.	-0.825 -0.925	-1.126 -1.140	+0.35 +0.29			
5500 5500 5500 5500 5500 5500	1.5 2.0 3.0 4.0 6.0 8.0	-1.992 -1.867 -1.691 -1.566 -1.390 -1.265 -1.168	197. 135. 83.9 61.6 41.3 31.8 26.3	113. 91.9 67.7 54.6 39.2 31.0 25.8	-0.870 -0.971 -1.090 -1.162 -1.247 -1.296 -1.332	-1.112 -1.139 -1.183 -1.214 -1.270 -1.310 -1.340	-0.25 -0.33 +0.02 +0.16 +0.12 +0.07 +0.04			
6000 6000 6000 6000 6000 6000	1.5 2.0 3.0 4.0 6.0 8.0	-1.992 -1.792 -1.615 -1.490 -1.314 -1.189 -1.093	133. 92.0 58.2 43.3 29.6 23.0 19.1	90.0 71.6 49.7 39.2 28.3 22.5 18.8	-0.927 -1.024 -1.136 -1.201 -1.279 -1.326 -1.358	-1.100 -1.133 -1.203 -1.244 -1.299 -1.336 -1.365	-0.73 -0.92 -0.69 -0.35 -0.15 -0.08			
6800 6900 7000 7100	1.0 1.0 1.0	-0.984 -1.971 -1.959 -1.946	131. 121. 113. 105.	83.8 81.6 79.6 77.5	-0.858 -0.874 -0.885 -0.898	-1.052 -1.045 -1.037 -1.030	+0.14 +0.00 -0.01 -0.10			
5800 5900 6000 6100	1.25 1.25 1.25 1.25	-2.025 -2.010 -1.996 -1.981	200. 185. 171. 158.	105. 101. 97.7 94.6	-0.833 -0.843 -0.857 -0.870	-1.113 -1.106 -1.100 -1.093	+0.13 +0.04 -0.00 -0.07			

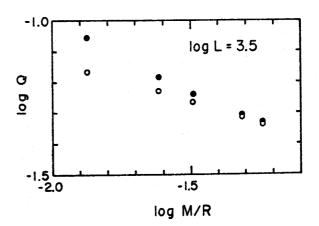


Fig. la. The pulsation constants versus the mass-radius ratios for $\log L = 3.5$. The open circles indicate the pulsation constants for linear adiabatic pulsations, and the filled circles express those for linear nonadiabatic pulsations. Only pulsationally unstable models are plotted.

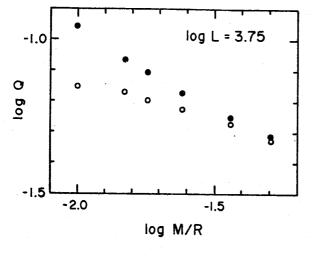


Fig. 1b. The pulsation constants versus the mass-radius ratios for $\log L = 3.75$. Notation is the same as in Fig. 1a.

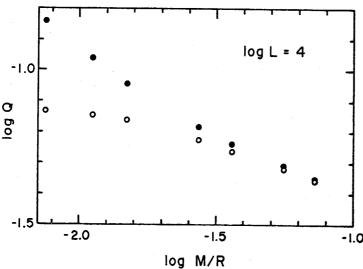


Fig. 1c. The pulsation constants versus the mass-radius ratios for $\log L = 4.0$. Notation is the same as in Fig. 1a.

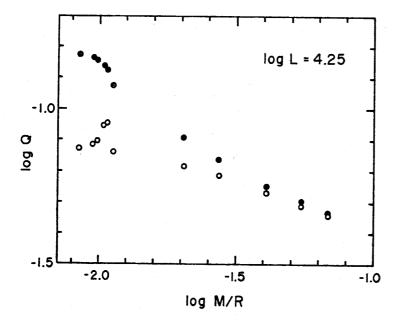


Fig. ld. The pulsation
 constants versus the
 mass-radius ratios for
 log L = 4.25. Notation
 is the same as in Fig.
 la.

seems to be a dependency of the constants on the effective temperature as well as on the mass-radius ratio.

Roughly speaking, in the range of the luminosity studied here, the differences between the adiabatic and nonadiabatic pulsation constants are -0.025 to -0.044 in dex at $\log(M/R) = -1.5$. They, however, amount to -0.20 to -0.24 dex at $\log(M/R) = -2.0$.

In summarizing theoretical linear periods of radial pulsations of stars, we note the following remarks:

- a) Periods of the stars whose M/R are large enough shouldn't be calculated by simple envelope models. The amplitudes are still large at the bottom of chemically homogeneous envelopes. Detailed full stellar models are required to determine their theoretical periods. This effect appears near the main-sequence, where the radial pulsation is usually stable.
- b) On the other hand, periods of the stars whose M/R are sufficiently small, the strange modes $^{11,12)}$ are appeared. Consequently the original Castor code for linear periods loses its adequacy. We choose the ordinary modes by careful examinations of the solutions found in the calculations.

§3. Periods Found in Hydrodynamic Models

Hydrodynamic models of yellow supergiants are investigated intensively by Fadeyev^{6,7)} in connection with the pulsation of FG Sge. It is noted that developed pulsations of these models show serious irregularities, and sometimes associated with strong shock waves. It seems that this is a characteristic of the pulsation of less-massive yellow supergiants, in contrast with that in classical cepheids and RR Lyrae variables which have definite limit cycles, in general.

As mentioned previously, non-linear pulsation periods found by Fadeyev are considerably smaller than those of the linear nonadiabatic approximation $^{13)}$.

Takeuti, Nakata and Aikawa¹⁴⁾ performed non-linear simulation for a model which is the same as a model in Fedeyev and Tutukov¹⁵⁾ focusing the problem on large reduction of the pulsation period in non-linear pulsation of supergiants. They found that strong shock waves were generated recurrently in every several periods, and the pulsation period of the model were reduced abruptly after the formation of extended atmospheres rendered by these shock waves. They suggested that the density inversion appeared even in static models of less-massive supergiants¹⁶⁾ were enhanced by the large amplitude pulsations with strong shock waves, and then the layer associated with the density inversion works as an effective reflection layer of acoustic waves. In general, the layer appears just below the photosphere of less-massive supergiant models, and thus when the layer works as a reflection layer, the total path length of acoustic waves between the center and the outer layer of the models are considerably reduced. It makes the pulsation period shorter^{17,18)}.

§4. Evolutionary Status of Bright Yellow Long-period Semi-variable Stars

There is a group of stars, named as UU Her type stars 19 . They are F-spectral type supergiants and have been confirmed to show light, color and radial-velocity variability of small amplitude with long periods (P > 40 days). The characteristics of the variability are the absence of strict periodicity, and in some stars there have been sudden epochs of quiescence (89 Her, HD161796) and changes of period (HD161796, UU Her). Arellano Ferro 20 , Fernie 21 , and Eggen 22) summarized the up-dated data of variability.

In spite of the Population I characteristics of spectroscopic observations, the evolutionary status has been unresolved for UU Her variables, because they are located in high Galactic latitude. HDl61796 was once suggested as a massive supergiant evolved off the main sequence 3). Aikawa 4) however, pointed out that the model found by Fernie is pulsationally stable, and only less-massive luminous supergiants may be pulsationally unstable at the observed effective temperature. In this case, the observed two periods should be caused temporarily by irregularities in less-massive supergiants. In fact, Fernie 21) reported the periods for HDl61796 in 1984, which differ a bit from the two previously found periods.

Bond, Carney and Grauer²³⁾ found a similar pulsational behaviour with the UU Her type stars in HD46703. It is a metal-deficient F-type, and unquestionably a halo star, as indicated by its low metallicity, high space velocity and large distance from the Galactic plane.

Recently, Parthasarathy and Pottasch²⁴⁾ found strong far infrared excesses in the far infrared IRAS measurements of HD161796. The excesses seem to be due to a large amount of dust around it, and the total shell masses are estimated between 0.3 and 1 solar masses. This evidence strongly suggests that the star is in phase of evolution after strong mass loss.

It is known that the a model having an inert carbon core with a very thin hydrogen envelope will be built for less massive stars. The model evolves horizontally towards higher temperature in the HR diagram increasing the core $ext{mass}^{24}$. The model, however, does not reach high luminosity stage like as L is more luminous than 10000 solar luminosities. The less massive model of high luminosity stars may be built under the assumption that the stars have been evolved from stars of which have their masses between 3 and 8 solar masses at the main sequence stage. A degenerate carbon core is formed during the helium shell burning of such a star. If the star lost most of its envelope, the carbon core will remain in the degenerate state, and only the double shell burning will supply the luminosity. The evolution is complicated by the occurrence of helium shell flashes. The detailed calculations have been performed by several authors $^{26)}$, and were discussed in relation to the evolutionary status of FG Sge $^{27)}$ and RV Tau variables 28). Fadeyev and Tutukov 15) and Fadeyev 29) indicated that the properties of FG Sqe is well explained by a stellar model with the doubleshell source surrounding the degenerate core.

These facts lead us to infer that some and perhaps all of the UU Her type stars are less-massive supergiants with irregular pulsations. The advantages of the less-massive models are that they may be pulsationally unstable, because the blue edge of the instability strip are drastically shifter towards higher effective temperature with increasing the luminosity for less-massive supergiants ^{12,13,30}. Zalewski ³¹⁾ also showed similar pulsational properties of less massive models.

Moreover, the characteristics of the observed pulsation in the UU Her type stars, in particular, the irregularities can be related with those in hydrodynamic models of less-massive supergiants, while we don't know precise physical mechanisms and detailed patterns of the irregularities in the models.

§5. Discussion

In the previous section, the evolved stars like as parent stars of planetary nebulae are proposed as the most satisfactory candidates of the bright yellow long-period, semi-variable stars. We shall discuss the masses of the UU Her stars on which massive models were suggested by Sasselov 19). The absolute magnitudes of 89 Her and HD161796 derived from the spectroscopic data are -7.1 and -6.4, respectively 32). By using these magnitudes, and the pulsation periods, the masses were estimated as 13-24 solar masses for 89 Her 33) and 14-24 solar masses for HD161796 8). The evolutionary status of these stars thus is assumed as the same of non-variable supergiants like as α Per. According to the lessmassive models of UU Her stars, we propose the non-variable stars to be massive. We thus believe that the UU Her stars are no cepheids, and absence of long period cepheids in the Galaxy is notable, as indicated by van Genderen 34).

Using a revised magnitude of HD161706, Burki, Mayor and Rufener questioned the runaway stars hypothesis proposed by Searle, Sargent and Jugaku to resolve the confliction between the location of high Galactic latitude and spectroscopic properties of population I for the UU Her stars. On the other hand, Fernie and Garrison and Aikawa estimated the absolute magnitude of HD161796 as -5.2 to -4.8, under the assumption of the less-massive models. In this case, the height above the Galactic plane is reduced to about 1 kpc. Furthermore, the life time at the main sequence stage of 3-5 solar mass stars which are original for less-massive models is longer by a factor of 5, compared with that of the massive models. Taking these into consideration, we conclude that the assumption of less-massive models for the UU Her stars is in favor of the funaway stars hypothesis.

We shall discuss possible mechanisms of the irregularities of the pulsation in less-massive supergiant models. As shown by Aikawa 2 , decreasing $\log(M/R)$ of the models, the pulsation constant of the fundamental radial mode approaches to the critical one. The latter is a pulsation constant below which the pulsations become progressive waves instead of standing waves. Even though the pulsation constants are above the critical ones, kinetic energy of the pulsation

motions can be transferred effectively to the atmospheres of less-massive models, because effective reflection of acoustic waves must occur near the photosphere. This is another contrast between classical cepheids and less-massive models.

We thus expect that the atmospheric motions in less-massive supergiants is complicated, even though the amplitude of the body pulsation is small. Probably, there are two possible mechanisms of the irregular motion. One is the generation of chaos in the atmospheric motion ³⁷⁾. The other is superposition of short wave motions on regular pulsations in the atmosphere.

We must perform hydrodynamic simulations of less-massive models near their blue edge to find detailed patterns of irregular motions in the atmospheres and to decide the physical mechanisms.

§6. Conclusions

The existence of nonadiabatic effect on the linear periods of radial pulsation, which was first published by Aikawa²⁾, is found generally for low surface-gravity stars.

Theoretically, the nonadiabatic effect works in two ways: first the superficial extended isothermal atmospheres make the periods longer, and the coupling of the acoustic waves and the radiative diffusion makes the period shorter. In the low surface-gravity models the latter is found as the dominant.

In numerical examinations, the nonadiabatic period decreases to a second or a third of the adiabatic period for some extremely low M/R models. Sometimes the disagreement between the periods derived from hydrodynamic models and the linear adiabatic periods are solved by the nonadiabatic effect on the periods.

Pulsationally unstable or neutral models of F sugergiant stars are only possible for the less massive models. HD161796 and ϵ Aur are the case. The yellow bright non-cepheid variables will be less massive stars with the double-shell source and degenerate core after a large amount of mass loss.

The aperiodic nature of bright yellow stars is still an enigma. The transfer of the oscillation energy in several pulsation modes is attractive hypothesis but the detail has not been studied. The turbulence or heat generated by the large amplitude shock waves is also a favourite candidate to change the structure of stellar atmospheres and envelopes. The interference between the occurrence of strong shock and the change of stratification of envelopes should be investigated more extensively.

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