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(Received April 22, 1985)

The vertical structure of steady accretion discs surrounding a neutron star is investigated with the effect of X-ray irradiation from the central part. Incident X-ray and thermal radiation are treated in two-colour Eddington's approximation. The structure of discs is modified by the X-ray irradiation, but the thickness is not increased remarkably.

Keywords: Accretion disc, X-ray irradiation, Two-colour Eddington's approximation.

§1. Introduction

The problem of the radiation from the surface of accretion discs is important to study the close binary system including the neutron star. The synthesized spectrum of the radiation from accretion discs was once calculated by Lyutyi and Sunyaev[1], under the consideration of the X-ray irradiation from the central body onto the disc surface. They demonstrated that the thickness of discs is not changed by the X-ray irradiation for the simplified model. Recently Pacharintanakul and Katz[2] have calculated the synthesized spectrum of accretion discs which radiate the light as the black body, taking the reflection effect into account. They also assumed the form of discs to be unchanged by the X-ray irradiation for simplicity.

The possibility that the form of accretion disc is modified by the X-ray irradiation from the central source was studied by Shakura and Suyaev[3] and was mentioned by Meyer and Meyer-Hofmeister[4,5]. To confirm the conclusion of Lyuti and Sunyaev[1], we shall study the structure of steady discs irradiated by X-ray from the central neutron star by using equations similar to those of Meyer and Meyer-Hofmeister[4,5]. We shall show that the structure of discs is modified but the thickness does not increase remarkably from that without the X-ray irradiation. The synthesized spectrum of Pacharintanakul and Katz[2] based on the unmodified disc, which has been used by Pedersen et al.[6] and Matsuoka et al.[7] for analysing Galactic X-ray sources, is confirmed more evidently in the present study. In next section, we shall study first the radiation field in the surface layers of accretion discs with the X-ray irradiation into consideration, for deriving the expression of the temperature distribution.

§2. Equations for Atmospheres Irradiated by X-ray

We construct equations for the radiation in two-colour approximation: the TH-component of radiation expresses the thermal radiation from the matter in the atmosphere and the X-component does incident X-ray and its scattered photons. We use here the semi-isotropic approximation like as the Eddington approximation for describing the radiation field. We choose the s_{TH}-axis in the direction of the maximum net flux of thermal radiation and the semi-infinite plane-parallel atmosphere perpendicular to the s_{TH}-axis. Along the s_{TH}-axis, the mean intensity $\bar{I}_{TH}$, the net flux F_{TH} and the K-integral K_{TH} are expressed in the following approximation.

$$\frac{dF_{TH}}{ds_{TH}} = 4\pi\kappa_{TH}\rho(-\bar{I}_{TH} + B) , \tag{1}$$

$$\frac{dK_{TH}}{ds_{TH}} = -\frac{\kappa_{TH}\rho F_{TH}}{4\pi} \tag{2}$$

where the Planck function $B = \sigma T^4$, κ_{TH} is the mean opacity for thermal radiation, and s_{TH} expresses the distance along the s_{TH}-axis.

Consider the s_X-axis in the direction to the disc centre along which X-ray is irradiated into the disc-surface. Putting the intensity of inward X-ray as I_X, we have

$$\frac{dI_X}{ds_X} = \kappa_X \rho I_X , \tag{3}$$

where κ_X is the mean opacity for X-ray. We assume that the intensity of incident X-ray is strong in the direction between θ_1 and θ_2 and scattered X-ray is nearly isotropic and weak. Geometrical relationship among θ_1 and θ_2 and other quantities is illustrated in Fig. 1, where z-axis indicates the direction perpendicular to the equatorial plane of the disc. The net flux of X-ray along the s_X-axis, F_X, is expressed as follows:

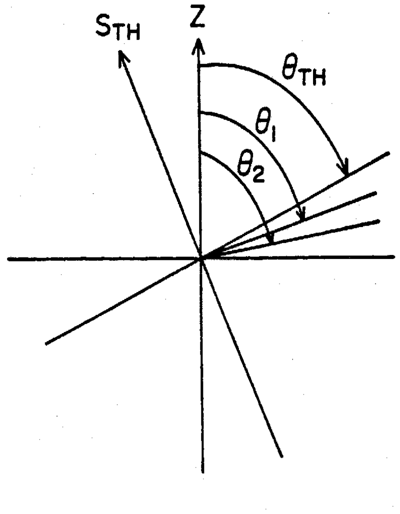

Fig. 1. Geometrical relationship among θ_{TH}, θ_1 and θ_2.

$$F_X = \pi\mu_X(\mu_1 - \mu_2)I_X , \tag{4}$$

Where μ_1, μ_2 and μ_X are in the following.

$$\mu_1 = \cos(\frac{\pi}{2} - \theta_{TH} + \theta_1) = -\sin(\theta_1 - \theta_{TH}) , \tag{5}$$

$$\mu_2 = \cos(\frac{\pi}{2} - \theta_{TH} + \theta_2) = -\sin(\theta_2 - \theta_{TH}) , \tag{6}$$

$$\mu_X = (\mu_1 + \mu_2)/2 , \tag{7}$$

$\theta_X = \cos^{-1}\mu_X - \pi/2 + \theta_{TH}$ is nearly equal to $(\theta_1 - \theta_2)/2$, which is usually negative, and the relationship among s_{TH}, s_X and z is illustrated in Fig. 2.

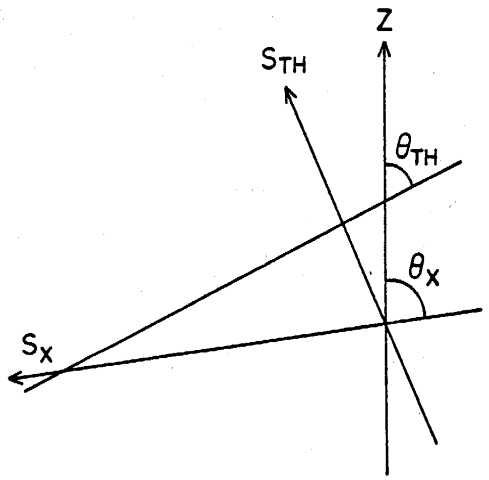

Fig. 2. Geometrical relationship among s_{TH}, s_X and z.

The negative value of F corresponds to the fact that the source of X-ray situates at $z = 0$ and the disc-surface receives the X-ray from lower side. Ignoring the absorption of scattered X-ray, we assumed that the resultant true absorption in multiple scattering is approximated by a considerable true absorption at the first encounter between the matter and the incident X-ray. In the case we have

$$I_X = I_X(0)\exp(-\tau_X) \tag{8}$$

and

$$F_X = F_X(0)\exp(-\tau_X) , \tag{9}$$

where

$$d\tau_X = -\kappa_X \rho ds_X . \tag{10}$$

Suppose the contribution of X-ray on the thermal flux as follows:

$$\frac{dF_{TH}}{ds_{TH}} = -(1 - \tilde{\omega}_X)\frac{dF_X}{ds_{TH}}, \tag{11}$$

where $\tilde{\omega}_X$ is the albedo of disc-surface for X-ray. By using eqs. (9) and (10), we have

$$\frac{dF_X}{ds_{TH}} = \frac{ds_X}{ds_{TH}} \cdot \frac{dF_X}{ds_X}$$

$$= - \frac{ds_X}{ds_{TH}} \kappa_X \rho \frac{d}{d\tau_X}[F_X(0)\exp(-\tau_X)]$$

$$= \frac{ds_X}{ds_{TH}} \kappa_X \rho F_X(0)\exp(-\tau_X) \ . \tag{12}$$

Then we have

$$4\pi\kappa_{TH}\rho(-\bar{I}_{TH} + B) = -\kappa_X\rho(1-\tilde{\omega}_X)F_X(0)\frac{ds_X}{ds_{TH}}\exp(-\tau_X) \ . \tag{13}$$

This means that atmospheres irradiated by X-ray is not in the radiative equilibrium for the thermal flux F_{TH}.

From the geometrical consideration, we have the following equations.

$$ds_{TH} = dz \sin\theta_{TH} \ , \tag{14}$$

$$ds_X = ds_{TH}[\sin(\theta_X - \theta_{TH})]^{-1}. \tag{15}$$

Putting

$$d\tau_{TH} = -\kappa_{TH}\rho ds_{TH} \ , \tag{16}$$

we have

$$\frac{dF_{TH}}{d\tau_{TH}} = (1-\tilde{\omega}_X)F_X(0)\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\exp(-\tau_X) \ , \tag{17}$$

$$\frac{d\bar{I}_{TH}}{d\tau_{TH}} = \frac{3F_{TH}}{4\pi}. \tag{18}$$

Eq. (17) is derived from eq. (13). The boundary condition for the case is that

$$I^-_{TH} = 0 \quad \text{at} \quad \tau_{TH} = 0 \ , \tag{19}$$

or

$$\bar{I}_{TH}(0) = \frac{F_{TH}(0)}{2\pi} \ , \tag{20}$$

where I^-_{TH} is the inward intensity of thermal radiation. I^+_{TH}, used later, is that of outward one.

Put $\kappa_X/\kappa_{TH} = \text{const.}$ for simplicity. In the case, eq. (17) is easily integrated and we have

$$F_{TH} = F_{TH}(\infty) - (1-\tilde{\omega}_X)F_X(0)\exp(-\tau_X) \ , \tag{21}$$

and

$$\tau_X = - \frac{\kappa_X}{\kappa_{TH}}\frac{ds_X}{ds_{TH}}\tau_{TH} \ . \tag{22}$$

$F_{TH}(\infty)$ is the value of F_{TH} at the infinitely great τ_X. Eq. (18) is integrated by using F_{TH}. Then we have

$$\bar{I}_{TH} = \frac{F_{TH}(\infty)}{2\pi}\left[1 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_{TH}(\infty)}\left\{1 + \frac{3}{2}\left(\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\right)^{-1}\right\} + \frac{3}{2}\tau_{TH} + \frac{(1-\tilde{\omega}_X)F_X(0)}{F_{TH}(\infty)}\frac{3}{2}\left(\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\right)^{-1}\exp(-\tau_X)\right], \tag{23}$$

and by using eq. (13)

$$B = \frac{F_{TH}(\infty)}{2\pi}\left[1 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_{TH}(\infty)}\left\{1 + \frac{3}{2}\left(\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\right)^{-1}\right\} + \frac{3}{2}\tau_{TH} + \frac{(1-\tilde{\omega}_X)F_X(0)}{F_{TH}(\infty)}\left\{\frac{3}{2}\left(\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\right)^{-1} - \frac{1}{2}\frac{ds_X}{ds_{TH}}\frac{\kappa_X}{\kappa_{TH}}\right\}\exp(-\tau_X)\right]. \tag{24}$$

The effect of X-ray irradiation is clearly demonstrated in above equations. At the outermost layers of accretion discs the source function for the thermal radiation B becomes very great since X-ray is absorbed in very thin layers above the photosphere, where the condition $ds_X/ds_{TH} \gg 1$ is satisfield. When X-ray comes perpendicularly to the photosphere or $ds_X/ds_{TH} = 1$, the increase of temperature is still found in sufficiently deep layers. The value of τ_X/τ_{TH} is also effective for determining the temperature distribution.

The radiation field of the case that $ds_X/ds_{TH} \gg 1$ and $\tau_X/\tau_{TH} = 1$ are described in the following.

$$F_{TH} = F_v - (1-\tilde{\omega}_X)F_X(0)\exp(-\tau_X), \tag{25}$$

$$\bar{I}_{TH} = \frac{F_v}{2\pi}\left[1 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH}\right], \tag{26}$$

where F_v expresses the radiative flux originated by the viscous stress. So I^+_{TH} and I^-_{TH} are

$$I^+_{TH} = \frac{F_v}{2\pi}\left\{2 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH} - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v}\exp(-\tau_X)\right\}, \tag{27}$$

$$I^-_{TH} = \frac{F_v}{2\pi}\left\{- \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH} + \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v}\exp(-\tau_X)\right\}. \tag{28}$$

when $\tau_{TH} \gg 1$, we may expect $\exp(-\tau_X) \to 0$. Then we have

$$F_{TH} \to F_v, \tag{29}$$

$$I^+_{TH} \to \frac{F_v}{2\pi}\left\{2 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH}\right\}, \tag{30}$$

$$I^-_{TH} \to \frac{F_v}{2\pi}\left\{- \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH}\right\}, \tag{31}$$

$$\bar{I}_{TH} \to \frac{F_v}{2\pi}\left\{1 - \frac{(1-\tilde{\omega}_X)F_X(0)}{F_v} + \frac{3}{2}\tau_{TH}\right\}. \tag{32}$$

We can see the increase of the mean intensity at the deep layers of accretion disc caused by the X-ray irradiation onto outermost layers. This means the increase of temperature at deep layers. It is also important to point out that the high temperature region is produced at the geometrically shallow layers, as investigated by Basko and Sunyaev[8] and Milgrom and Salpeter[9]. The radiation emitted from these hot layers increases the temperature, and also the total pressure, of inner layers of the disc.

§3. Model Construction

We describe equations for constructing the steady accretion discs in the following.

$$\frac{dP}{dz} = -\Omega^2 \rho z , \tag{33}$$

$$\frac{dF_{TH}}{dz} = \frac{4}{3} \alpha \Omega^2 \rho v_s H_p - (1-\tilde{\omega}_X) F_X(0) \frac{ds_X}{ds_{TH}} \frac{ds_{TH}}{dz} \kappa_X \rho \exp(-\tau_X) , \tag{34}$$

$$\frac{dT}{dz} = \frac{T}{P} \frac{dP}{dz} \nabla , \tag{35}$$

$$\frac{d\tau_{TH}}{dz} = -\kappa_{TH} \rho \frac{ds_{TH}}{dz} , \tag{36}$$

$$\frac{d\tau_X}{dz} = -\kappa_X \rho \frac{ds_X}{ds_{TH}} \frac{ds_{TH}}{dz} , \tag{37}$$

$$\frac{dm}{dz} = \rho , \tag{38}$$

where R is the radius from the disc-centre, P, the total pressure, $\Omega^2 = GMR^{-3}$, the square of the angular velocity, v_s, the velocity of sound, H_p, the pressure scale height, α, the constant for viscous heat generation, ∇, the temperature gradient, and m, the mass of the column of unit cross section measured from the equatorial plane of disc. The temperature gradient ∇ is ∇_R for the radiative flux, which is described as follows:

$$\nabla_R = \frac{3}{16\sigma} \frac{\kappa_{TH} P}{\Omega^2 z T^4} \frac{ds_{TH}}{dz} [F_v - (1-\tilde{\omega}_X) F_X(0) \left\{ 1 - \frac{1}{3} \left(\frac{ds_X}{ds_{TH}} \frac{\kappa_X}{\kappa_{TH}} \right)^2 \right\} \exp(-\tau_X)] . \tag{39}$$

F_v is the flux generated by the viscous stress. In the convection zone, ∇_{conv} is given by the usual mixing-length theory.

For the starting values of the system of equations, we put

$$P_G = \left[\frac{GMz}{\kappa_{TH} R^2} - \frac{1}{c} \left\{ (F_v - F_X(0) \frac{\kappa_X}{\kappa_{TH}}) \sin \theta_{TH} - F_X(0) \frac{\kappa_X}{\kappa_{TH}} \cos \theta_X \right\} \right] \tau^* , \tag{40}$$

where τ^* is small value, taking the anisotropy of radiation into consideration. The first term of the R.H.S. of eq. (34) is ignored at the top of photosphere. Then we have

$$F_{TH}(\tau^*) = F_v - (1-\tilde{\omega}_X)F_X(0)\exp(-\tau_X^*) , \tag{41}$$

at τ^*. The vertical structure is not affected remarkably by the switching of α between the radiative zone and convective zone, as appeared in the works of Smak[10] and Meyer and Meyer-Hofmeister[4], and moreover much machine time is required when we use the switching. So we put $\alpha = 1.0$ throughout the present work. ds_{TH}/dz is assumed to be 1.0 and the integration is started inward from the point $z = z^*$ with assumed $\theta_X - \theta_{TH}$. z^* is chosen for satisfying the condition $F_v = 0$ at $z = 0$. The opacity κ_{TH} is calculated by Stellingwerf's formula and we put $\kappa_{TH} = 0.2(1 + X)$ when the formula gives the opacity under this value in the small density layers, taking the Thomson scattering into account. κ_X will be discussed later. Chemical composition (X, Y, Z) = (0.7, 0.28, 0.02) is chosen in the usual expressions.

The model-disc is constructed for the mass of the central object of 1.0 $M_\odot$ and the radius $R_0 = 10^6$ cm. The accretion rate $\dot{M} = 1.6 \times 10^{-9}$ $M_\odot$ yr^{-1} ($= 10^{17}$ gr sec^{-1}) is chosen with the critical accretion rate, 2.2×10^{18} gr sec^{-1}. The luminosity of the central star L_X is given by the following equation,

$$L_X = GM\dot{M}/(2R_0) , \tag{42}$$

and inserting above parameters into this equation, we have $L_X = 6.65 \times 10^{36}$ erg sec^{-1}. Because the effective temperature of the central source is 9.83×10^6 K for these parameters, the wavelength of the maximum intensity is 2.95×10^{-8} cm or the frequency 1.02×10^{18} Hz. The observational frequency of X-ray from Sco X-1 is 1.2×10^{18} or 2.4×10^{18} Hz. The effective temperature obtained above is calculated on the assumption that X-ray is emitted uniformly from the whole surface of the central compact object. So the observed frequency seems not so strange when X-ray from the central source is emitted from a restricted area near the equator. We assume the effective wavelength of X-ray from the central source to be 2×10^{-8} cm in the following calculation.

The effective opacity for X-ray is calculated by using the following equation. The photo-electric effect of OVIII is assumed and the equation given in Basko and Sunyaev[8] is used. The Thomson scattering is also assumed for the electron scattering.

$$\kappa_X = \frac{X}{m_P}\frac{N_{OVIII}}{N_O}\sigma_0 + 0.2(1 + X) , \tag{43}$$

where X is the mass fraction of hydrogen, m_P, the mass of proton, N_{OVIII}, the number of oxygen nuclei which have the K-shell electron, N_O, the total number of oxygen nuclei, σ_0, the cross section of OVIII ion (cm^2 per hydrogen nucleus), and

$$\sigma_0 = 2.5 \times 10^{-22}\ (2.42 \times 10^{17}/\nu)^{2.4} . \tag{44}$$

ν is the effective frequency of X-ray. The fractional abundance of OVIII to the total number of oxygen nuclei is calculated by using the following form.

$$\frac{N_{OVIII}}{N_O} = a_0 N_e N_O / (a_0 N_e N_O + N_H \sigma_0 \cdot \frac{4\pi \bar{I}_X}{h\nu}) , \tag{45}$$

where a_0 is the recombination rate, N_e, the number density of free electron, N_H, the number density of hydrogen nuclei, respectively. The X-ray irradiation produces the OIX zone, where only the Thomson scattering is effective for X-ray, at the small optical depth layers of disc-atmospheres, as pointed out by Basko and Sunyaev[8] on the close binary system and also by Milgrom and Salpeter[9]. The appearance of OIX zone allows X-ray to penetrate into deeper layers and increases the photospheric pressure. So, the accretion disc becomes thinner than those without the X-ray irradiation, as written by Lyutyi and Sunyaev[1].

§4. Results and Discussions

The steady discs without the X-ray irradiation, constructed in previous section, are tabulated in Table 1. The height of the layer at $\tau_{TH} = 2/3$ from the equatorial plane, z_1, is expressed in the form:

$$z_1 = AR^{1.12} , \tag{46}$$

where A is a constant. T_{eff0}, the effective temperature in the case without X-ray irradiation, is given in the following relation because we assume the Keplerian discs.

Table 1. The accretion disc without X-ray irradiation

log R	z_1/R	log T_{e0}	log m_C
7.5	0.0373	5.966	4.037
8.0	0.0414	5.601	3.715
8.5	0.0481	5.231	3.356
9.0	0.0559	4.859	2.976
9.5	0.0639	4.486	2.574
10.0	0.0711	4.111	2.223

R : in cm.

m_C : in gr cm^{-2}.

$$T_{eff0}^4 = \frac{3}{8\pi\sigma} \frac{GM\dot{M}}{R^3} \left\{ 1 - \left(\frac{R_0}{R}\right)^{1/2} \right\} . \tag{47}$$

m_C, the surface density of discs, is also tabulated. Results presented here agree well with those of Meyer and Meyer-Hofmeister[4].

For considering the effect of the X-ray irradiation on the structure of discs, the model-discs are calculated with various $\theta_X - \theta_{TH}$. Results for $R = 10^8$ and 10^{10} cm are tabulated in Table 2. The relative thickness of disc z_1/R

Table 2. Changes of the thickness and the gradient of surface with the X-ray irradiation

Radius	$\theta_X - \theta_{TH}$	z_1/R	dz_1/dR	$F_{TH}(0)/F_v(0)$
10^8 cm	0	0.0414	0.0414	0.0
	0.001	0.0415	0.0425	$1.85 \cdot 10^{-2}$
	0.01	0.0413	0.0513	$1.85 \cdot 10^{-1}$
	0.1	0.0410	0.1410	1.81
10^{10} cm	0	0.0711	0.0711	0.0
	0.001	0.0715	0.0725	1.67
	0.01	0.0702	0.0801	$1.67 \cdot 10$
	0.1	0.0682	0.1682	$1.63 \cdot 10^2$

increases little bit for very small $\theta_X - \theta_{TH}$ and decreases slightly with the increase of $\theta_X - \theta_{TH}$. This coincides well with that of Lyutyi and Sunyaev[1), and Meyer and Meyer-Hofmeister[5)] remarked that the results (described in Meyer and Meyer-Hofmeister[4)]) which was not identical with ours was not correct. The hot plateau appears at the top of atmosphere, as the result of absorption of X-ray. The total pressure is increases in this plateau. The temperature and pressure at the equatorial plane of disc is hardly changed, although the increase in the pressure is important to evaluate the thickness of disc. Because we assume the dissipation of kinetic energy to be proportional to the total pressure, the increase of photospheric pressure, caused by the hot plateau, makes the thickness of discs thinner to keep the total dissipation constant.

From the geometrical consideration, we have

$$\theta_X - \theta_{TH} = \frac{dz_1}{dR} - \frac{z_1}{R}. \tag{48}$$

By using eq. (46), we obtain

$$\theta_X - \theta_{TH} = 0.12\left(\frac{z_1}{R}\right), \tag{49}$$

or $\theta_X - \theta_{TH}$ is 0.004-0.009. The decrease of thickness corresponding with these values is less than 1 % compared with the case $\theta_X - \theta_{TH} = 0$. The assumption used by Pacharintanakul and Katz[2)], that the form of discs is kept unchanged under the interactive radiation exchange, is confirmed in the present calculation. We have calculated model-discs by using the Thomson scattering only for X-ray opacity, and found that the thinning is slightly remarkable for small $\theta_X - \theta_{TH}$ and a little bit of thickening for large value of $\theta_X - \theta_{TH}$.

We have restricted the study of the steady discs with the constant X-ray irradiation, although the irradiation changes usually its intensity with the time. Probably the pulsation of X-ray from the central source produces the change of the photospheric pressure and the accoustic wave propagating inward. When the pulsation period of X-ray irradiation coincides to the period of vertical oscillation at the overstable annulus, the amplitude of vertical pulsation of disc would increase and would make the thickness of discs a considerably great. The behaviour of the annulus having the surface temperature of 10000 K seems interesting the study of close binary system including the neutron star.

§5. Conclusion

In the present study, we have investigated the vertical structure of accretion discs with the effect of X-ray irradiation from the central neutron star. High-temperature layers were produced at the upper layers of atmospheres and the temperature and pressure near the surface is increased but the physical quantities at the equatorial plane are scarcely changed. The thickness of discs has been modified only less than 1 %. The thickning of discs surrounding the neutron star should be studied in considering the dynamical effect of X-ray pulsation.

The author wishes to thank for the support in part by the Grant-in-Aid for Cooperative Research of the Ministry of Education, Science and Culture under Grant No. 58340013.

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