Hydrodynamic Models of Classical Cepheids. II. Two-mode Resonance

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Two-mode Resonance

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The two-mode coupling in hydrodynamic models of classical cepheids is investigated. The centre of resonance is situated at the period ratio $P_{2O}/P_F = 0.49$ for a model sequence having 5091 $L_\odot$ and 5 $M_\odot$. The phase of bump of the luminosity- and photospheric velocity-curves coincides with the analytical results of two-mode resonance theory. The decrease of amplitude at the resonance centre is also found in the model sequence.

Keywords: Stellar pulsation, Hydrodynamic model, Resonance.

§1. Introduction

The remarkable relationship between the phase of a secondary bump in the light curve and the period of classical cepheids pointed out by Hertzsprung has been a subject of the study on pulsating variable stars. Since hydrodynamic models were constructed for investigating pulsating stars, the Hertzsprung relation on the phase of bump has been discussed based on model classical cepheids. Christy argued that the secondary bump seems to be an echo of the primary bump 1.4 periods earlier[1)]. Stobie found that the phase of the secondary bump changes differentially with the changes of model parameters and that the relation between the phase of bump and the period of models coincides with the observation[2)]. Very recently, Fadeyev[3)] succeeded in demonstrating the result in agreement with Christy's one[4)], both matching with the observational relationship.

In view of the resonance theory of pulsating stars, the Hertzsprung relation is a favourite subject. Simon and Schmidt found that the phase of secondary bump depends on the period ratio of the fundamental mode to that of the second overtone[5)]. Recently, van der Pol's equation for self-exciting oscillators is applied to investigate resonance in stellar pulsation by Takeuti and Aikawa[6)]. Resonant oscillations of self-exciting system show interesting properties that not only the phase of secondary bumps but also the amplitude of oscillation are functions of the period ratio. The multiplicity of the limit-cycle are also found. So, we shall study possible effects of the resonance in hydrodynamic models. The model used here is the DYN-code, originally constructed by Castor et al.[7], the same as that used in a previous paper (Paper I)[8]. The limit-cycle of radial pulsation is calculated by the technique described in Paper I. The decrease of the amplitude at the resonance centre will be demonstrated and the behavior of non-linear period will be also discussed. The multiplicity of the limit-cycle is beyond the scope of the present paper. The problem will be discussed in a future work.

§2. Models

The hydrodynamic models we have constructed are tabulated in Table 1. The models have the luminosity of 5091 $L_\odot$ and the mass of 5 $M_\odot$ to compare with the hydrodynamic model studied by Karp[9]. The model 3 has the same physical properties as Karp. Linear adiabatic periods for the fundamental mode and the second overtone have been calculated with initial static models constructed by the DYN-code itself. The boundary conditions for the non-linear oscillations are used to calculate the linear adiabatic pulsation function and the eigen-value. The model sequence covers the resonance centre of the fundamental mode to the second overtone, i.e. $P_F = 2P_{2O}$. In the non-linear calculation a constant in the formula of the artificial viscosity, C_Q, is fixed as unity through the present paper.

Table 1. Properties of models

Model	T_{eff}	P_F	P_{2O}	P_{2O}/P_F	P	P_F/P
1	6000	10.42	5.32	0.5105	stable	
2	5880	11.21	5.65	0.5040	11.03	1.016
3	5730	12.30	6.10	0.4960	12.19	1.009
4	5600	13.36	6.53	0.4890	13.29	1.005
5	5525	14.03	6.80	0.4850	13.99	1.003
6	5450	14.73	7.08	0.4808	14.75	0.999
7	5300	16.27	7.69	0.4726	16.37	0.994

* P_F and P_{2O} express the linear adiabatic period for the fundamental mode and the second overtone, respectively. (in days)

** P is the period of the limit-cycle oscillation. (in days)

§3. Limit-cycle Oscillation

We have searched the limiting-amplitude of the models by using the technique described in Paper I. Then the oscillation has been continued to reach the limit-cycle. The period of the limit-cycle oscillation, tabulated in Table 1, has been determined from the time interval of corresponding kinetic energy maxima (Kmax). The light-curves and photospheric velocity-curves are illustrated in Figs. 1 - 6. The limit-cycle of Model 3, which is shown in Fig. 2, is very similar to Karp's result.

The light- and photospheric velocity-variation of models shows successive changes like as the Hertzzprung relation of classical cepheids. Bumps in models whose period ratio P_{2O}/P_F is greater than 0.49 are situated at the descending branches and those in models whose period ratio is less than 0.485 are situated at the ascending branches. Although bumps situated at the ascending branch of latter models are the highest in both their luminosity and photospheric velocity,

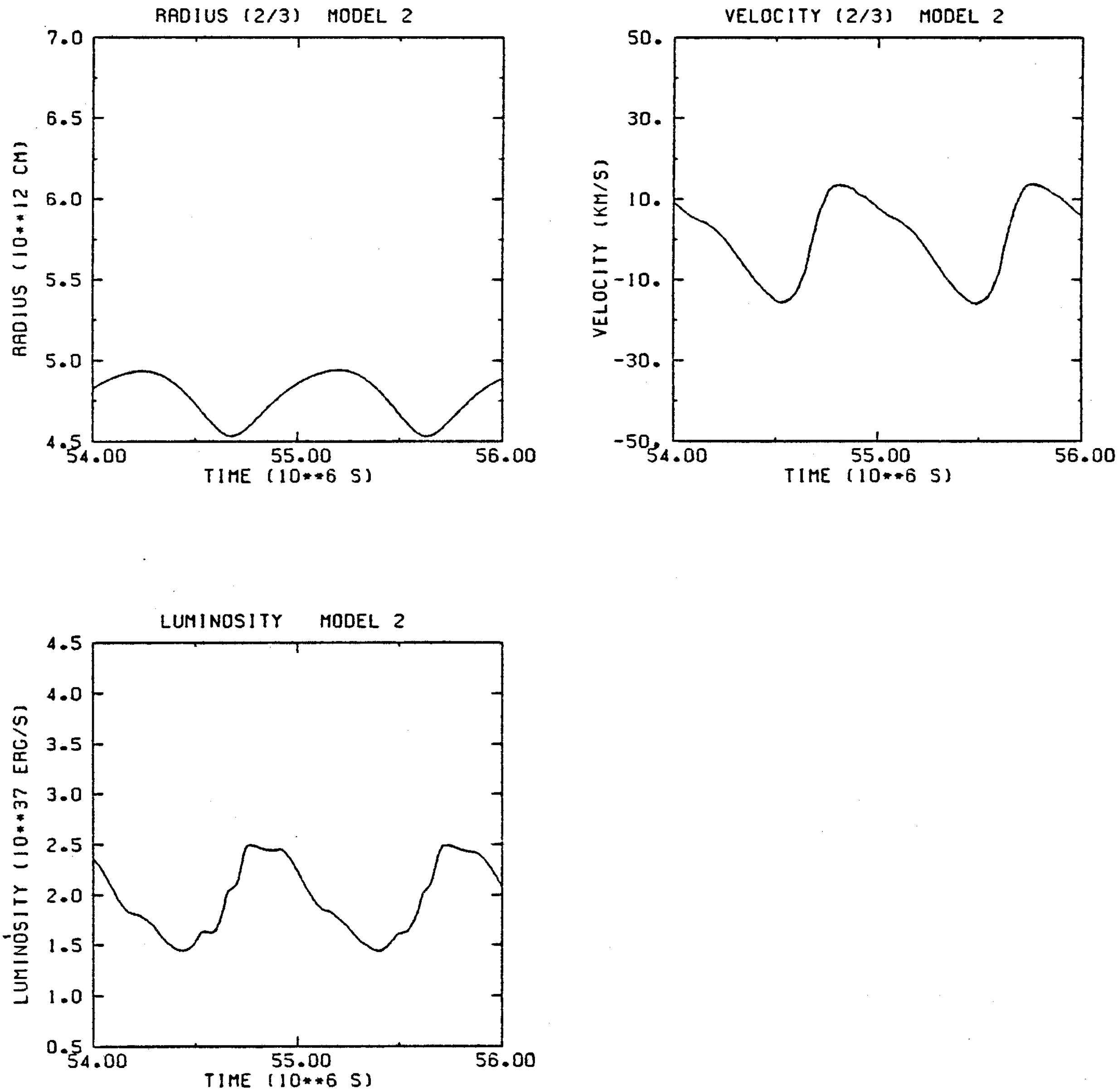

Fig. 1. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 2.

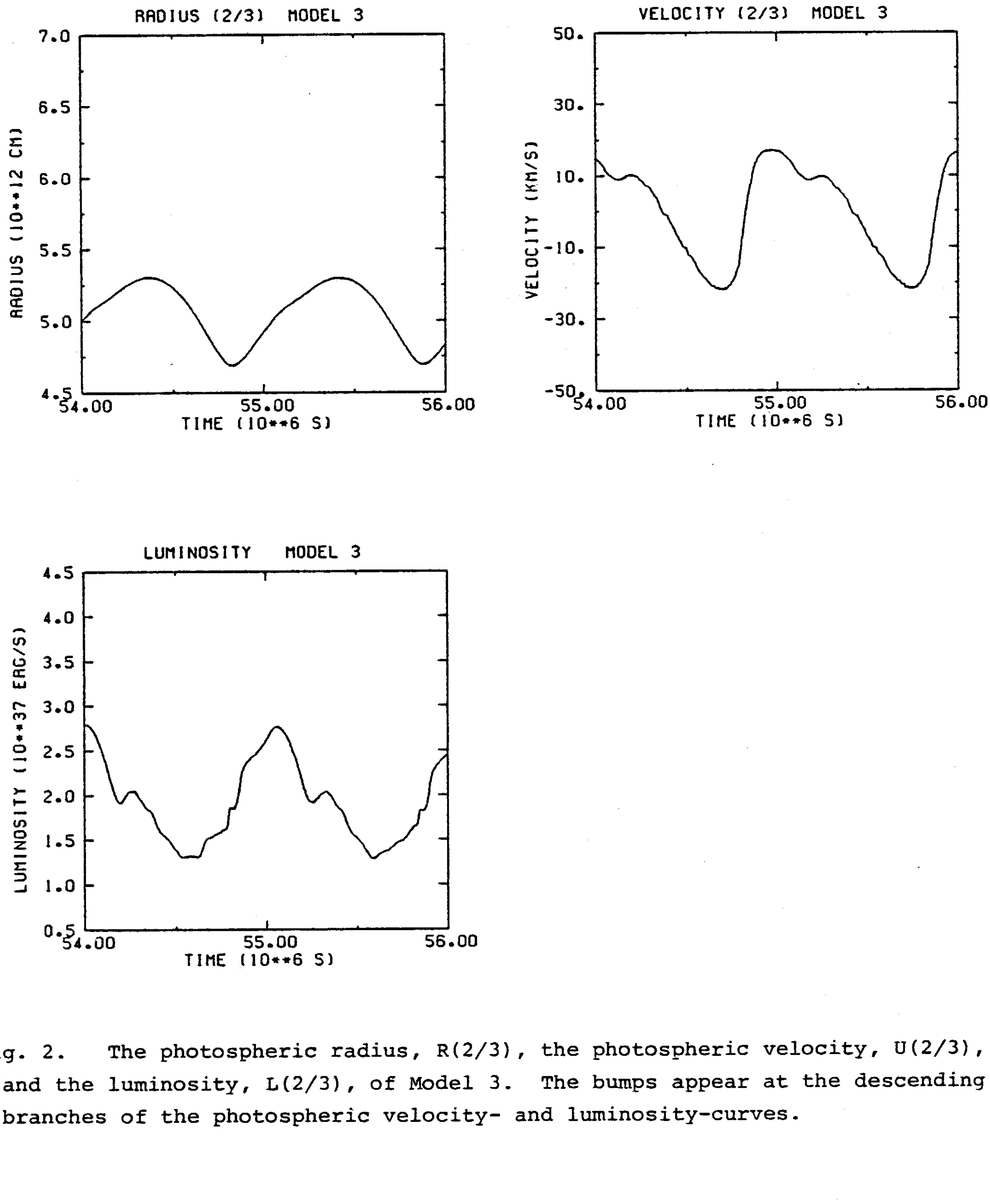

Fig. 2. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 3. The bumps appear at the descending branches of the photospheric velocity- and luminosity-curves.

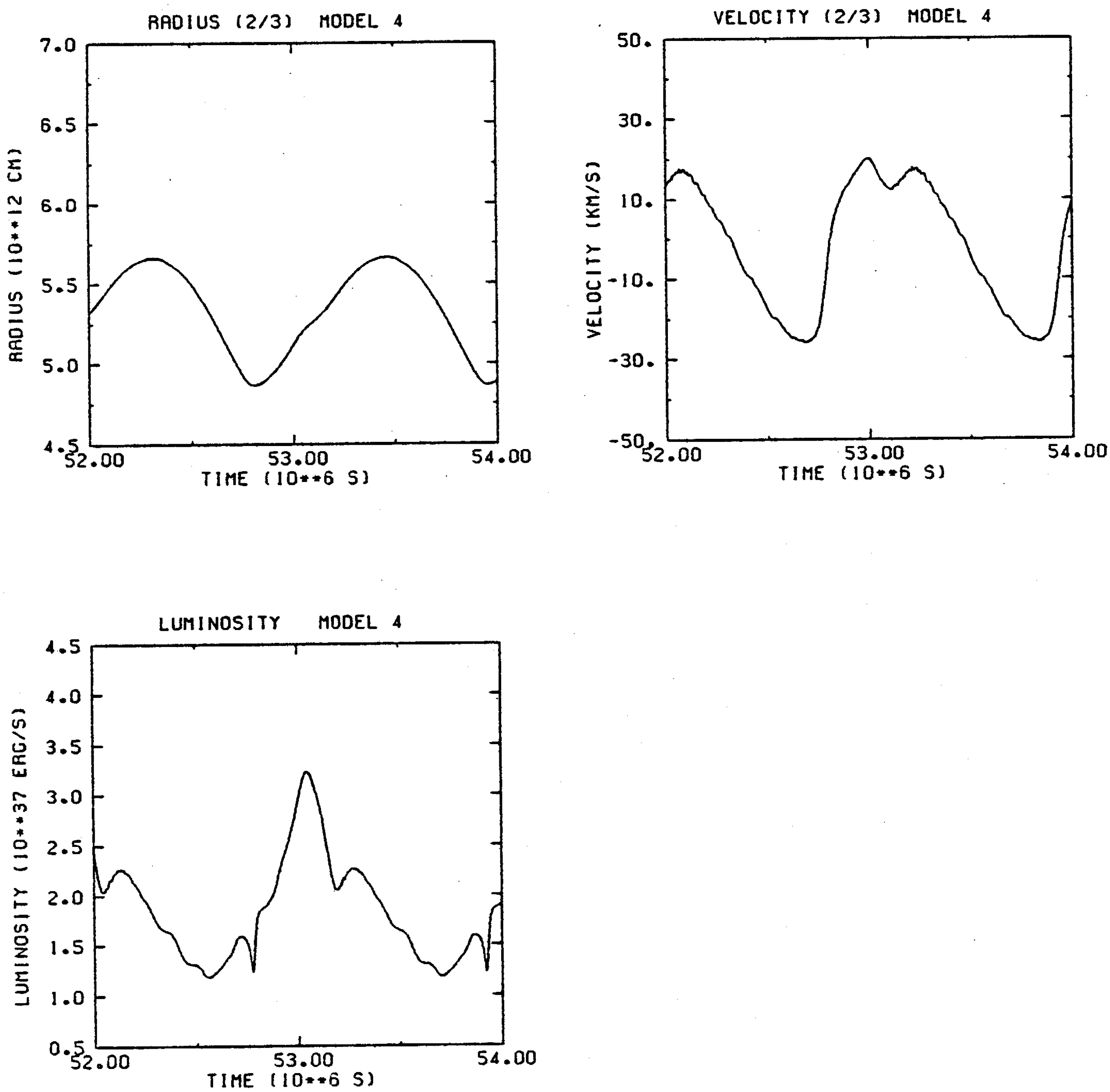

Fig. 3. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 4. The oscillation is near the resonance centre. The kinetic energy maximum occurs at the middle of two peaks of the photospheric velocity-curves.

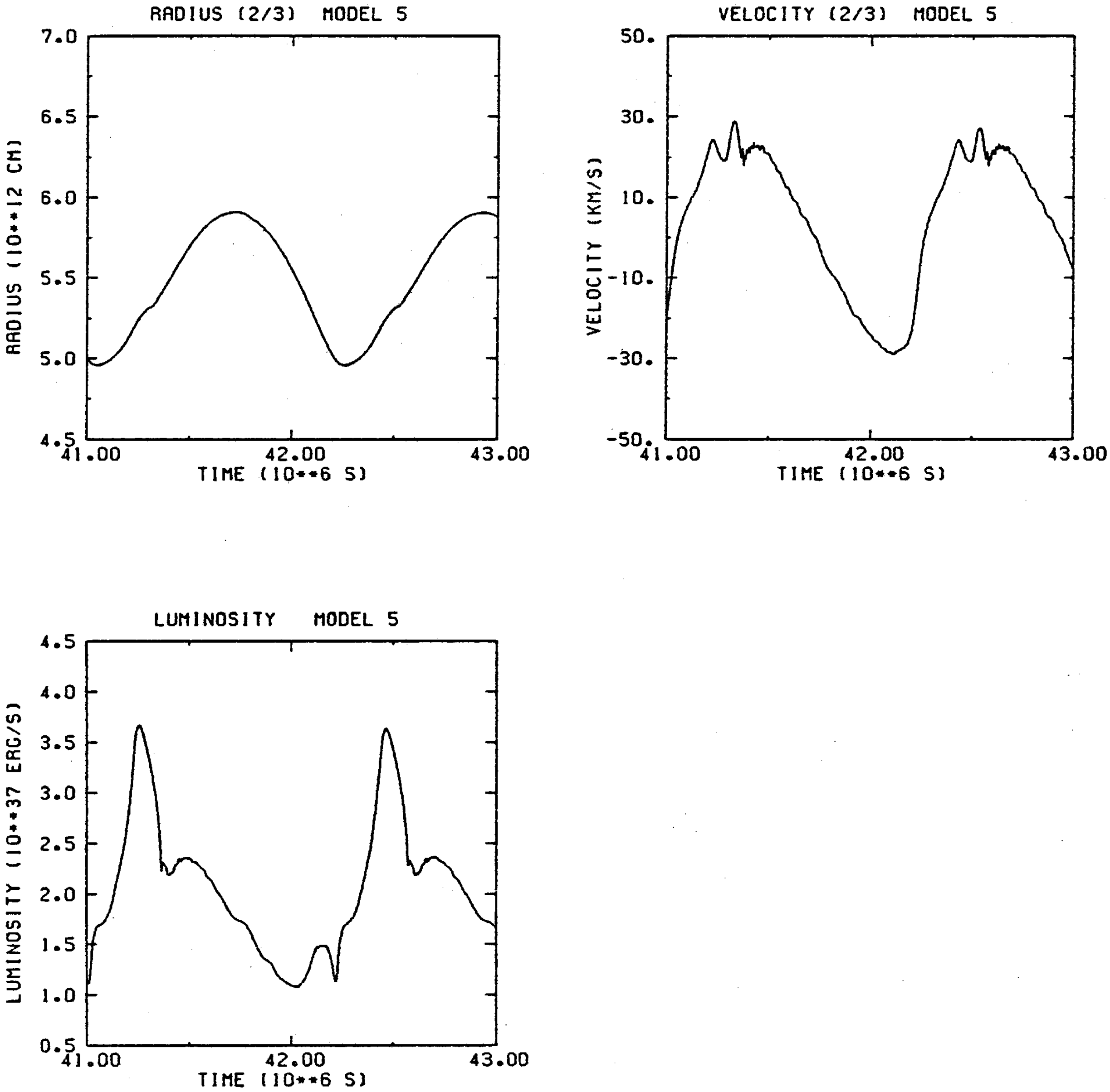

Fig. 4. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 5. The spikes of the photospheric velocity-curve are bumps. The maximum of the kinetic energy for outward movement corresponds to the third peak of the velocity-curve and to the second peak of the luminosity-curve.

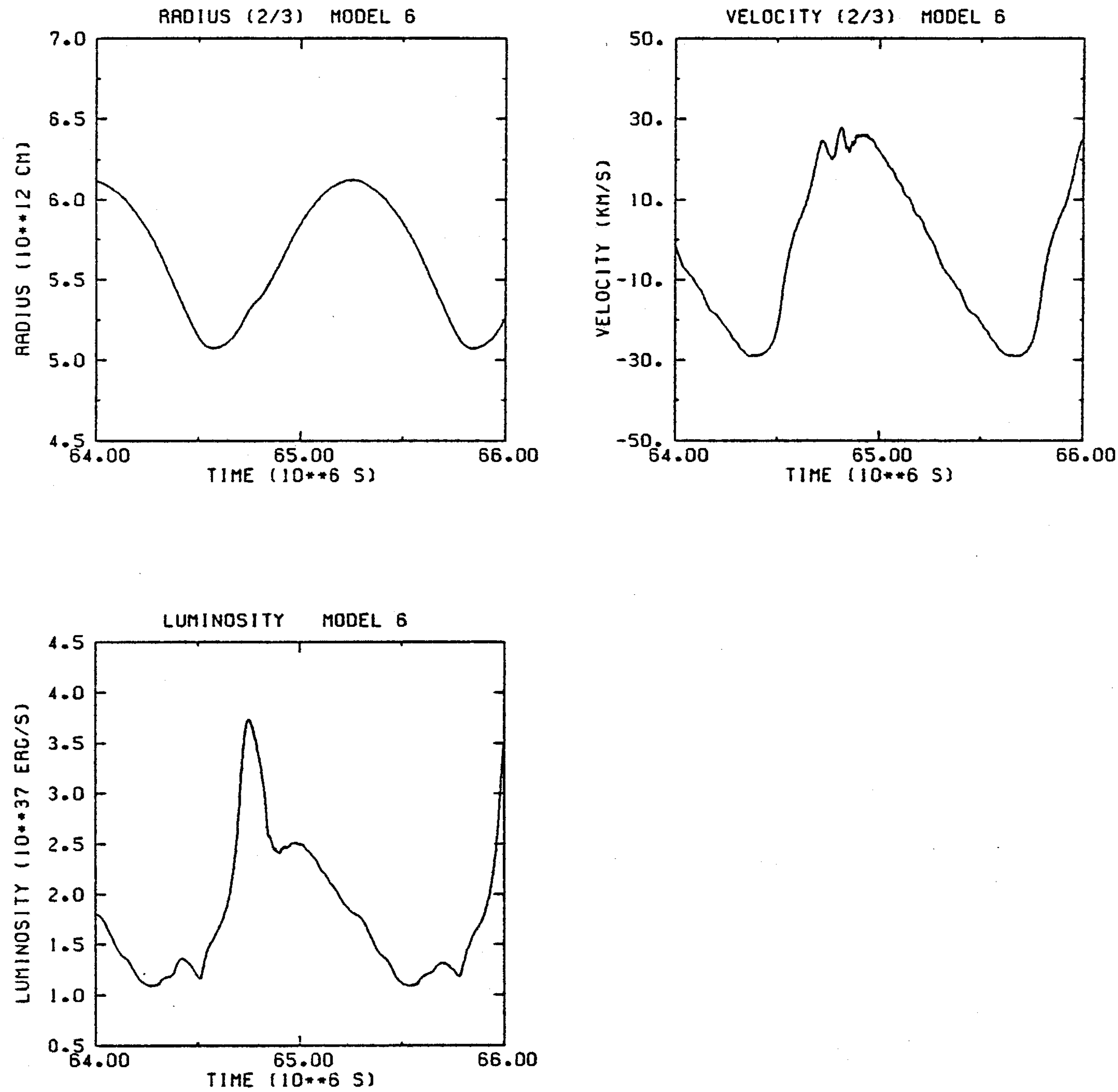

Fig. 5. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 6.

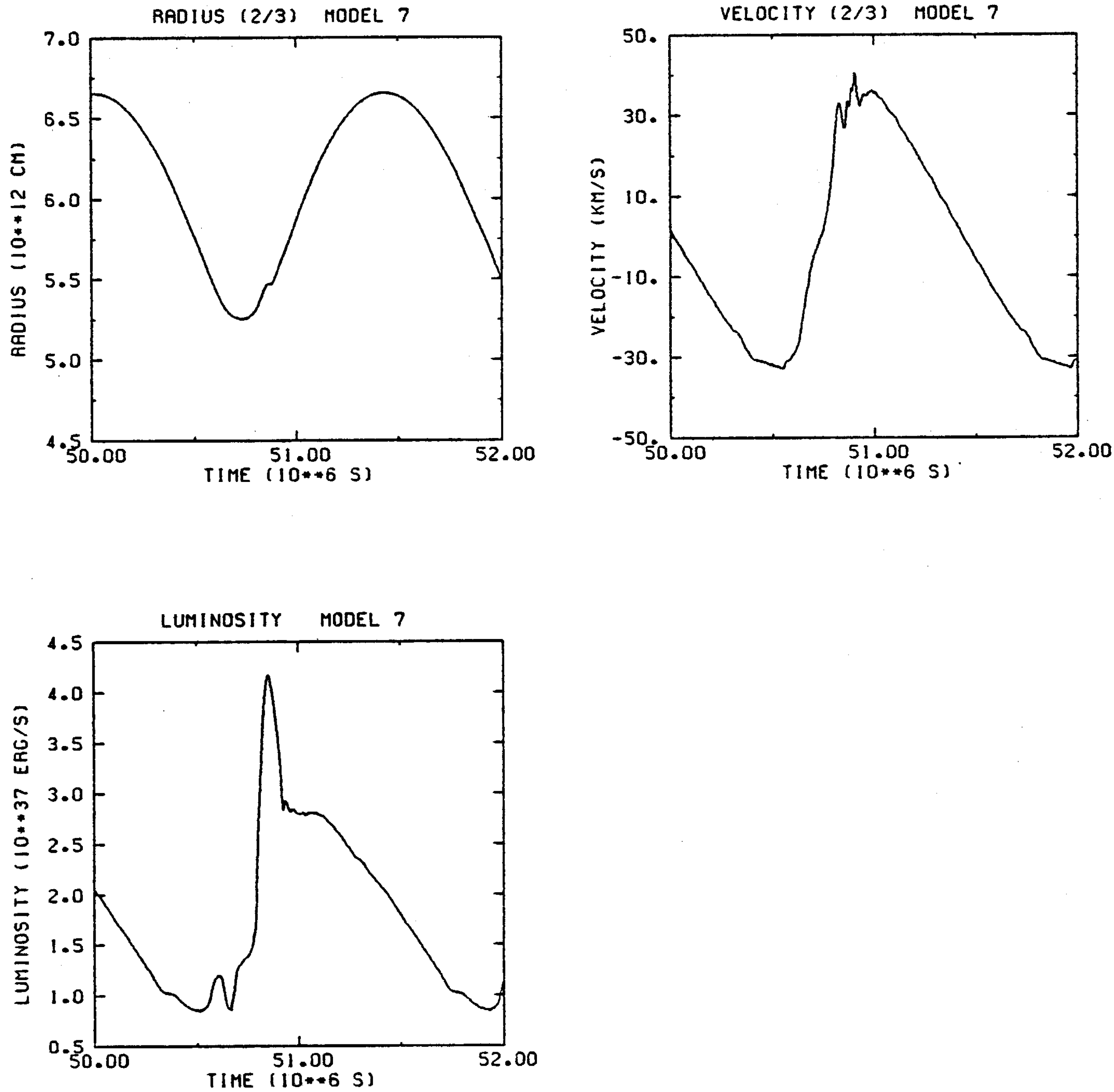

Fig. 6. The photospheric radius, R(2/3), the photospheric velocity, U(2/3), and the luminosity, L(2/3), of Model 7.

they are the secondary maximum because their kinetic energy is smaller than that of lower peaks. The model near the period ratio of 0.49 shows two nearly equal bumps and the kinetic energy maximum occurs in the middle of their peaks. So we can guess Model 4 ($P_{2O}/P_F = 0.489$) the most resonant. The term, the most resonant, means that the kinetic energy of oscillation transfers to the second overtone mode by resonance at the greatest efficiency. Therefore, the amplitude should be the smallest with the period ratio P_{2O}/P_F of 0.49. The fact that the effective resonance centre is found with a period ratio different from 0.50 is not so unusual compared with the results of Carson et al. and King et al.[10].

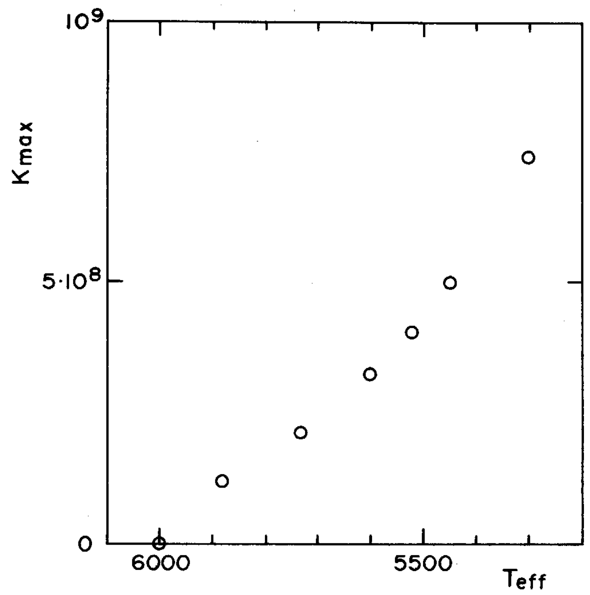

Fig. 7. The effective temperature, T_{eff}, vs. the maximum kinetic energy, Kmax, for Models 1 - 7.

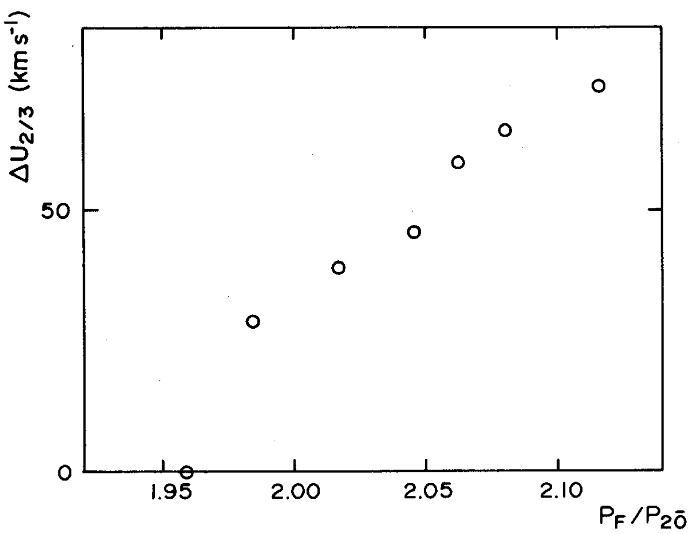

Fig. 8. The linear adiabatic period ratio, P_F/P_{20}, vs. the amplitude of the photospheric velocity, $\Delta U(2/3)$.

The relation of the maximum kinetic energy Kmax to the effective temperature of models is illustrated in Fig. 7. While Kmax shows no decrease at the resonance centre, the amplitude of the photospheric velocity has a dip at $P_F/P_{20} = 2.05$ (see Fig. 8).

Compared with the shape of the surface velocity of stellar models in resonance derived from the Takeuti-Aikawa oscillator model[6], the feature of the hydrodynamic models described above fits well to their analytical results. So we may conclude that the model oscillator system proposed by Takeuti and Aikawa expresses the stellar oscillation approximately.

§4. Discussions

The periodic solutions of resonant oscillations have been given by Takeuti and Aikawa[6]. We shall discuss the results of the hydrodynamic models compared with the analytical theory more precisely. The behavior of model oscillating system of Takeuti and Aikawa has some remarkable features in addition to the phase of bump-to-period ratio relation and the decrease of the amplitude. The example presented in the Takeuti-Aikawa paper has shown the multiplicity of limit-cycle. The hydrodynamic models studied here show no evidence for such multiplicity. We examined the growth rate with various amplitudes, but it is not found that the limit-cycle has the multiplicity. Because the check of the multiplicity may be very delicate, we hardly decide that the multiplicity of limit-cycle does not appear near the resonance centre. The width of loop on the $\sigma_2/\sigma_0 - a_0$

plane depends on the damping of the second-overtone oscillator and the strong demping suppresses usually such a complicated feature, so the model with the small artificial viscosity should be investigated.

The reality of multiplicity of limit-cycle has to be confirmed more precisely. Although the idea that the limit-cycle has the multiplicity is quite favourable on explaining the small amplitude cepheids as proposed by Auvergne et al.[11], the branch with very small amplitude appeared in Takeuti-Aikawa's paper is unstable[12]. If so, the small amplitude cepheids are not evidence for the resonance phenomenon.

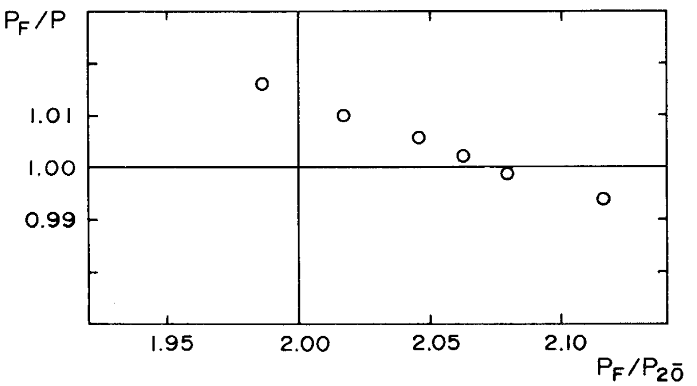

Fig. 9. The linear adiabatic period ratio, P_F/P_{2O}, vs. the non-linear period P compared with the linear adiabatic period P_F.

Next, we shall discuss the relation of non-linear period P to the period ratio P_{2O}/P_F. The ratio P/P_F in non-resonant models is usually greater than unity for non-linear oscillation. Resonance makes P/P_F small with great values of the period ratio P_{2O}/P_F. In Fig. 9, we demonstrate the P_F/P - P_F/P_{2O} relation to compare with the result of Takeuti-Aikawa's paper. The effect of resonance appears near the resonance centre as the increase of P_F/P. The kink or S-bend appeared in the analytical study, corresponding to the multiplicity of limit-cycle, is also unclear in Fig. 9.

The effect of coupling between the fundamental mode and the second overtone is described by coefficients, C(0;0,2), C(2;0,0), C(0;2,2) and C(2;0,2), and the coefficient C(0;0,0) indicates the coupling between the fundamental mode and its first harmonic mode, in the Takeuti-Aikawa theory[6]. A great C(0;0,0) generally causes σ_2/σ_0 or P_F/P_{2O} at the resonance centre to be small in the analytical theory. The coupling between two modes possibly makes P_F/P_{2O} at the resonance centre great. So the fact that P_F/P_{2O} at the resonance centre seems to be 2.05 makes us an implication that the coupling between the fundamental mode and the second overtone is very strong and the self-coupling in the fundamental mode is weak. General trends as P_F/P is greater than unity seems to be also evidence for weak self-coupling.

Although the increase of P_F/P looks like the effect of resonance, it is possible that the decrease of P_F/P for redder models might be caused from the increasing effect of shock wave in the envelope layers. The investigation with small artificial viscosity will be also needed in this point of views.

§5. Concluding Remarks

In conclusion, we may state here that hydrodynamic models near 12 day-period behave like as the result of resonance theory. The DYN-code we used in the present study works very smoothly, although the degree of the internal viscosity of model seems very important. One of the most important problem to be solved in probably the multiplicity of limit-cycle near the resonance centre. Whitney[13] has recently demonstrated the equivalency of the resonance theory with the idea of propagating pulse, often called as "echo" phenomenon. In the hydrodynamics the wave can be expressed through the superposition of propagating and reflecting pulses. We may derive, however, many useful results applicable to non-linear phenomenon in cepheid variables from the oscillation theory, when the concept "resonance" is accepted. Hydrodynamic models can play important role for the investigation on non-linear stellar oscillations.

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PS After completing the manuscript the paper on the non-linear properties of classical cepheids by N.R. Simon and C.G. Davis was published[14]. Their study confirms the analytical results of Takeuti and Aikawa. The relationship between P_F/P_{2O} and P_F/P, which is the most difficult to detect in the course of the present study, is also unclear in their paper, but P_F/P is generally greater than unity just like as the present result. They stressed the advantage of the Fourier decompositions for examining the phase of bump. In the present paper, we distinguished carefully the main peaks and bumps based on the variation of the kinetic energy of oscillation, however. The resonance centre estimated here may be reliable. (MT)

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