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On the Models of Early Main Sequence Evolution of Moderately Massive Stars with growing Inhomogeneous Convection Zone

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(Received November 29, 1983)

The models similar to those proposed by Sakashita and Hayashi to represent the main sequence evolution of massive stars have been studied for the evolution of moderately massive star. Each of the models consists of the inhomogeneous convection zone developing between the outer radiative envelope of initial chemical composition and the homogeneous convection core of the diminishing hydrogen content. In actual computation, such an inhomogeneous convection zone is approximated by the treatment of Sakashita and Hayashi's semiconvection zone. The models are examined for two cases of simplified opacity law. Along the evolutionary sequence of such models, the intermediate inhomogeneous convection zone extends steadily inward and chemically homogeneous central core shrinks without changing the outermost boundary of the convection core. Though it has not been made sure exactly yet, the central homogeneous core disappears when the central hydrogen content becomes a critical value which is still appreciably high and depends on the adopted opacity law and initial chemical composition. In general, it may be said that the central hydrogen content at such a critical stage should be smaller for the star of larger mass. Until arriving at the critical stage defined here, the present scheme of evolution does not bring about any appreciable change in stellar luminosity, radius and the internal run of physical quantities such as mass, pressure and density except the temperature which increases in a way of just compensating the increase of the mean molecular weight in the equation of state for pressure.

Some speculations have been briefly made of the model characters of the subsequent evolution.

Keywords: Stellar structure, Stellar evolution, Main sequence, Convection core, Semiconvection zone.

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§1. Introduction

The structure and its evolution of the star with convection core has been computed under the supposition that the complete mixing of the chemical elements in the convection core is a very good approximation. Thus the early evolution of main sequence stars with a convection core has been studied by many authors. Especially for the stars of moderately large masses ranging from $2M_\odot$ to $10M_\odot$ at least, the general character of evolution has been regarded as well established. Namely, since Tayler[1] and Kushwaha[2] had initially shown, the evolutionary behaviour of such moderately massive stars has been characterized by that the convection core decreases in its mass as evolution proceeds. Recently Maeder[3-6] has computed the evolutionary models for moderately massive stars by using non-local expressions of the mixing length formalism to treat the convection in the stellar core. He has shown that, if the mixing length is longer, the more extensive overshooting from the convection core may occur during the main sequence evolution. In his computation, it is assumed that mixing between the usual convection core and the overshot region is so complete that the overshot region is kept chemically homogeneous with the same abundance as that in the full convection core at any instant. Except that the well mixed homogeneous central core is largely extended, however, the mass fraction in such a central core decreases as evolution proceeds and the essential evolutionary characters do not change from those of usual one so far as the star of mass larger than $2M_\odot$ and less than $10M_\odot$ at least is concerned. The idea of instantaneous complete mixing in the convection core and in adjacent overshot region comes from understanding that the mixing is taking place in the mechanical time scale of convection element which is much shorter than that of evolution. So long as one keeps this assumption, however, there appear difficult situations under which the hypothetical semiconvection zone has to be introduced when one considers the stars of masses larger than about $10M_\odot$.

In principle, the sweeping effect of the convection against an inhomogeneity of chemical composition depends on the divergence of material flow and may proceed with different time scale in each layer of a convective region. According to the mixing length theory, this time scale may be expected to be the order of local Kelvin time scale due to convective energy transport and, therefore, much longer than that of the mechanical time scale of the convective element. From this point of view, Suda[7] and Suda and Uchida[8,9] have proposed and developed a method to treat the evolving convection core and investigated the logical scheme to solve the problem. These investigations have been made basing on the early evolution of the massive main sequence stars and led them to the conclusion that the structure with an inhomogeneous outer region inside the convection core, which may be approximated by the Sakashita and Hayashi's first model[10], can be formed at an early epoch of the massive main sequence evolution. This conclusion has been actually confirmed by the test computation[11].

This is expected to be the case also for the early main sequence evolution of the moderately massive star. Namely, also for the moderately massive star, it is expected that the inhomogeneous convection zone similar to Sakashita and Hayashi's semiconvection zone can be formed just inside of the outer boundary of the convection core at a quite early epoch of the main sequence evolution. Therefore, it should be important to attempt to construct the evolutionary models similar to those of Sakashita and Hayashi[10] to represent the structural evolution of the moderately massive star in the main sequence. To get a preliminary information of the problem is a main purpose of the present work.

§2. General Consideration for Constructing the Models

To facilitate the investigation, we use here a classical way by using the standard dimensionless variables to construct the stellar models[12]. Since we are considering the case of the moderately massive star, we neglect the small effects of the radiation pressure in equation of state. We further assume that the opacity κ and the energy generation rate ε can be simply approximated by power laws of density and temperature

$$\kappa = \kappa_0 \rho^{\alpha} T^{-\beta} , \tag{1}$$

$$\varepsilon = \varepsilon_0 \rho^{\delta} T^{\nu} . \tag{2}$$

Hereinafter, except special specifications, usual definitions and notations are used.

Each of the models which are expected here to represent the internal structure for the early period of main sequence evolution consists of the radiative envelope and the convection core which is divided into the inner homogeneous core and the outer inhomogeneous zone. In some quantities which will appear in following investigation, attached subscript 1 denotes the outer boundary of the convection core. In the same way, subscript 2 indicates the outer boundary of the central homogeneous core. In other words, subscript 1 and 2 are used to denote the outer and inner boundary of the inhomogeneous convection zone, respectively.

Not only in the central homogeneous core but also in the inhomogeneous zone, the basic equation proper to describe that the region is in convective equilibrium is

$$\frac{d\,\ln\rho}{d\,\ln P} \equiv \nabla_\rho = \nabla_{\rho,ad} \equiv \frac{N}{N+1} \quad (N = 3/2 \text{ in the present case}) \tag{3}$$

and the relation

$$P = K\rho^{1+1/N} \quad \text{with } N = 3/2 \tag{4}$$

is applicable in common to both of the central homogeneous core and the inhomogeneous zone. Thus, the whole structure of the convection core must be expressed basically by the Emden solution $\theta(\xi)$ of $N = 3/2$.

For a specified opacity law, a particular solution for the radiative envelope is characterized by the value of C defined as

$$C \equiv \frac{3}{4ac}\left(\frac{k}{HG}\right)^{4+\beta}\left(\frac{1}{4\pi}\right)^{\alpha+2}\frac{\kappa_{os}}{\mu_s^{\beta+4}}\frac{LR^{\beta-3\alpha}}{M^{\beta-\alpha+3}} . \tag{5}$$

Here and hereinafter the subscript s is used to denote the surface values of the attached quantities.

The fitting conditions at the interface between the radiative envelope and the convection core can be expressed as

$$U_{1o} = U_{1i}, \quad V_{1o} = V_{1i} \tag{6}$$

and

$$\nabla_{rad,1o} = \nabla_{rad,1i} = \nabla_{ad} = 1/(N+1) = 0.4 \tag{7}$$

where subscripts o and i are used to denote the just outer and inner side of the interface, respectively. These conditions are completely the same as those being used to construct the corresponding chemically homogeneous initial model. Since, furthermore, the homogeneous envelope solution which can satisfy these fitting conditions at its inner boundary and, then, can contact with Emden solution of $N = 3/2$ in the U,V-plane is the unique one, the models along the sequence under consideration must share the same envelope solution. Thus, it may be thought that the outer boundary of the convection core does not move in both of mass and radius fractions following the sequence of the present models.

In the inhomogeneous convection zone, the temperature gradient ($\nabla = d\,\ell n\,T/d\,\ell n\,P$) and the composition gradient must be so formed that the relations

$$\nabla = \nabla_{ad} + d\,\ell n\,\mu/d\,\ell n\,P \tag{8}$$

$$\nabla_{rad} > \nabla > \nabla_{ad} \tag{9}$$

are satisfied. The first equation itself is identical with the equation (3) in its meaning. Second condition is necessary one in order that the region is convective one. Following Suda and Uchida[8,9,11], however, ∇ is very close to the radiative one ($= \nabla_{rad}$) in such an inhomogeneous convection zone. Therefore we use here the approximation given as

$$\nabla_{rad} = \nabla_{ad} + d\,\ell n\,\mu/d\,\ell n\,P. \tag{10}$$

This is the equation introduced originally by Sakashita and Hayashi[10] to represent the so-called semiconvection zone in their evolutionary models for massive main sequence stars. In constructing their models, Sakashita and Hayashi have completely neglected the effect of nuclear energy generation in their semiconvection zones. This would be good approximation for the case of massive stars as they treated, mainly because of that the original size of the homogeneous convection core is so large that the location of the semiconvection zone is in the sufficiently outer part where the nuclear energy generation is negligible. For the case of the moderately massive star, this should not be the case since the original convection core is not so large when we compare it with the cases of massive stars. Therefore, in this work, we take into account the effect of nuclear energy generation even in the inhomogeneous convection zone. Here, following Schwarzschild[12], we introduce the composition functions defined as

$$\ell = \frac{\mu}{\mu_s}, \quad j = \ell\frac{\kappa_o}{\kappa_{os}}, \quad i = \ell\frac{\varepsilon_o}{\varepsilon_{os}}. \tag{11}$$

Then the run of Schwarzschild's dimensionless variables in the inhomogeneous convection zone is given by the relations

$$\frac{d\,\ln \ell}{d\,\ln \xi} = -(\nabla_{rad} + \nabla_{\rho,ad} - 1)V = -(\nabla_{rad} - 0.4)V, \tag{12}$$

$$\frac{d\,\ln f}{d\,\ln \xi} = \frac{Di\ell p^2 t^{\nu-2} x^3}{f}, \tag{13}$$

$$x = x_1(\xi/\xi_1), \tag{14}$$

$$q = q_1(\xi^2\frac{d\theta}{d\xi})/(\xi^2\frac{d\theta}{d\xi})_{\xi=\xi_1}, \tag{15}$$

$$p = p_1(\frac{\theta}{\theta_1})^{2.5}, \tag{16}$$

$$t = \frac{t_1}{\theta_1}\ell\theta, \tag{17}$$

$$\nabla_{rad} = C\frac{j}{\ell^{1-\alpha}}\frac{fp^{\alpha+1}}{qt^{\alpha+\beta+4}}, \tag{18}$$

together with the conditions

$$\ell = 1, \quad j = 1, \quad i = 1 \quad \text{and} \quad f = 1$$

at the outer boundary of this zone. The parameter D in the right hand side of equation (13) is defined as

$$D = (\frac{HG}{k})^{\nu}(\frac{1}{4\pi})^{\delta}\varepsilon_{os}\mu_s^{\nu}\frac{M^{\delta+\nu+1}}{LR^{\nu+3\delta}} \tag{19}$$

Now, we suppose that a value is specified for ℓ_c and/or X_c which may be regarded as the parameter specifying the evolutionary phase of the model. Let us further suppose that a value is arbitrarily chosen for D. Then the equations from (12) to (18) provide the necessary values for the variables at the inner boundary of the inhomogeneous zone.

Next, with these boundary values, the runs of Schwarzschild's variables in the homogeneous core are given as

$$\ell_c = \ell_2, \quad i_c = i_2, \tag{20}$$

$$p_c = p_2/\theta_2^{2.5}, \tag{21}$$

$$t_c = t_2/\theta_2, \tag{22}$$

$$p = p_c \theta^{2.5} \tag{23}$$

$$t = t_c \theta \tag{24}$$

$$x = \left(\frac{5}{2}\frac{t_c^2}{p_c}\right)^{1/2} \frac{1}{\ell_c} \xi , \tag{25}$$

$$q = \frac{t_c^2 (2.5)^{3/2}}{\ell_c^2 \sqrt{p_c}} \left(-\xi^2 \frac{d\theta}{d\xi}\right), \tag{26}$$

$$f = (5/2)^{3/2} D i_c \ell_c^{\delta-3} p_c^{\delta-1/2} \; t_c^{\nu-\delta+2} \int_0^{\xi} \theta^{1.5\delta+\nu+1.5} \xi^2 d\xi , \tag{27}$$

Let us denote f_{2i} and f_{2o} the values of f evaluated (at the interface between the inhomogeneous zone and the homogeneous core) from outward integration of (27) and inward integration of (13), respectively. Then the condition

$$f_{2o} = f_{2i} \tag{28}$$

must be satisfied. The value of parameter D which appears in equations (13) and (27) must be so searched that this condition is satisfied. Thus, since there must be an unique correspondence between X_c and D of the model, we may regard as that the hydrogen content in the homogeneous central core is the parameter characterizing the evolutionary phase of the model under consideration.

As mentioned above, the characteristic value of the radiative envelope does not change along the sequence of the models as considered here and the outer boundary of the convection core does not move in both of mass and radius fractions. The outer inhomogeneous zone will grow from just inside of such outer boundary of the convection core toward the center as the hydrogen content in the central homogeneous core decreases. Until what phase can we construct this kind of evolutionary models ? This is a main problem to be studied in the present work.

§3. Properties of the Models

In order to see the effects of the different opacity formulae on the sequence of the models under consideration, two cases of the electron scattering opacity ($\alpha = 0.0$, $\beta = 0.0$) and the Kramer's opacity ($\alpha = 1.0$, $\beta = 3.5$) are considered. As regards the energy generation rate, corresponding to the CNO-cycle reaction at the temperature appropriate to the center of the moderately massive star, we adopt $\delta = 1.0$ and $\nu = 16.0$ in the approximate expression (2). Thus, two series of models with decreasing central hydrogen content were constructed. The mathematical characteristics of these models are given in Table 1.

i) Critical situations against the construction of the models

Table 1a). Characteristics of the models (electron scattering opacity)

log C = -3.2981

x_c	0.8	0.7	0.6	0.5	0.4	0.33
x_1			0.2832			
$\log q_1$			-0.5059			
$\log p_1$			1.1977			
$\log t_1$			-0.2991			
f_1			1.0			
$\log U_1$			0.3589			
$\log V_1$			0.3412			
$(n+1)_1$			2.5			
x_2	—	0.1628	0.1110	0.0715	0.0381	0.0118
$\log q_2$	—	-1.1219	-1.5924	-2.1515	-2.9648	-4.4881
$\log p_2$	—	1.5092	1.5894	1.6299	1.6504	1.6577
$\log t_2$	—	-0.1423	-0.0753	-0.0212	0.0286	0.0632
f_2	—	0.8884	0.6746	0.4254	0.1765	0.0120
$\log U_2$	—	0.4405	0.4604	0.4702	0.4752	0.4769
$\log V_2$	—	-0.1590	-0.4953	-0.8794	-1.4274	-2.4449
$(n+1)_{20}$	—	1.4933	1.0892	0.7627	0.4567	0.2859
$\log p_c$	1.6585	1.6585	1.6585	1.6585	1.6585	1.6585
$\log t_c$	-0.1148	-0.0825	-0.0477	-0.0097	0.0318	0.0635
$\log D$	1.6352	1.1970	0.8036	0.4832	0.2638	0.1638

Table 1b). Characteristics of the models (Kramers' opacity)

log C = -5.9887

x_c	0.8	0.7	0.66
x_1		0.1700	
$\log q_1$		-0.8345	
$\log p_1$		1.7346	
$\log t_1$		-0.1514	
f_1		1.0	
$\log U_1$		0.4122	
$\log V_1$		0.0864	
$(n+1)_1$		2.5	
x_2	—	0.0552	0.0165
$\log q_2$	—	-2.2177	-3.7829
$\log p_2$	—	1.9683	1.9930
$\log t_2$	—	-0.0257	-0.0022
f_2	—	0.2830	0.0138
$\log U_2$	—	0.4706	0.4765
$\log V_2$	—	-0.9019	-1.9523
$(n+1)_{20}$	—	1.4355	1.1265
$\log p_c$	1.9954	1.9954	1.9954
$\log t_c$	-0.0471	-0.0148	-0.0012
$\log D$	0.3268	0.0326	-0.0089

In general, for a specified value for D, there is a minimum critical value for X to which the corresponding integration for the inhomogeneous zone can attain. Namely, when we proceed the integration toward the center, X decreases monotonically at first. But the integration arrives at a critical point where the right hand side of equation (12) becomes just equal to zero. And, when we further proceed the integration beyond this critical point, the right hand side of equation (12) becomes positive because of that ∇_{rad} becomes less than ∇_{ad} and, then, X begins to increase. Such behaviours are illustrated in (X, log q) -diagram of Figure 1 for various values of D. As shown there, the smaller is the value of D, the lesser is the attainable minimum critical value of X.

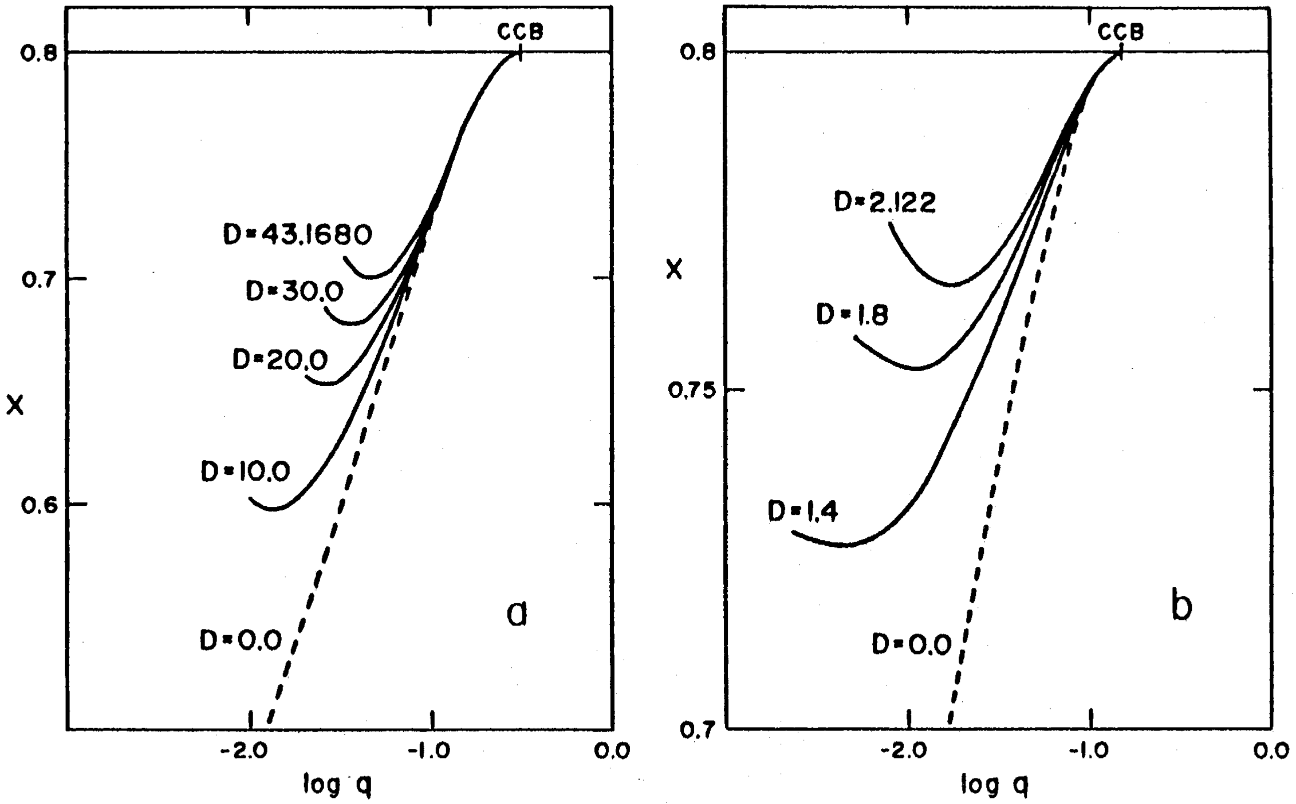


Fig. 1. Profiles of the hydrogen distributions calculated in the inhomogeneous convection zones for various specified values of D; a) the case of electron scattering opacity; b) the case of Kramers' opacity. CCB is the outer boundary of the convection core. Smaller is the value of D, lesser is the attainable minimum value of X.

From the point of view to construct the model with the specified value of X_c, the behaviours as mentioned above may be stated as follows. To attain a specified value of X along the inward integration, there is an upper critical value for D. Hereinafter we term this as D_{crit}. With the value of D larger than this D_{crit}, the corresponding prescribed value of X can not be attained along the integration. So, in order to construct the model with the specified value of X_c, we have to choose the value of D which is smaller than corresponding D_{crit}. Since, as described above, the lesser is the value of X_c, the smaller is the value of corresponding D_{crit}, the range of D adoptable to construct the model becomes narrower as we consider the model of more advanced phase.

When a value is specified for X_c, the final value of D has to be so searched that the condition (28) is satisfied. In Figure 2a, as an example, for $X_c = 0.4$ in the case of the electron scattering, the values of log f on the both sides of the interface between the homogeneous core and the inhomogeneous zone are plotted against the corresponding values of log D. In the case of electron scattering, the model could be constructed until the phase corresponding to $X_c = 0.33$. But, when X_c becomes 0.32 in this case, two curves of f_{20} and f_{2i} can never cross each other as shown in Figure 2b. Thus, there is a limiting phase along the sequence of the models under consideration. In the case of Kramer's opacity, such a change of fitting situation appears for a value of X_c between 0.66 and 0.65. Then the model presented in the last column of Table 1b with $X_c = 0.66$ is very close to the limiting model in this sense.

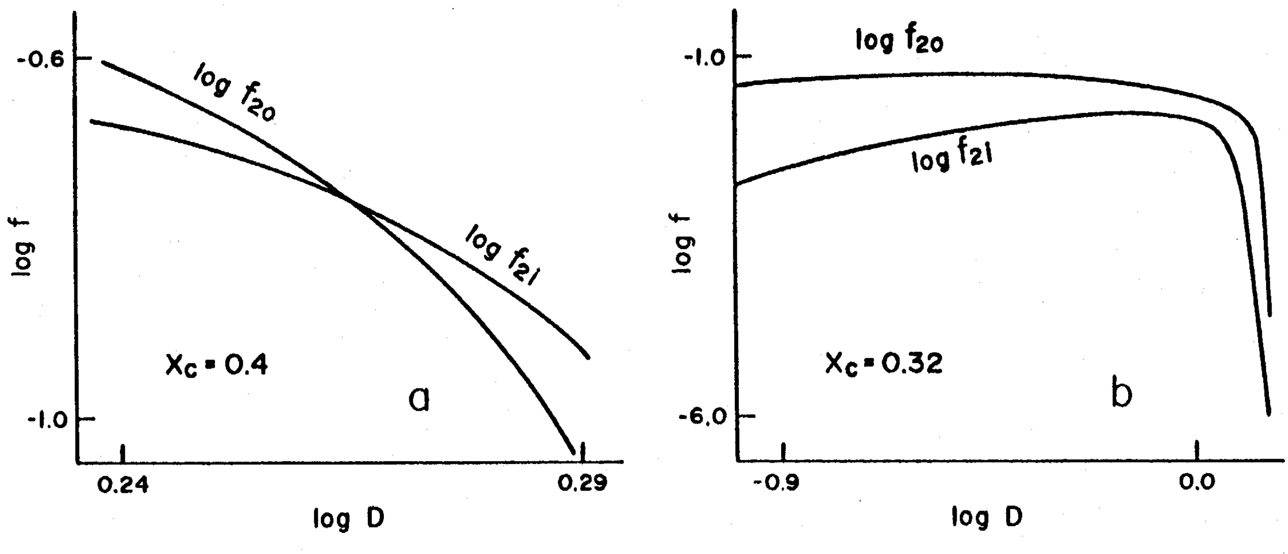


Fig. 2. In the case of the electron scattering opacity and for the specified values of X_c, the values of log f at the both sides of the interface between the homogeneous core and the inhomogeneous zone are plotted against the corresponding values of log D. f_{2i} and f_{20} are values of f at just inside and outside, respectively, of the interface. a) $X_c = 0.4$, f is continuous across the interface; b) $X_c = 0.32$, f can not be continuous across the interface for any value of D.

It should be noted, however, that, for both cases of opacity, the fractions of the masses contained in the homogeneous cores of the last models in Table 1 are already very small and the inner boundaries of the inhomogeneous zones are very close to the center. As a more plausible possibility, it might be thought that the limiting phase will be represented by the model in which the inhomogeneous zone has grown to reach to the center. For such a limiting model, the value of D must be so that the value of f following the corresponding inward integrations of (12) and (13) just becomes zero at the center. However, as well known, it is a difficult problem to attempt to obtain a particular solution which satisfies the conditions at the center by directly integrating the equations toward the center. Namely, as integration proceeds nearer and nearer to the center, solutions of equations (12) and (13) should be more sensitively influenced by the accumulated mathematical inaccuracies. From this point of view, it may be rather natural to consider that the situation as shown in Figure 2b would come from the mathematical inaccuracies. At any rate, in each last one of the models given in Table 1, the size of the central homogeneous core is quite small and practically negligible. Even if the limiting models were correctly constructed, the corresponding values of X_c should not be differ appreciably from those of the latest models presented here.

ii) Evolutionary characters of the models

Evolutionary changes of hydrogen distributions in the stars according to the present model sequences are illustrated in (X, log q)-diagrams of Figure 3. The central homogeneous core shrinks and the intermediate inhomogeneous zone extends inward. And, though it has not been exactly ascertained yet, the central homogeneous core should disappear with finite central hydrogen content which is still appreciably high and larger for the case of Kramer's opacity in comparison with the case of the electron scattering. This difference appearing on the value of critical hydrogen content at the center may be understood as being due to mainly the difference of the size in mass fraction of the original convection core.

As can be easily understood by the relations from (14) to (26) and the fact that the characteristic variables in the radiative envelope and at the outer boundary 1 of the convection core do not change along each of the model sequences under consideration, the runs of p, x and $\rho/\bar{\rho}(R)$ against the coordinate q within the convection core do not change throughout the phases under consideration. The effect of growing inhomogeneous distribution of X as illustrated in (X, log q)-diagrams of Figure 3 appears mainly on the run of t. Thus, the values of p_c and $\rho_c/\bar{\rho}(R)$ are kept to be unchanged respectively throughout the phases of the consideration though the value of t_c monotonously increases. Since, as seen later, stellar radius does not change appreciably, the changes in the central pressure and density are negligible. The increase of the central temperature is almost completely compensated by the increase of mean molecular weight.

From equations (5) and (9), the luminosity, radius and effective temperature of present model star can be expressed in units of those of the

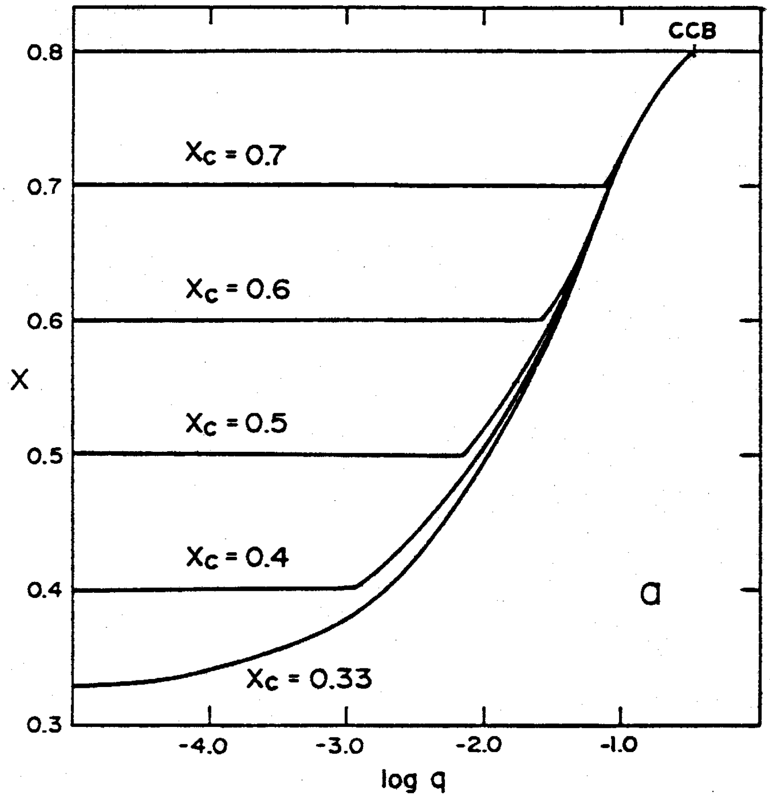
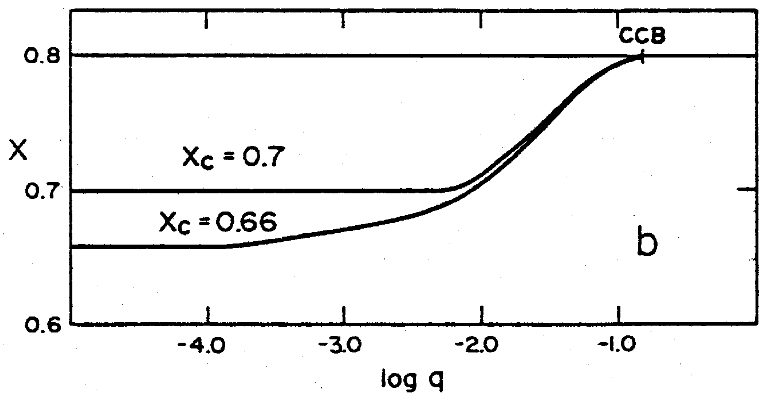


Fig. 3. Evolutionary changes of hydrogen distribution in the convection cores of the stars according to the present model sequences. a) the case of electron scattering opacity; b) the case of Kramers' opacity. CCB is the outer boundary of the convection core.



corresponding starting initial homogeneous model as follows:

for the case of Kramer's opacity

$$\log L/L_0 = (19/18.5)\log C/C_0 + (0.5/18.5)\log D/D_0 \tag{29}$$

$$\log R/R_0 = -1/18.5(\log C/C_0 + \log D/D_0) \tag{30}$$

$$\log T_e/T_{e,0} = (21/74)\log C/C_0 + (2.5/74)\log D/D_0 \tag{31}$$

for the case of electron scattering opacity

$$\log L/L_0 = \log C/C_0 \tag{32}$$

$$\log R/R_0 = -1/19(\log C/C_0 + \log D/D_0) \tag{33}$$

$$\log T_e/T_{e,0} = (21/76)\log C/C_0 + (1/38)\log D/D_0\ . \tag{34}$$

In these expressions, the subscript 0 is used to refer to the initial homogeneous model. These expressions show that the values of L/L_0, R/R_0 and $T_e/T_{e,0}$ depend only quite weakly on the value of D/D_0. Furthermore, since, for each model of the sequence, the value of C equals to that of the corresponding initial homogeneous model, L/L_0, R/R_0 and $T_e/T_{e,0}$ should not change appreciably from 1. Applying the initial and the last model listed in Table 1 to the star of 5 $M_\odot$, we give in Table 2 the corresponding values of luminosity and effective temperature. As shown there, these quantities do not differ appreciably from those of the corresponding homogeneous model.

Table 2. Luminosities and effective temperatures of the star of 5 $M_\odot$ by the initial and last model considered here

	a) electron scattering		b) Kramers' opacity	
	initial model	last model	initial model	last model
$\log L/L_\odot$	2.91	2.91	2.35	2.34
$\log T_{eff}$	4.34	4.30	4.15	4.14

§4. Summary and Remarks

Although the simple approximation formulas for the opacity and the energy generation rate were used, it was shown that the models similar to those proposed by Sakashita and Hayashi for massive stars can be constructed to represent the internal structure for the early period of main sequence evolution of the moderately massive star. Along the evolutionary sequence of such models, the outer inhomogeneous zone grows steadily inward and chemically homogeneous central core shrinks without bringing about any appreciable changes in stellar luminosity and radius and the internal spacial distributions of mass, pressure and density except the distribution of temperature which increases following a way of just compensating the increase of the mean molecular weight in the convective region. Though it has not been made sure exactly yet, the central homogeneous core should disappear when the central hydrogen content would become the finite critical value. Such a critical value would be still high and should be smaller for the star of larger mass. This character is in a contrast to that for the case of massive stars as treated by Sakashita and Hayashi. In their case, there still remains a central homogeneous core of fairly large size in the mass fraction even when the central hydrogen content becomes nearly equal to zero. In constructing their models, Sakashita and Hayashi have completely neglected the effect of the nuclear energy generation in the inhomogeneous zone. This effect, however, would be negligibly slight in the case of massive stars as treated by them. A main reason causing the difference as mentioned above should be due to the character that the radiation pressure is important in the massive stars but not so important in the moderately massive stars as being considered here.

At any rate, according to our scheme of models for the early main sequence evolution of moderately massive star, there should exist a critical stage at which the growing inhomogeneous zone just reaches to the center with a finite central hydrogen content. Until this phase, the star should not show any appreciable movement of its position on the H-R diagram. Due to a gentle slope of hydrogen distribution, the time elapsed until this critical stage should be fairly shorter than that estimated, at the phase of the same central hydrogen content, from the corresponding usual standard model sequence.

How is the character of the subsequent evolutionary model sequence ? We proceed the description using a simplified opacity law and following a classical way of model construction as adopted here. In each model along such a subsequent evolution, the value of C characterizing its radiative envelope must be larger than that used in the corresponding models as treated here and increase as evolutionary phase proceeds. Then, we have to introduce a radiative inhomogeneous zone between the radiative homogeneous envelope and the inhomogeneous convection core which is basically represented by the Emden solution for polytrope of $N = 1.5$. This means that the outermost part of the inhomogeneous convection core will change to be radiative. As evolution proceeds, such an inhomogeneous radiative region will extend inward and, at the same time, the profile of the hydrogen distribution in the inhomogeneous convection core will be steadily modified. (It should be noted here that, in an innermost part of the radiative inhomogeneous zone thus expected to extend inward should be overstable in a sense as pointed out by Kato[13]). Such a modification will continue until the hydrogen exhausted central core will be formed. If such an expectation would be realized, further subsequent evolutionary model sequence should be the refined modification of the sequence of the models with isothermal cores as studied by Hitotuyanagi and Suda[14,15]. The sequence of the models as mentioned above will represent the extremely poor case of the mixing efficiency of convection against the standard evolutionary model sequence which has been usually adopted.

We would like to thank Miss. M. Nakamura for her help in preparation of the manuscript.

References

1) R.J. Tayler: Astrophys. J. 120 (1954) 332.
2) R.S. Kushwaha: Astrophys. J. 125 (1957) 242.
3) A. Maeder: Astron. Astrophys. 32 (1974) 177.
4) A. Maeder: Astron. Astrophys. 40 (1975) 303.
5) A. Maeder: Astron. Astrophys. 43 (1975) 61.
6) A. Maeder: Astron. Astrophys. 47 (1976) 389.
7) K. Suda: Sci. Rep. Tôhoku Univ. Ser.I 52 (1969) 10 = Sendai Astronomiaj Raportoj Nr.108.

8) K. Suda and J. Uchida: Sci. Rep. Tôhoku Univ. Ser.I 56 (1973) 117 = Sendai Astronomiaj Raportoj Nr.143.

9) K. Suda and J. Uchida: Sci. Rep. Tôhoku Univ. Ser.I 58 (1975) 113 = Sendai Astronomiaj Raportoj Nr.163.

10) S. Sakashita and C. Hayashi: Prog. Theor. Phys. 22 (1959) 830.

11) K. Suda and J. Uchida: Sci. Rep. Tôhoku Univ. Ser.I 61 (1979) 220 = Sendai Astronomiaj Raportoj Nr.195.

12) See, for example, M. Schwarzschild: Structure and Evolution of the Stars, Princeton Univ. Press (1958).

13) S. Kato: Publ. Astron. Soc. Japan 18 (1966) 374.

14) Z. Hitotuyanagi and K. Suda: Publ. Astron. Soc. Japan 9 (1957) 7 = Sendai Astronomiaj Raportoj Nr.53.

15) Z. Hitotuyanagi and K. Suda: Publ. Astron. Soc. Japan 10 (1958) 8 = Sendai Astronomiaj Raportoj Nr.60.