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Hydrodynamic Models of Classical Cepheids. I.
A Technique to Search the Limiting-amplitude Oscillation

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The limiting-amplitude of hydrodynamic models for cepheid variables is searched by checking the intrinsic growth-rate of the oscillation derived by the pumping-up. The procedure is scarcely affected by the initial values. The modulation of oscillation appeared at the earlier stage may be enhanced by the overtones in resonance.

Keywords: Stellar pulsation, Hydrodynamic model, Resonance.

§1. Introduction

The hydrodynamic models have been used to investigate the stellar pulsation since Christy's pioneer work on RR Lyrae stars[1]. So far hydrodynamic models themselves have been developed by several authors and the main features of classical cepheids pulsation have been reproduced precisely. Very recently A.N. Cox and his collaborators have the DYNSTAR code, Stothers and his collaborators have another code, and the DYN code is used by Davis and his collaborators. Fadeyev and Tutukov have made their own code.

It has been a problem that the possible change of opacities could affect the properties of models. So it seems difficult to compare directly the theoretical models with observation for evaluating physical and chemical properties of cepheids accurately without thorough considerations. On the other hand, the non-linear behavior possibly concerning with resonance has not been studied so sufficiently except for bump features of models which may be interpreted as resonance between the fundamental mode and second overtone. So in the papers of present series, we shall focus our study on the resonance phenomenon of the hydrodynamic models for classical cepheids. In the first paper, we shall describe a technique to search the limiting-amplitude promptly and discuss the modulation in the maximum kinetic energy of pulsation during a period briefly. The behavior of models in resonance will be presented in later papers.

§2. Models

The hydrodynamic models we used here is one constructed originally by Castor, Davis, and Davison[2] and modified partly by Adams and Castor[3]. As described by Davis, Moffett, and Barnes[4], the code has been constructed by using the non-Lagrangian, continuous rezoning on dynamic zoning method. The procedure makes the light- and the radial velocity-curves very smooth. Although convection is ignored in the code it is not so serious for our purpose, because our study is not intent to compare the theoretical result with observation directly. The Los Alamos opacities are used in the form of Stellingwerf's analytical expression. The problem caused by the difference between the Los Alamos ones and Carson's one is beyond the scope of our purpose.

The artificial viscosity expressed in usual von Neuman-Richtmyer's formula (e.g. see Christy[5]) is used.

The outer atmospheres are solved by using the diffusion approximation. Our code works quite well and so no difficulty is found in simulating the pulsation with the amplitude nearly equal or less than the limiting amplitude, for classical cepheids.

§3. Intrinsic Growth-rate

The cycle-to-cycle behavior of hydrodynamic model was investigated carefully by several authors. Christy[1] has found that the RR Lyrae pulsation converges smoothly to the limit-cycle and Cox et al.[6] regarded the small changes appeared cycle by cycle as non-essential in the course of getting the stable limit-cycle. Stobie[7] used the pubp up technique to shorten the time for computation, but the cycle-to-cycle change of the kinetic energy of pulsation was only used to judge whether the pulsation reaches to the limiting-amplitude or overshootes it. In the present work, we shall study the growth-rate in the individual cycle carefully.

The total kinetic energy of pulsation K of hydrodynamic models is described as following:

$$K = \Sigma \frac{1}{2} u_i^2 \Delta M_i \ , \tag{1}$$

where u_i and ΔM_i are the velocity and the mass of the i-th shell respectively. We denote the maximum of K by K_{max}. The maximum appears twice a cycle, but we pick up K_{max} only for the case that $U(\tau = 2/3) > 0$.

In search for the limiting-amplitude, we usually use the pumping up that u_i is multiplied by the factor $(1 + \eta_e)^{1/4}$ at the instant that K reaches the maximum. For a cycle the pulsation is pumped twice and so the growth-rate of K_{max} associated with the pumping up procedure is given by η_e. We define the resultant growth-rate η' by the ratio of successive K_{max} minus one. In general, η' is not identical with η_e because the model has the excitation or dumping according to the characteristics of model. So we have the intrinsic growth-rate η defined as

$$1 + \eta = (1+\eta')/(1+\eta_e). \tag{2}$$

The extremum η at the infinitesimal amplitude is the vibrational or pulsational instability studied in the linear theory. When we let the model pulsate without pumping up, $\eta_e = 0$ and then $\eta = \eta'$.

When we find the pulsation with $\eta = 0$, it is in the limit-cycle. In the case of $d\eta/dK_{max} > 0$, the limit-cycle is unstable, and on the contrary, $d\eta/dK_{max} < 0$, the limiting-cycle is stable.

Even though Stobie[7] and Karp[8] used the pumping up technique, they checked η after putting η_e to zero, i.e. in the state that the pumping is stopped tentatively. If the diagnostic used in the state of pumping-off will be applicable in that of pumping-on, we can survey η for the wide range of K_{max} promptly. So we shall see the behavior of η accompanied by the pumping and compare it with the result in the pumping-off case.

§4. Results and Discussion

In Fig. 1 the result obtained from the model E AQL 4A2 of Castor, Davis, and Davison[2] is described. The physical properties of the model are as follows:

the stellar mass, $M_s = 1.84 \times 10^{34}$ g,

the liminosity, $L = 1.59 \times 10^{37}$ ergs/sec,

the effective temperature, $T_{eff} = 5610$ K,

and the chemical composition,

$X = 0.7$, $Y = 0.28$, and $Z = 0.02$.

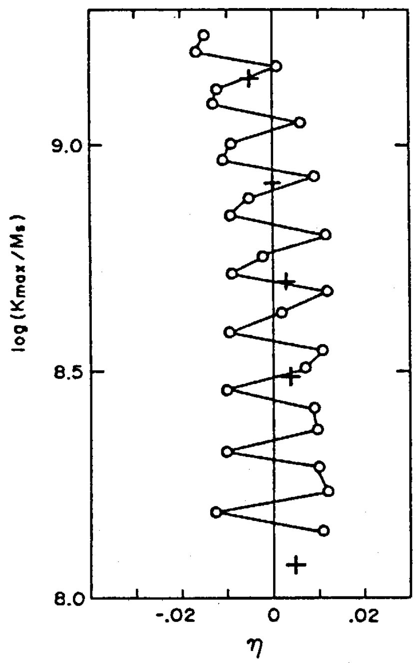

Fig. 1. Intrinsic growth-rate η. The open circles combined with solid lines indicate the successive results for pumping up. The crosses show η averaged for the results without pumping. For the case of R1MAX = 0.1, n = 6.3 and $C_Q = 1$.

The inner boundary of model is calculated by putting the parameter of program, R1MAX to be 0.10. This means that the radius of the innermost shell is just less than 0.10 of the stellar radius. The initial value of u_i is given by the formula,

$$u_i = u_o(r_i/R)^n, \qquad (3)$$

where r_i is the radius of the i-th shell and u_o is the parameter and R, the radius of the outermost shell. $n = 6.3$ and $u_o = -10$ km/sec are chosen for Fig. 1. The ordinate is $\log(K_{max}/M_s)$ where K_{max} is expressed in ergs. The abscissa is the intrinsic growth-rate η. The open circles indicate the growth-rate η for a cycle with pumping ($\eta_e = 0.1038$). They are combined in order by solid lines to indicate the successive results and have the values of K_{max} in resultant state. The crosses show η averaged for a number of cycles without pumping ($\eta_e = 0$).

η with pumping is spread compared with the averaged pumping-off value. The diagram, however, gives us a clear implication that η is not affected by the pumping-up procedure in its global appearence. Near the pulsation of $\log(K_{max}/M_s) = 8.5$, η is so close to zero that the e-folding time of K is more than 30 periods, and near $\log(K_{max}/M_s) = 8.9$, the e-folding time reaches 230 periods and more. So the search for the limiting-amplitude requires rather great computation time unless we use pumping-up.

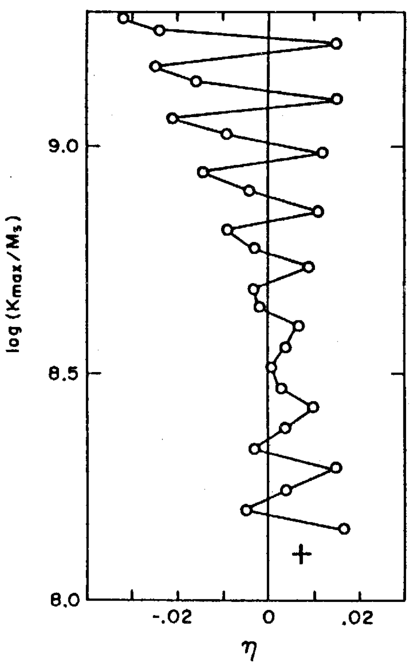

Fig. 2. The same as Fig. 1 except for R1MAX.
R1MAX = 0.05, n = 6.3 and $C_Q = 1$.

In Fig. 2 the result of another model, which has a different parameter R1MAX, is illustrated to check the result shown in Fig. 1. R1MAX of 0.05 is chosen here. The model has a little smaller radius of the innermost core for the wave. The difference of the time required to travel between the envelopes may cause the different result in the η-K_{max} diagram. Although the diagram shows certainly different pattern compared with the former one, the correspondence between the pumping-on and -off cases is still held quite well. The modulation appeared in the η-log K_{max} diagram will be discussed later.

We shall study the dependency of η on the initial distribution of u_i to confirm the adequacy of the technique described above. First, the index n in equation (3) is changed to check its effect. $n = 6.0$ and 6.6 are chosen in Figs. 3 and 4. The increase of n enhances the amplitude of

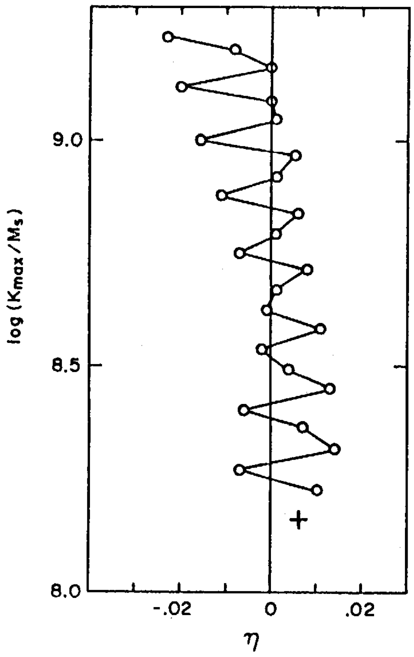

Fig. 3. The same as Fig. 2 except for n.
R1MAX = 0.05, n = 6.0 and C_Q = 1.

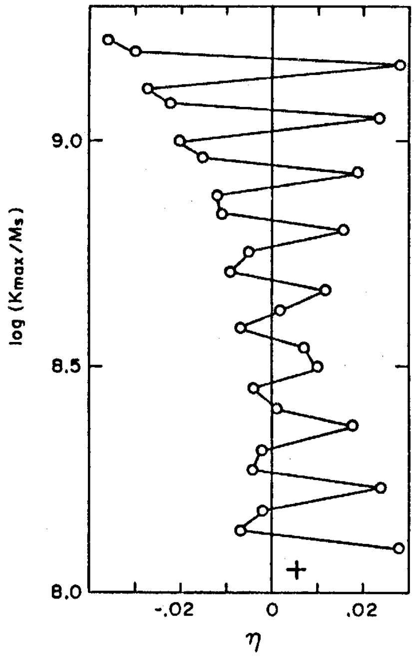

Fig. 4. The same as Fig. 2 except for n.
R1MAX = 0.05, n = 6.6 and C_Q = 1.

modulation. General trend of the diagram is, however, unchanged. Although the effect of the initial distribution of u_i should not be ignored, the above technique works efficiently to search the limiting-amplitude and the limit-cycle oscillation will be found after sufficient numbers of trial oscillation starting near the limiting-amplitude oscillation. The shortage of machine time depends on the value of η_e.

Next problem on which we shall discuss is what the modulation appeared in the η-log K_{max} diagram is. The modulation shows the cycle-to-cycle change of K_{max} strictly, so the problem is none other than the modulation in K_{max}. A similar modulation in K_{max} was found for model 4e by Stobie[9]. We have tried computation over 50 periods for the model we used for Fig. 1. The $r(\tau = 2/3)$ - $u(\tau = 2/3)$ diagram is illustrated in Fig. 5. The diagram is apparently similar to the $x - \dot{x}$, phase-plane diagram in the oscillation theory. The diagram shows the three-period modulation just like the modulation in η-log K_{max} diagram. The behavior for each three cycle is quite similar, so the limit-cycle is approximately established involving nearly three periods.

The eigen-period of the model used in the present study is tabulated in Table 1 with both 0.10 and 0.05 for R1MAX. The difference in the depth of model does not seriously affect the periods. The commensurability in the periods is checked as follows:

$4P_1/3P_0$ = 0.99818, 1.00342,
$5P_2/3P_0$ = 0.99348, 1.00315,
$6P_3/3P_0$ = 0.95959, 0.97154.

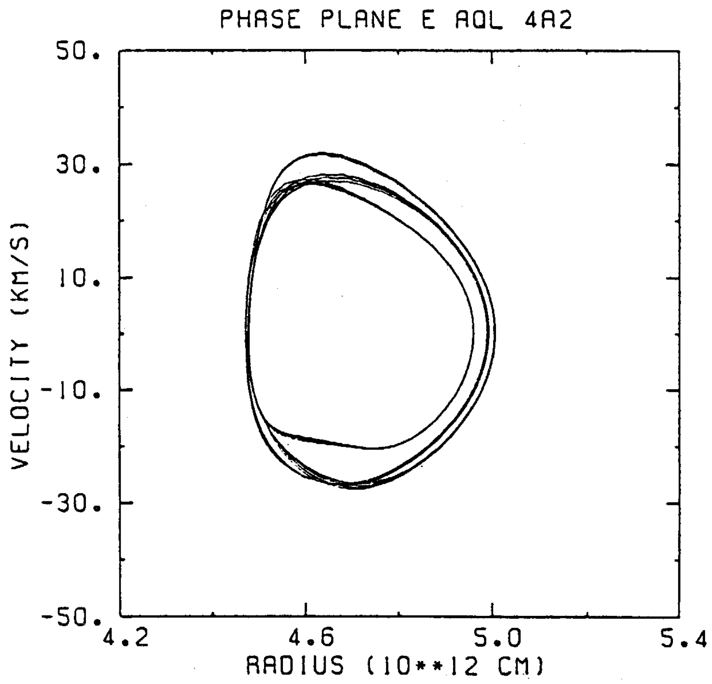

Fig. 5. Phase-plane diagram for the shell of $\tau = 2/3$. It comes from the results without pumping which are indicated by a cross near $\log(K_{max}/M_s) = 8.9$ in Fig. 1

Table 1. Eigen-periods of the models.

R1MAX of model	0.10	0.05
P_0	619.208	620.587
P_1	463.563	467.031
P_2	369.102	373.526
P_3	297.093	301.464

*the period is in 10^{-3} sec.

Two values in the right-hand side are the ratios for R1MAX = 0.10 and 0.05, respectively. The result suggests that resonance among the fundamental mode, the first overtone, and the second overtone may be realized. The modulation in K_{max} which has the period of $3P_0$ coincides with these resonance.

To conclude whether the modulation results from resonance or not, we should study the models with in- and off-resonance. Some trial shows that the remarkable modulation is yielded only in the model with in-resonance. Details of the study will be reported in the future paper.

In Fig. 6, the η-K_{max} diagram is shown for the model with $n = 6.3$ and $C_Q = 4$.

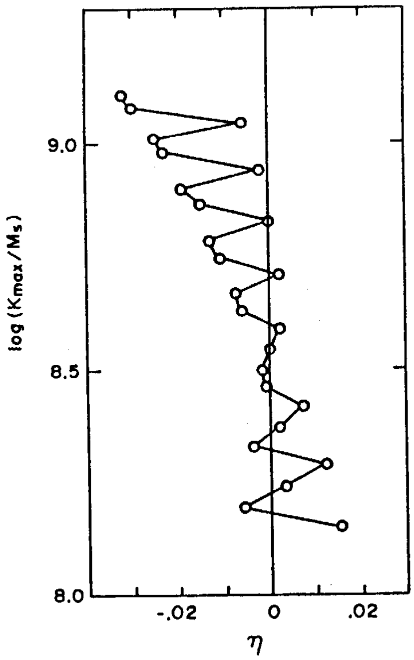

Fig. 6. The same as Fig. 2 except for C_Q. R1MAX = 0.05, n = 6.3 and C_Q = 4.

The difference between Figs. 2 and 6 is that in the constant C_Q of the artificial viscosity. The increase of the artificial viscosity makes the limiting-amplitude smaller and the modulation to be less significant. The enhancement of the first overtone may be generally affected by the artificial viscosity term. This should be checked to study the multiperiodicity in pulsating stars.

In conclusion, the intrinsic growth-rate calculated from the oscillation with pumping is useful to search the limiting-amplitudes of hydrodynamical models. Although the modulation in η probably caused from resonance is appeared, general trends of η coincide with the growth-rate without pumping. By using the present technique we shall investigate the properties of hydrodynamical models, especially the effect of initial distribution of the velocity.

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