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On the Global Structure of Stellar Magnetospheres with Stellar Winds

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The solutions for the magnetic field and stellar wind in axially symmetric stellar magnetospheres are obtained through perturbational method in three artificial extreme situations, i.e., the limits of weak magnetic field, strong magnetic field and weak electromagnetic coupling. The set of basic equations are derived from MHD equations in the two-fluid approximation of a plasma by assuming quasi-neutrality and small mass ratio of electrons to protons. It is emphasized in this treatment that, since the inertial term in the generalized Ohm's law has generally a non-zero rotation, the 'violation of flux-freezing' arises even in a perfectly conducting plasma. This fact makes it possible for a stellar wind to blow across the magnetic lines of force. The global structure of a stellar magnetosphere is inferred from the results obtained in the three extreme situations. It is suggested that the magnetosphere have generally the closed magnetic lines of force and the stellar wind blows across them forming a current sheet which may result in a very elongated shape of the lines of force at about the equatorial plane.

Keywords: Stellar magnetosphere, Stellar (Solar) wind, MHD, Violation of flux-freezing.

§1. Introduction

Although more than two decades have passed since Parker[1] predicted the existence of the solar wind, no satisfactory solution to the combined problem of stellar wind and magnetospheric structures has been obtained yet.

Waber and Davis[2] considered the solar wind solution in the presence of the magnetic field and solar rotation by restricting their attention only to the equatorial plane and showed that there appear two more critical points in addition to the sonic point. Mestel[3] removed this restriction and calculated the angular momentum loss by the stellar wind. Under the assumptions of axial symmetry and asymptotically radial structure,Pneuman and Kopp[4] calculated numerically a concrete magnetic field structure. However, only a few authors have discussed on the three dimensional structure of the stellar wind and magnetic field. Though all these works are based on the assumption of the strict MHD condition,

$$\mathbb{E} + \frac{1}{c}\ \mathbb{V} \times \mathbb{B} = 0, \tag{1}$$

(where $\mathbb{E}$, $\mathbb{B}$ and $\mathbb{V}$ are the electric, magnetic and velocity fields, respectively, and c is the light velocity) Kuo-Petravic et al.[5] have carried out a numerical computation in the two-fluid approximation. Their solution has, interesting enough, closed magnetic lines of force and plasma outflow across them suggesting the "violation of frozen-in condition".

Eq. (1) serves as a special form of the generalized Ohm's law in a perfectly conducting plasma. Recently, however, the importance of including the inertial terms in the generalized Ohm's law has been suggested by Wang[6] and Wright[7] for a charge-separated plasma and by Kaburaki[8] for a quasi-neutral plasma. If the inertial terms and other forces such as pressure and gravity are included, Eq. (1) becomes

$$\mathbb{E} + \frac{1}{c}\ \mathbb{V} \times \mathbb{B} = \mathbb{K}. \tag{2}$$

The explicit expression for $\mathbb{K}$ in the one-fluid approximation is given in section 2. The ratio of this extra-term to the motional field,

$$\lambda = \frac{|\mathbb{K}|}{|\mathbb{V} \times \mathbb{B}/c|} \tag{3}$$

is very small for a quasi-neutral plasma and this fact makes one use Eq. (1), but the ratio may become comparable to unity in a charge-separated plasma.[6] Moreover, it is not $\mathbb{K}$ itself but $\nabla\times\mathbb{K}$ that contribute to the violation of flux-freezing. Since $\nabla\times\mathbb{E}=0$ in a steady state, the motional field should be rotation-free, $\nabla\times(\mathbb{V}\times\mathbb{B}/c)=0$, as far as $\mathbb{K}$ is neglected. However, if the inertial term is included (i.e. $\mathbb{K}\neq 0$) it becomes generally non-zero, $\nabla\times(\mathbb{V}\times\mathbb{B}/c)=\nabla\times\mathbb{K}\neq 0$ (c.f. section 3), so that the frozen-in condition is violated. It must also be noted that $\nabla\times\mathbb{K}$ has generally the same order of magnitude as $\nabla\times(\mathbb{V}\times\mathbb{B}/c)$, although $\mathbb{K}$ itself may very smaller than $\mathbb{V}\times\mathbb{B}/c$ (i.e. $\lambda\ll 1$ in a quasi-neutral plasma). The current density induced by the extra-electric field $\mathbb{K}$ can be large enough to affect the magnetic field structure.

We first derive in section 1 a set of basic equations for a quasi-neutral plasma in the one-fluid description by neglecting electron masses. An explicit expression for the generalized Ohm's law of the form (2) is given in this approximation. In section 3 the solutions of these equations are obtained in three extreme situations through perturbational method. These solutions are synthesized in section 4 to infer the complete solutions in actual situations. We have a picture similar to that of Kuo-Petravic et al.[5] The effects of finite electron masses are also discussed.

§2. Derivation of Basic Equations

We assume that the plasma around a star consists only of electrons and protons whose masses, electric charges and number densities are denoted by m_e, m_p, $-e$, e, n_e and n_p, respectively. The conservation equations for mass and momentum of each species are written in the two-fluid approximation as

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \boldsymbol{V}_e) = 0, \tag{4a}$$

$$m_e n_e \{\frac{\partial \boldsymbol{V}_e}{\partial t} + (\boldsymbol{V}_e \cdot \nabla)\boldsymbol{V}_e\} = -en_e(\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}_e \times \boldsymbol{B}) - \nabla P_e + m_e n_e \nabla\frac{GM}{R} + \boldsymbol{F} \tag{4b}$$

and

$$\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \boldsymbol{V}_p) = 0, \tag{5a}$$

$$m_p n_p \{\frac{\partial \boldsymbol{V}_p}{\partial t} + (\boldsymbol{V}_p \cdot \nabla)\boldsymbol{V}_p\} = en_p(\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}_p \times \boldsymbol{B}) - \nabla P_p + m_p n_p \nabla\frac{GM}{R} - \boldsymbol{F}, \tag{5b}$$

where $\boldsymbol{V}_e$, $\boldsymbol{V}_p$, P_e and P_p are the flow velocities and the scalar pressures of electrons and protons, respectively, $\boldsymbol{F}$ is the momentum exchange rate between the two species, G, M and R are the gravitational constant, the mass of a central star and the distance from the stellar center. Since our main interest is in the inertial effects in a 'quasi-neutral plasma', it is convenient to employ a one-fluid description. Introducing the mass density, charge density, mean fluid velocity and current density by the relations

$$\rho = m_p n_p + m_e n_e, \tag{6}$$

$$q = e(n_p - n_e), \tag{7}$$

$$\boldsymbol{V} = \frac{m_p n_p \boldsymbol{V}_p + m_e n_e \boldsymbol{V}_e}{m_p n_p + m_e n_e}, \tag{8}$$

$$\boldsymbol{j} = e(n_p \boldsymbol{V}_p - n_e \boldsymbol{V}_e), \tag{9}$$

we first obtain the mass and charge conservation laws from Eqs. (4a) and (5a);

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0, \tag{10}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \boldsymbol{j} = 0. \tag{11}$$

Equation of motion in this scheme is obtained by adding Eqs. (4b) and (5b);

$$\frac{\partial \rho \boldsymbol{V}}{\partial t} + \nabla \cdot [\frac{1}{(1+\alpha\delta)(1-\alpha)}\{(1-\alpha+\alpha\delta)\rho\boldsymbol{V}\boldsymbol{V} - \alpha\delta\rho(\boldsymbol{V}\frac{\boldsymbol{j}}{en} + \frac{\boldsymbol{j}}{en}\boldsymbol{V}) + \delta\rho(\frac{\boldsymbol{j}}{en})(\frac{\boldsymbol{j}}{en})\}]$$
$$= en(\alpha\boldsymbol{E} + \frac{1}{c}\frac{\boldsymbol{j}}{en} \times \boldsymbol{B}) - \nabla P + \rho\nabla\frac{GM}{R}, \tag{12}$$

where $\alpha \equiv q m_p / e\rho$ is the neutrality parameter of the plasma, $\delta \equiv m_e/m_p$ is the mass ratio of an electron to a proton, $n \equiv \rho/m_p = n_p + \delta n_e$, and $P \equiv P_e + P_p$ is the total gass pressure. On the other hand, subtraction of Eq. (4b)$\times(e/m_p)$ from Eq. (5b)$\times(e/m_e)$ yeilds the generalized Ohm's law;

$$\delta\frac{\partial}{\partial t}(\rho\frac{\boldsymbol{j}}{en}) + \delta\nabla\cdot[\frac{1}{(1+\alpha\delta)(1-\alpha)}\{-\alpha\rho\boldsymbol{V}\boldsymbol{V} + \rho(\frac{\boldsymbol{j}}{en}\boldsymbol{V} + \boldsymbol{V}\frac{\boldsymbol{j}}{en}) - (1-\delta+\alpha\delta)\rho(\frac{\boldsymbol{j}}{en})(\frac{\boldsymbol{j}}{en})\}]$$

$$= en[\{1-\alpha(1-\delta)\}\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}\times\boldsymbol{B} - \frac{1-\delta}{enc}\boldsymbol{j}\times\boldsymbol{B} + \frac{1}{en}\nabla(P_e - \delta P_p) + \alpha\delta\frac{m_p}{e}\nabla\frac{GM}{R} - \frac{1}{\sigma}(\boldsymbol{j} - q\boldsymbol{V})], \tag{13}$$

where we have evaluated the momentum exchange rate as $\boldsymbol{F} = e^2 n_e n_p(\boldsymbol{V}_p - \boldsymbol{V}_e)/\sigma$ (σ represents the electric conductivity). The set of Eqs. (10), (11), (12) and (13) are equivalent to that of Eqs. (4a), (4b), (5a) and (5b).

As is well known, the mass ratio δ is very small and the neutrality parameter α is also much less than unity for a quasi-neutral plasma. This fact surves to simplify Eqs. (12) and (13). In the most part of this paper we neglect electron masses entirely (i.e. $\delta\to 0$) and charge separations (i.e. $\alpha\to 0$) except in the electromagnetic terms. Therefore, the terms come from $\alpha\boldsymbol{E}$ remain in Eqs. (14b) and (14g). The effects of finite δ are discussed in section 4. Further for simplicity, we hereafter restrict our problem only to a steady state (therefore, at least magnetic and rotational axes of the star must be coincide) and assume an infinite conductivity (i.e. $\sigma\to\infty$) and isothermal surrounding plasma. Thus simplified equations, together with Maxwell's equations and the equation of state, form the following closed set of equation;

$$\nabla\cdot(\rho\boldsymbol{V}) = 0, \tag{14a}$$

$$\rho(\boldsymbol{V}\cdot\nabla)\boldsymbol{V} = q\boldsymbol{E} + \frac{1}{c}\boldsymbol{j}\times\boldsymbol{B} - \nabla P + \rho\nabla\frac{GM}{R}, \tag{14b}$$

$$\nabla\times\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j}, \tag{14c}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{14d}$$

$$\nabla\times\boldsymbol{E} = 0, \tag{14e}$$

$$\nabla\cdot\boldsymbol{E} = 4\pi q, \tag{14f}$$

$$\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}\times\boldsymbol{B} = \frac{1}{e(\rho/m_p)}(q\boldsymbol{E} + \frac{1}{c}\boldsymbol{j}\times\boldsymbol{B} - \nabla P_e), \tag{14g}$$

$$P = f(\rho, T_0), \tag{14h}$$

where T_0 is the assumed constant temperature of the plasma. Eq. (14g) is the generalized Ohm's law and the first two terms of its right-hand side can be expressed in terms of the inertial term $\rho(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}$ by using Eq. (14b).

Here we non-dimensionalize the set of basic equations for the convenience of deriving their limiting forms in various extreme situations. To this end, the units of normalization for various physical quantities are introduced and some non-dimensional parameters are defined in terms of them. They are summarized in Tables I(a) and (b).

Table I(a). Introduced symbols

Physical quantities	Normalizing units	Non-dimensional quantities	Meanings
$\mathbf{R}$ (, ∇)	r_0	$\mathbf{r}$ (, ∇_r)	position vector (Laplacian)
$\mathbf{V}$	v_0	$\mathbf{u}$	velocity of fluid
ρ	ρ_0	y	mass density
P (, P_e, P_p)	p_0	p (, p_e, p_p)	total gass pressure (partial pressures of electrons and protons)
$\mathbf{B}$	B_0	$\mathbf{b}$	magnetic field
$\mathbf{E}$	E_0	$\mathbf{e}$	electric field
$\mathbf{j}$	j_0	$\mathbf{i}$	current density
q	q_0	η	charge density

Table I(b). Non-dimensional parameters

Concerned with the unit of velocity, v_0.		
$M_s^2 = \dfrac{v_0^2}{p_0/\rho_0}$	$K = \dfrac{cE_0/B_0}{v_0}$	
Concerned with the unit of current density, j_0.		
$I = \dfrac{j_0}{cB_0/4\pi r_0}$	$s = \dfrac{j_0}{e(\rho_0/m_p)v_0}$	
Ratios among energy densities		
$\beta = \dfrac{p_0}{B_0^2/8\pi}$	$g = \dfrac{GM/r_0}{2p_0/\rho_0}$	$h = \dfrac{B_0^2/8\pi}{\rho_0 c^2}$
Neutrality parameter		
$\xi = \dfrac{q_0}{e(\rho_0/m_p)}$		

Referring to these tables, we obtain the non-dimensional equations

$$\nabla_r \cdot (y\boldsymbol{u}) = 0, \tag{15a}$$

$$y(\boldsymbol{u} \cdot \nabla_r)\boldsymbol{u} = \frac{1}{M_s^2}\{\frac{2I}{\beta}\cdot\frac{K\xi}{s}\eta\boldsymbol{e} + \frac{2I}{\beta}\boldsymbol{i}\times\boldsymbol{b} - \nabla_r p + 2gy\nabla_r(\frac{1}{r})\}, \tag{15b}$$

$$\nabla_r \times \boldsymbol{b} = I\boldsymbol{i}, \tag{15c}$$

$$\nabla_r \cdot \boldsymbol{b} = 0, \tag{15d}$$

$$\nabla_r \times \boldsymbol{e} = 0, \tag{15e}$$

$$\nabla_r \cdot \boldsymbol{e} = \frac{\xi I}{M_s^2 K s h \beta}\eta, \tag{15f}$$

$$K\boldsymbol{e} + \boldsymbol{u}\times\boldsymbol{b} = \frac{\beta}{2I}s\frac{1}{y}(\frac{2I}{\beta}\cdot\frac{K\xi}{s}\eta\boldsymbol{e} + \frac{2I}{\beta}\boldsymbol{i}\times\boldsymbol{b} - \nabla_r P_e), \tag{15g}$$

$$p = f(y, T_0), \tag{15h}$$

where ∇_r represents the non-dimensional gradient operator. We have left all the normalizing units independent of each other, because there is a proper choice of the set of units (some relations among the units) for each extreme situation which we shall consider below.

§3. Consideration of Extreme Situations

In this section, we infer the behavior of the complete stellar-wind solution of Eqs. (15) by examining some artificial extreme situations in each of which some effect is dominant over other effects. As such situations, three limiting cases are taken up; the limits of 1) weak magnetic field, 2) strong magnetic field and 3) weak electromagnetic coupling. Though in the former two cases the effects of inertia are taken into account only through the perturbational method, the latter is a large inertia limit in itself and perturbation acts to reduce the inertial effects.

3-1. Limit of weak magnetic field

First, we consider the limit of weak magnetic field. In order to clarify the limiting procedure, we replace B_0 by $B_0\varepsilon$ and let $\varepsilon\to 0$ in this limit. Further, we must choose suitably a set of normalizing units to obtain the proper form of non-dimensional equations for this limit. The unit for pressure, p_0, is specified by

$$p_0 = \frac{\rho_0 k T_0}{m_p \mu_0} \tag{16}$$

where k is the Boltzman constant and $\mu_0=1/2$ is the mean molecular weight of the plasma. Therefore, the non-dimensional version of the ideal gas law becomes

$$p = y. \tag{17}$$

This specification is used also in other two extreme cases. Since in the absence of magnetic field a purely thermal wind is expected,[1] provided that the stellar rotation is negligible, we adopt the sound speed as the unit of velocity (i.e. $v_0=\sqrt{kT_0/m_p\mu_0}$ or $M_s=1$). In this limiting case the stretching of magnetic lines of force due to this wind results in the current density of the order of $j_0=cB_0\varepsilon/4\pi r_0$ (i.e. $I=1$), which therefore is the convenient unit for the current density. The unit for the electric field is suitably given by $E_0=kT_0/er_0\mu_0$, a typical value of the electric field to preserve the quasi-neutrality of plasma under the gravity of a star.[9] A typical value of the motional field $\boldsymbol{V}\times\boldsymbol{B}/c$ is not suitable for the unit since, if it is adopted, non-dimensional electric field $\boldsymbol{e}$ diverges in the limit of weak magnetic field. All other values of units are specified at a pole on the stellar surface (Table II(a)). For this set of units, the non-dimensional parameters are rewritten as listed in Table II(b).

Table II(a). Units for the limit of weak magnetic field

Dependent units	
$v_0 = \dfrac{\sqrt{kT_0}}{\sqrt{m_p\mu_0}}$	$j_0 = \dfrac{cB_0}{4\pi r_0}\varepsilon$
$E_0 = \dfrac{kT_0}{er_0\mu_0}$	$q_0 = \dfrac{E_0}{4\pi r_0}$
Independent units	
r_0 : radius of the star ρ_0 : density on the stellar surface T_0 : constant temperature of plasma	

Table II(b). Non-dimensional parameters

$M_s = 1$	$K = \sqrt{\dfrac{\xi}{2h_0}}\cdot\dfrac{1}{\varepsilon}$	
$I = 1$	$s = \dfrac{2}{\beta_0}\sqrt{\dfrac{\xi}{2h_0}}\,\varepsilon$	
$\beta = \beta_0\dfrac{1}{\varepsilon^2}, (\beta_0 = \dfrac{\rho_0 kT_0/m_p\mu_0}{B_0^2/8\pi})$	$h = h_0\varepsilon^2, (h_0 = \dfrac{B_0^2/8\pi}{\rho_0 c^2})$	$g = \dfrac{GM/r_0}{2p_0/\rho_0}$
$\xi = \dfrac{m_p}{4\pi(\rho_0/m_p)e^2}\cdot\dfrac{kT_0/m_p\mu_0}{r_0^2}$		

Making use of these units and non-dimensional parameters, we rewrite the non-dimensionalized basic Eqs. (15) as

$$\nabla\cdot(y\boldsymbol{u}) = 0, \tag{18a}$$

$$y(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla y + 2gy\nabla(\frac{1}{r}) + \xi\eta\boldsymbol{e} + \frac{2}{\beta_0}\varepsilon^2\boldsymbol{i}\times\boldsymbol{b}, \tag{18b}$$

$$\nabla\times\boldsymbol{b} = \boldsymbol{i}, \tag{18c}$$

$$\nabla\cdot\boldsymbol{b} = 0, \tag{18d}$$

$$\nabla\times\boldsymbol{e} = 0, \tag{18e}$$

$$\nabla\cdot\boldsymbol{e} = \eta, \tag{18f}$$

$$\boldsymbol{e} + \frac{\nabla y}{2y} - \xi\frac{\eta}{y}\boldsymbol{e} + \sqrt{\frac{2h_0}{\xi}}\,\varepsilon\boldsymbol{u}\times\boldsymbol{b} - \frac{2}{\beta_0}\varepsilon^2\frac{\boldsymbol{i}\times\boldsymbol{b}}{y} = 0, \tag{18g}$$

where subscript r in ∇_r has been omitted and the pressure p has been eliminated with the aid of Eq. (17). Of course these equations are exactly equivalent to Eqs. (15) as long as $\varepsilon=1$. In the following we neglect the term $\xi\eta e$, since ξ is very small in actual stellar winds (e.g. $\xi\sim 10^{-22}$ for solar parameters). Owing to this assumption we obtain from Eqs. (18e) and (18g) another subsidiary equation in the form

$$\nabla\times(\boldsymbol{u}\times\boldsymbol{b} - \frac{2}{\beta_0}\sqrt{\frac{\xi}{2h_0}}\,\varepsilon\frac{\boldsymbol{i}\times\boldsymbol{b}}{y}) = 0, \tag{18h}$$

which plays inportant roles in the following discussion.

In the limit of small ε, all physical quantities can be expressed by a first few terms of a power series in ε;

$$f = f_0 + f_1\varepsilon + f_2\varepsilon^2 + \cdots\cdots, \tag{19}$$

where f may be a scalar or any component of vectors. Substituting this type of expansions into Eqs. (18), we have the following equations in the lowest order approximation;

$$\nabla\cdot(y_0\boldsymbol{u}_0) = 0, \tag{20a}$$

$$y_0(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_0 = -\nabla y_0 + 2gy_0\nabla(\frac{1}{r}), \tag{20b}$$

$$\nabla\times\boldsymbol{b}_0 = \boldsymbol{i}_0, \tag{20c}$$

$$\nabla\cdot\boldsymbol{b}_0 = 0, \tag{20d}$$

$$\nabla\times\boldsymbol{e}_0 = 0, \tag{20e}$$

$$\nabla\cdot\boldsymbol{e}_0 = \eta_0, \tag{20f}$$

$$\boldsymbol{e}_0 + \frac{\nabla y_0}{2y_0} = 0, \tag{20g}$$

and

$$\nabla \times (\boldsymbol{u}_0 \times \boldsymbol{b}_0) = 0. \tag{20h}$$

Eqs. (20a) and (20b) show that, as expected, the plasma flows as a thermal wind freely from the electromagnetic force. The solution is given by the algebraic equations in the absence of stellar rotation;[10)]

$$\frac{1}{2}\{ u_0^2(r) - 1\} - 2 \ln (\frac{r}{g}) - \ln u_0(r) + 2(1 - \frac{g}{r}) = 0, \tag{21}$$

$$y_0(r) = \frac{u_0(1)}{u_0(r) r^2}, \tag{22}$$

where $u_0(r)$ is the radial component of $\boldsymbol{u}_0$ and the density on the stellar surface is unity (i.e. $y_0(1)=1$) due to the normalization we have adopted. The electric field is given by $\boldsymbol{e}_0 = -\nabla(\ln y_0(r)/2)$ and this satisfies Eq. (20e). Although the magnetic field is vanishingly small in the limit we are considering, the non-dimensionalized field $\boldsymbol{b}_0$, which remains finite, is calculated from Eq. (20h). Eq. (20h) implies that the magnetic field is frozen into the plasma. The correction due to the inertial term does not appear in the lowest order. Taking into account the boundary condition on the stellar surface, $\boldsymbol{b}_0(1) = (b_0 \cos\theta, 0, 0)$, we have

$$\boldsymbol{b}_0 = \begin{pmatrix} b_0 \cos\theta/r^2 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

in the spherical polar coordinate (r, θ, ϕ). Associated current density is calculated from Eq. (20c) as

$$\boldsymbol{i}_0 = \begin{pmatrix} 0 \\ 0 \\ b_0 \sin\theta/r^3 \end{pmatrix} \tag{24}$$

Thus in the lowest order approximation, the magnetic lines of force have a radially extending structure blown away by the thermal wind. This structure is maintained by a toroidal current distribution weakly concentrated to the equatorial plane.

In the next step of approximation we have the following first order equations;

$$\nabla\cdot(y_1 \boldsymbol{u}_0 + y_0 \boldsymbol{u}_1) = 0, \tag{25a}$$

$$y_0(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_1 + y_0(\boldsymbol{u}_1\cdot\nabla)\boldsymbol{u}_0 + y_1(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_0 = -\nabla y_1 + 2gy_1\nabla(\frac{1}{r}), \tag{25b}$$

$$\nabla \times b_1 = i_1, \tag{25c}$$

$$\nabla \cdot b_1 = 0, \tag{25d}$$

$$\nabla \times e_1 = 0, \tag{25e}$$

$$\nabla \cdot e_1 = \eta_1, \tag{25f}$$

$$e_1 + \frac{1}{2y_0}(\nabla y_1 - \frac{y_1}{y_0}\nabla y_0) + \sqrt{\frac{2h_0}{\xi}}\, u_0 \times b_0 = 0, \tag{25g}$$

$$\nabla \times (u_1 \times b_0 + u_0 \times b_1 - \frac{2}{\beta_0}\sqrt{\frac{\xi}{2h_0}}\, \frac{i_0 \times b_0}{y_0}) = 0. \tag{25h}$$

The solutions of Eqs. (25a), (25b), (25e), (25f) and (25g) are $y_1=0$, $u_1=0$, $e_1=0$ and $\eta_1=0$, because they have no inhomogeneous term. Substituting the lowest order solution into Eq. (25h), we have

$$b_1 = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{\beta_0}\sqrt{\frac{\xi}{2h_0}}\, \frac{b_0^2}{u_0(1)r^3}\{\frac{u_0(1)}{u_0(r)} r^2 - 1\} \sin\theta \cos\theta \end{pmatrix}, \tag{26}$$

which gives the current density, i_1, as

$$i_1 = \frac{2}{\beta_0}\sqrt{\frac{\xi}{2h_0}} \begin{pmatrix} \frac{b_0^2}{u_0(1)r^4}\{\frac{u_0(1)}{u_0(r)} r^2 - 1\}(3\cos^2\theta - 1) \\ \frac{b_0^2}{u_0(1)r^2}\{\frac{u_0(1)}{u_0(r)}\frac{r}{u_0(r)}\frac{du_0(r)}{dr} - \frac{2}{r^2}\} \sin\theta \cos\theta \\ 0 \end{pmatrix}. \tag{27}$$

The stream lines of the current density i_1 are shown in Fig. 1.

In the first order correction to the lowest order solution, there appear the toroidal magnetic field and the poloidal loop current while the flow is still radial in this approximation. Thus, the violation of flux-freezing is realized due to the contributions of the inertial term in Eq. (25h) (note that the force field $i_0 \times b_0 / y_0$ is not rotation-free). In general, the frozen-in condition is violated whenever the rotation of inertial force, $\nabla\times\{(u\cdot\nabla)u\}$, does not vanish. Therefore, Pneuman and Kopp's solution[4)] obtained under the assumption of the frozen-in condition can be self-consistent only when their velocity field satisfies $\nabla\times\{(u\cdot\nabla)u\}=0$. Since the inertial effects are expected to be stronger in the actual situations in which $\varepsilon=1$, the results obtained here suggest the violation of flux-freezing also in the solutions for actual cases.

Further, we consider the second order effects in ε. The set of equations

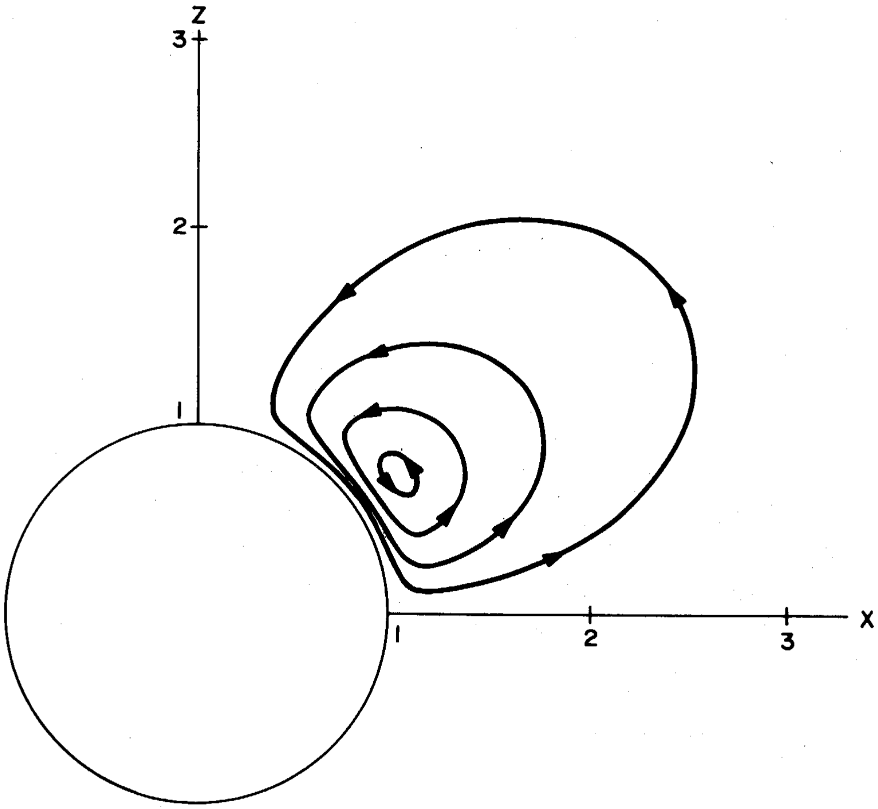


Fig. 1. Stream lines are shown for the poloidal current density i_1 calculated in the first order approximation. In the limit of weak magnetic field, the inertial term in the generalized Ohm's law, $i_0 \times b_0 / y_0$, is not rotation-free. This fact causes this poloidal loop current resulting in a weak violation of frozen-in condition.

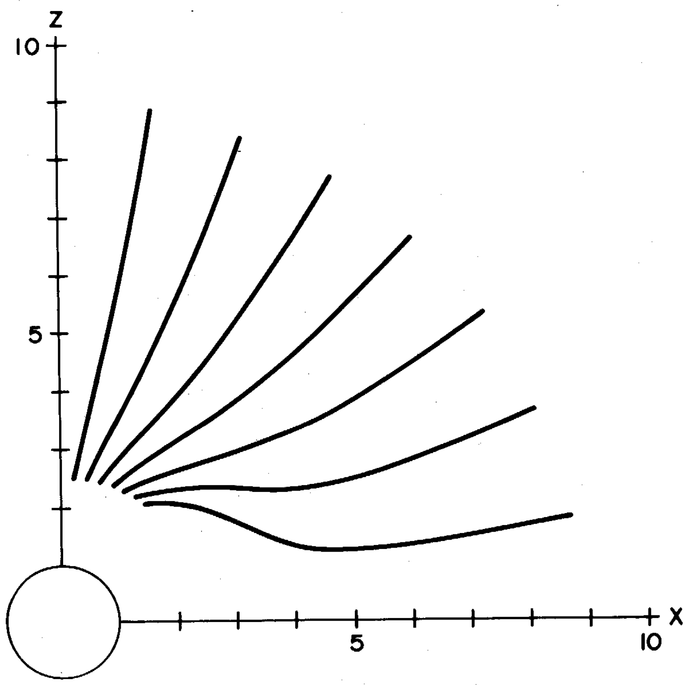


Fig. 2. Stream lines are shown for a thermal wind solution, $u_0 + u_2 \varepsilon$, obtained within the second order perturbation. The distance is normalized to the radius of a central star, and the values of non-dimensional parameters are fixed to $g=5$, $\beta_0=1$ and $\varepsilon=0.25$ for the solar case as a typical example. The second order effect causes the wind to bend toward the equatorial plane.

are obtained as

$$\nabla\cdot(y_0 \boldsymbol{u}_2 + y_2 \boldsymbol{u}_0) = 0, \tag{28a}$$

$$y_0(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_2 + y_0(\boldsymbol{u}_2\cdot\nabla)\boldsymbol{u}_0 + y_2(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_0 = -\nabla y_2 + 2gy_2\nabla(\frac{1}{r}) + \frac{2}{\beta_0}\boldsymbol{i}_0\times\boldsymbol{b}_0, \tag{28b}$$

$$\nabla\times\boldsymbol{b}_2 = \boldsymbol{i}_2, \tag{28c}$$

$$\nabla\cdot\boldsymbol{b}_2 = 0, \tag{28d}$$

$$\nabla\times\{\boldsymbol{u}_2\times\boldsymbol{b}_0 + \boldsymbol{u}_0\times\boldsymbol{b}_2 - \frac{1}{y_0}\frac{2}{\beta_0}\sqrt{\frac{\xi}{2h_0}}(\boldsymbol{i}_0\times\boldsymbol{b}_1 + \boldsymbol{i}_1\times\boldsymbol{b}_0)\} = 0, \tag{28e}$$

where we have omitted the equations for the electric field since they are completely decoupled from other equations. In order to calculate the second order corrections, we express the angular dependent parts of all physical quantities in terms of the Legendre polinomials and obtain a set of ordinary differential equations. A non-dimensional parameter appears in the set of equations;

$$L(r) = \frac{2}{3}\frac{\varepsilon^2 b_0^2}{\beta_0 u_0(1)g^2}\frac{y_0(r)}{y_0(g)}, \tag{29}$$

which represents the importance of the electromagnetic force compared with the pressure gradient force at a distance r. We must approximately specify the representative value of ε with the aid of Eq. (29), in order to infer the electromagnetic effects in actual situations from the perturbational solution up to the second order which is correct only for very small ε's. To this end, the Alfvén radius r_A is first evaluated from the dipole magnetic field and the lowest order solution for the plasma flow. Then, we find a suitable value of ε by setting $L(r_A)=1$. For the solar parameters, we have $g=5$, $\beta_0=1$ and $r_A=3.5$, and therefore have $\varepsilon=0.25$. The stream lines are shown in Fig. 2 for the value of ε thus evaluated. We can see in this picture that the second order effect lead the flow to bend toward the equational plane due to the electromagnetic force. Since we set $\beta_0=1$, the situation is no longer the weak magnetic-field case in the region near the surface. Therefore, the accuracy of approximation is not so good there, but is fairly good outside the Alfvén surface $r_A<r$.

3-2. Limit of strong magnetic field

Next we examine the behavior of solutions in the limit of strong magnetic field. This time B_0 is replaced by B_0/ε and the limit is taken by making $\varepsilon\to 0$. In this situation, the magnetic field is expected to have a current-free configuration almost suppressing the plasma motion across its lines of force, provided that there is no mechanism in the stellar interior which drives a non-negligible field-aligned current. Neglecting the inertial and gravitational terms, we have, from the equation of motion and the generalized Ohm's law, the order-of-magnitude estimations as

$$\nabla P \sim \frac{1}{c} \mathbf{j} \times \mathbf{B} , \tag{30}$$

$$\mathbf{E} \sim \frac{1}{c} \mathbf{V} \times \mathbf{B} \sim \frac{m_p}{\rho_e} \nabla P_p , \tag{31}$$

which can be used to arrange the suitable units for this limit as shown in Table III(a). In this choice of the units the fluid velocity and current density, which are both expected to be very small in this limit, have finite normalized values because their units, v_0 and j_0, also approach to zero as $\varepsilon \to 0$. Making use of the renewed units we again rewrite the non-dimensional parameters (Table III(b)).

Table III(a). Units for the limit of strong magnetic field

Dependent units	
$v_0 = \dfrac{kT_0 c}{e r_0 \mu_0 B_0} \varepsilon$	$j_0 = \dfrac{c \rho_0 k T_0}{r_0 B_0 m_p \mu_0} \varepsilon$
$E_0 = \dfrac{kT_0}{e r_0 \mu_0}$	$q_0 = \dfrac{E_0}{4\pi r_0}$
Independent units	
r_0: radius of the star ρ_0: density on the stellar surface T_0: constant temperature of plasma	

Table III(b). Non-dimensional parameters

$M_s^2 = \dfrac{\xi}{2h_0} \varepsilon^2$	$K = 1$	
$I = \dfrac{\beta_0}{2} \varepsilon^2$	$s = 1$	
$\beta = \beta_0 \varepsilon^2$, $\left(\beta_0 = \dfrac{\rho_0 k T_0 / m_p \mu_0}{B_0^2/8\pi}\right)$	$h = \dfrac{h_0}{\varepsilon^2}$, $\left(h_0 = \dfrac{B_0^2/8\pi}{\rho_0 c^2}\right)$	$g = \dfrac{GM/r_0}{2p_0/\rho_0}$
$\xi = \dfrac{m_p}{4\pi(\rho_0/m_p)e^2} \cdot \dfrac{kT_0/m_p\mu_0}{r_0^2}$		

The non-dimensional equations becomes for this choice of the units

$$\nabla \cdot (y\mathbf{u}) = 0, \tag{32a}$$

$$\frac{\xi \varepsilon^2}{2h_0} y(\mathbf{u} \cdot \nabla)\mathbf{u} = \xi\eta\mathbf{e} + \mathbf{i} \times \mathbf{b} - \nabla y + 2gy\nabla(\frac{1}{r}), \tag{32b}$$

$$\nabla \times \mathbf{b} = \frac{\beta_0}{2} \varepsilon^2 \mathbf{i}, \tag{32c}$$

$$\nabla \cdot \mathbf{b} = 0, \tag{32d}$$

$$\nabla \times \mathbf{e} = 0, \tag{32e}$$

$$\nabla \cdot \mathbf{e} = \eta, \tag{32f}$$

$$\mathbf{e} + \mathbf{u} \times \mathbf{b} - \frac{1}{y}(\xi\eta\mathbf{e} + \mathbf{i} \times \mathbf{b} - \frac{1}{2}\nabla y) = 0, \tag{32g}$$

$$\nabla \times \{\mathbf{u} \times \mathbf{b} - \frac{\xi}{2h_0}\varepsilon^2 (\mathbf{u} \cdot \nabla)\mathbf{u}\} = 0. \tag{32h}$$

With the aid of expansion formula (19), we obtain the lowest order equations as

$$\nabla \cdot (y_0\mathbf{u}_0) = 0, \tag{33a}$$

$$\xi\eta_0\mathbf{e}_0 + \mathbf{i}_0 \times \mathbf{b}_0 - \nabla y_0 + 2gy_0\nabla(\frac{1}{r}) = 0, \tag{33b}$$

$$\nabla \times \mathbf{b}_0 = 0, \tag{33c}$$

$$\nabla \cdot \mathbf{b}_0 = 0, \tag{33d}$$

$$\nabla \times \mathbf{e}_0 = 0, \tag{33e}$$

$$\nabla \cdot \mathbf{e}_0 = \eta_0, \tag{33f}$$

$$\mathbf{e}_0 + \mathbf{u}_0 \times \mathbf{b}_0 - \frac{1}{y_0}(\xi\eta_0\mathbf{e}_0 + \mathbf{i}_0 \times \mathbf{b}_0 - \frac{1}{2}\nabla y_0) = 0, \tag{33g}$$

$$\nabla \times (\mathbf{u}_0 \times \mathbf{b}_0) = 0. \tag{33h}$$

At first sight, it may appear somewhat curious that Eqs. (33b) and (33g) contain the current density $\mathbf{i}_0$ while Eq. (33c) insists on a current-free field. However, this means merely that the current density is too small to affect the magnetic field structure in the lowest order approximation. Nevertheless, it can contribute to the balancing of the small terms in Eqs. (33b) and (33g), which give the functional form of the vanishingly small current density and velocity.

As a current-free magnetic field well representing the stellar magnetization, we adopt the dipole field;

$$\boldsymbol{b}_0 = \begin{pmatrix} b_0 \cos\theta/r^3 \\ b_0 \sin\theta/2r^3 \\ 0 \end{pmatrix}. \tag{34}$$

This satisfies the same boundary condition for b_{0r} (i.e. $b_{0r}(1)=b_0 \cos\theta$) as in the previous subsection. From Eq. (33h) we obtain the velocity field as

$$\boldsymbol{u}_{0p} \,/\!/\, \boldsymbol{b}_0 , \tag{35}$$

$$u_{0\phi} \mp u_{eq} r^{a/2} \sin\theta^{3-a}, \tag{36}$$

where $\boldsymbol{u}_{0p}$ and $u_{0\phi}$ are the poloidal and toroidal components of $\boldsymbol{u}_0$, respectively, and u_{eq} and a are constants to be determined at the stellar surface. Eq. (36) implies that if the stellar surface rotate rigidly (i.e. $a=2$), the surrounding plasma corotates with the star in the whole space. This type of solution is allowed since the inertial effects of the plasma do not appear in the lowest order approximation. Further, if we neglect the pressure and gravity terms, the equation of motion (33b) and the generalized Ohm's law (33g) reduce to

$$\boldsymbol{i}_0 \times \boldsymbol{b}_0 + \xi\eta_0\boldsymbol{e}_0 = 0, \tag{37}$$

and

$$\boldsymbol{e}_0 + \boldsymbol{u}_0 \times \boldsymbol{b}_0 = 0, \tag{38}$$

respectively. Combining these equations, we obtain the current density

$$\boldsymbol{i}_0 = \xi\eta_0\boldsymbol{u}_0 + J\boldsymbol{b}_0, \tag{39}$$

where J is a scalar function satisfying $(\boldsymbol{b}_0\cdot\nabla)J=0$. This situation have been examined by many authors as the force-free approximation.[11] It must be noted, however, that in general we should take the effects of pressure and gravity terms into account in determining the current density (see Eqs. (33b) and (33g)), even if in the limit of strong magnetic field. If we take the gravity and pressure terms into account, y_0, $\boldsymbol{i}_0$, $\boldsymbol{e}_0$ and η_0 cannot be determined uniquely due to the lack of the knowledge about the outer boundary condition and the decoupling of $\boldsymbol{b}_0$ and $\boldsymbol{i}_0$ in Eq. (33c).

Coupling among the magnetic field, the current density and the velocity field first appears in the second order perturbation. The second order equations are

$$\nabla \cdot (y_0\boldsymbol{u}_2 + y_2\boldsymbol{u}_0) = 0, \tag{40a}$$

$$\frac{\xi}{2h_0} y_0(\boldsymbol{u}_0\cdot\nabla)\boldsymbol{u}_0 = \xi(\eta_2\boldsymbol{e}_0 + \eta_0\boldsymbol{e}_2) + (\boldsymbol{i}_0 \times \boldsymbol{b}_2 + \boldsymbol{i}_2 \times \boldsymbol{b}_0) - \nabla y_2 + 2gy_2\nabla(\frac{1}{r}), \tag{40b}$$

$$\nabla \times \boldsymbol{b}_2 = \frac{\beta_0}{2}\boldsymbol{i}_0, \tag{40c}$$

$$\nabla \cdot b_2 = 0, \tag{40d}$$

$$\nabla \times e_2 = 0, \tag{40e}$$

$$\nabla \cdot e_2 = \eta_2, \tag{40f}$$

$$e_2 + (u_0 \times b_2 + u_2 \times b_0) = \frac{1}{y_0}(i_0 \times b_2 + i_2 \times b_0 - \frac{1}{2}\nabla y_2 + \xi\eta_2 e_0 + \xi\eta_0 e_2)$$
$$- \frac{y_2}{y_0^2}(i_0 \times b_0 - \frac{1}{2}\nabla y_0 + \xi\eta_0 e_0), \tag{40g}$$

$$\nabla \times \{u_0 \times b_2 + u_2 \times b_0 - \frac{\xi}{2h_0}(u_0 \cdot \nabla)u_0\} = 0. \tag{40h}$$

If we assume the current-free magnetic field and hydrostatic equilibrium as a lowest order solution (i.e. $u_0=0$, $i_0=0$, $e_0=0$ and $\eta_0=0$), the inhomogeneous terms in the second order equations all vanish. Therefore the second order corrections do not appear. We can show that this lowest-order solution is in fact an exact solutions for the original equations (i.e. $\varepsilon=1$). This fact corresponds to the existence of discontinuity between the dead zone and wind zone, which can be seen in Pneuman and Kopp's model.[4] Even if we introduce a toroidal flow reflecting the stellar rotation, the inhomogeneous term in Eq. (40h) also vanish because the centrifugal force due to corotation is rotation-free. Therefore, the corotation of plasma does not remove the discontinuity between the dead and wind zones. The self-consistent solutions in this stage of approximation (in the actual situation, i.e. $\varepsilon=1$) have obtained by Hill and Carbary[12] and Kaburaki[8] on somewhat different physical basis.

We can show in the following, an example in which the violation of flux-freezing actually occur and therefore the discontinuity between the dead and wind zones is removed. To this end, we introduce the field-aligned flow as well as the corotational velocity;

$$u_0 = \begin{pmatrix} w_0 b_0 \cos\theta / r^3 y_0(r) \\ w_0 b_0 \sin\theta / 2r^3 y_0(r) \\ u_{eq}\, r \sin\theta \end{pmatrix}, \tag{41a}$$

where w_0 is a constant. The magnetic field, plasma density and current density are assumed to be

$$b_0 = \begin{pmatrix} b_0 \cos\theta / r^3 \\ b_0 \sin\theta / 2r^3 \\ 0 \end{pmatrix}, \tag{41b}$$

$$y_0(r) = \exp[2g(\frac{1}{r} - 1)], \tag{41c}$$

$$\boldsymbol{i}_0 = 0, \tag{41d}$$

respectively, where we have ignored the convection current for simplicity. The second order corrections to the above lowest-order solutions are

$$u_{2\phi} = -\frac{\xi}{2h_0} w_0^2 b_0 g \frac{\exp[-2g(\frac{1}{r} - 1)]}{r^7} \sin\theta, \tag{42a}$$

$$u_{2r}\frac{b_0 \sin\theta}{2r^3} - u_{2\theta}\frac{b_0 \cos\theta}{r^3} = \frac{3}{2}\frac{\xi}{h_0}\frac{w_0^2 b_0^2 u_{eq}}{y_0(r) r^3} \sin\theta\cos\theta, \tag{42b}$$

$$\boldsymbol{i}_{2p} = \frac{\boldsymbol{u}_{2p}}{y_0(r)}, \tag{42c}$$

where u_{2r}, $u_{2\theta}$ and $u_{2\phi}$ are the components of $\boldsymbol{u}_2$ in the spherical polar coordinate and $\boldsymbol{u}_{2p}$ and $\boldsymbol{i}_{2p}$ are the poloidal components of $\boldsymbol{u}_2$ and $\boldsymbol{i}_2$, respectively. Thus, the flow across the magnetic field appears as a drift motion caused by the toroidal component of the inertial term.

We can also obtain a similar flow pattern by introducing a field-aligned current and taking account of the effects of finite electron masses.

3-3. Limit of weak electromagnetic coupling

In this last subsection we consider the limit of weak electromagnetic coupling. Since the strength of the electromagnetic coupling between charged particles and field is prescribed by the electric charge unit e, we replace e by εe and let $\varepsilon\to 0$ in order to realize this extreme situation. In this limit the stellar plasma behaves like a neutral gas and blows as a thermal wind unaffected by the presence of the magnetic field, which therefore has a current-free configuration. Therefore, the suitable unit for velocity is the thermal velocity as in the case of weak magnetic field (i.e. $v_0=\sqrt{kT_0/\mu_0 m_p}$). Since the current, which arises as a result of electromagnetic interaction between the magnetic field and the plasma flow, is expected to be very small, we use the unit $j_0=\varepsilon e n v_0$, which is also very small in this limit.

For the case of a thermal wind, decoupling between the plasma and electromagnetic field generally takes place in two steps as ε tends to zero; (1) Decoupling between plasma and magnetic field. When ε decreases to reach $\varepsilon\sim\sqrt{\xi_0/2h}$ ($\sim 10^{-8}$ for the solar parameters), where ξ_0 and h are given in Table IV(a), the inertial effects gradually increase and the violation of flux-freezing becomes cruicial. However, in this step the electric field which tends to preserve the charge neutrality of the plasma still works well. (2) Decoupling between two species of plasma particles. When ε reaches

$\varepsilon \sim \sqrt{\xi_0}$ ($\sim 10^{-11}$), protons and electrons come to behave as two independent species of a neutral gas with different masses. Since our interest is concentrated on the first step, the suitable unit for the electric field is given by $E_0 = kT_0/e\mu_0 r_0 \varepsilon$ and subsequently $q_0 = E_0/4\pi r_0$. All the units in this limit are listed in Table IV (a). Then we can rewrite the non-dimensional parameters (Table IV (b)) and the set of equations.

Table IV(a). Units for the limit of weak electromagnetic coupling

Dependent units	
$v_0 = \dfrac{\sqrt{kT_0}}{\sqrt{m_p \mu_0}}$	$j_0 = \dfrac{e\rho_0 v_0}{m_p} \varepsilon$
$E_0 = \dfrac{kT_0}{e\mu_0 r_0} \dfrac{1}{\varepsilon}$	$q_0 = \dfrac{E_0}{4\pi r_0}$
Independent units	
r_0 : radius of the star	
ρ_0 : density on the stellar surface	
T_0 : constant temperature of plasma	

Table IV(b). Non-dimensional parameters

$M_s^2 = 1$	$K = \sqrt{\dfrac{\xi_0}{2h}} \dfrac{1}{\varepsilon}$	
$I = \dfrac{\beta}{2} \sqrt{\dfrac{2h}{\xi_0}}$	$s = 1$	
$\beta = \dfrac{\rho_0 kT_0/m_p \mu_0}{B_0^2/8\pi}$	$h = \dfrac{B_0^2/8\pi}{\rho_0 c^2}$	$g = \dfrac{GM/r_0}{2P_0/\rho_0}$
$\xi = \dfrac{\xi_0}{\varepsilon^2}$, $\left(\xi_0 = \dfrac{m_p kT_0}{4\pi\rho_0 e^2 r_0^2 \mu_0}\right)$		

Eqs. (15) become

$$\nabla \cdot (y\boldsymbol{u}) = 0, \tag{43a}$$

$$y(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla y + 2gy\nabla\left(\frac{1}{r}\right) + \sqrt{\frac{2h}{\xi_0}}\, \varepsilon \boldsymbol{i} \times \boldsymbol{b} + \frac{\xi_0}{\varepsilon^2} \eta \boldsymbol{e}, \tag{43b}$$

$$\nabla \times \boldsymbol{b} = \frac{\beta}{2}\sqrt{\frac{2h}{\xi_0}}\ \varepsilon \boldsymbol{i}, \tag{43c}$$

$$\nabla \cdot \boldsymbol{b} = 0, \tag{43d}$$

$$\nabla \times \boldsymbol{e} = 0, \tag{43e}$$

$$\nabla \cdot \boldsymbol{e} = \eta, \tag{43f}$$

$$\boldsymbol{e} + \frac{\nabla y}{2y} + \sqrt{\frac{2h}{\xi_0}}\ \varepsilon\left(\boldsymbol{u} \times \boldsymbol{b} - \frac{\boldsymbol{i} \times \boldsymbol{b}}{y} + \sqrt{\frac{\xi_0}{2h}}\ \frac{\xi_0}{\varepsilon^3}\ \eta\, \boldsymbol{e}\right) = 0, \tag{43g}$$

$$\nabla \times \left\{\sqrt{\frac{2h}{\xi_0}}\, \varepsilon\, \boldsymbol{u} \times \boldsymbol{b} - (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right\} = 0. \tag{43h}$$

Eq. (43h) shows that if ε becomes as small as $\sqrt{\xi_0/2h}$, the frozen-in situation breaks down.

We have in the lowest order approximation

$$\nabla \cdot (y_0 \boldsymbol{u}_0) = 0, \tag{44a}$$

$$y_0(\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u}_0 = -\nabla y_0 + 2g y_0 \nabla\left(\frac{1}{r}\right), \tag{44b}$$

$$\nabla \times \boldsymbol{b}_0 = 0, \tag{44c}$$

$$\nabla \cdot \boldsymbol{b}_0 = 0, \tag{44d}$$

$$\nabla \times \boldsymbol{e}_0 = 0, \tag{44e}$$

$$\nabla \cdot \boldsymbol{e}_0 = \eta_0, \tag{44f}$$

$$\boldsymbol{e}_0 + \frac{\nabla y_0}{2y_0} = 0, \tag{44g}$$

$$\nabla \times \left(\boldsymbol{u}_0 \times \boldsymbol{b}_0 - \frac{\boldsymbol{i}_0 \times \boldsymbol{b}_0}{y_0}\right) = 0, \tag{44h}$$

by assuming $\sqrt{\xi_0} \ll \varepsilon \ll \sqrt{\xi_0/2h}$. It must be emphasized that the inertial terms remain in the lowest order equations in contrast with previous two limits. This limit can also be called the large inertia limit since the ratio of the inertial term to the electromagnetic one, which include the factor m_p/e, becomes very large in this limit. We can see in this large inertia limit, that the radial thermal wind, Eq. (21), and the dipole magnetic field, Eq. (34), are indeed compatible with each other as the lowest order solutions. Using these solutions, we obtain from Eq. (44h) the lowest order current density

$$\boldsymbol{i}_{0p} = y_0 \boldsymbol{u}_0 + J\boldsymbol{b}_0, \tag{45a}$$

$$i_{0\phi} = 0, \tag{45b}$$

where J is a scalar function which satisfies $(\boldsymbol{b}_0 \cdot \nabla) J = 0$. Thanks to the charge conservation, we have from Eqs. (22), (34) and (45a)

$$\boldsymbol{i}_0 = \begin{pmatrix} \frac{u_0(1)}{r^2}\{1 - \frac{\delta(\theta) + \delta(\theta - \pi)}{\sin\theta}\} \\ 0 \\ 0 \end{pmatrix} \tag{46}$$

where $\delta(\theta)$ is the Dirac δ-function. Positive charges, which are lost by the radial outflow of protons across the magnetic lines of force in all directions, are compensated by the field-aligned line current carried by electrons emitted from the two poles of the star. Although this field-aligned current has a diverging strengh, this is merely a consequence of our artificial limiting procedure $\varepsilon \to 0$.

The first order equations are

$$\nabla \cdot (y_0 \boldsymbol{u}_1 + y_1 \boldsymbol{u}_0) = 0, \tag{47a}$$

$$y_0(\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u}_1 + y_0(\boldsymbol{u}_1 \cdot \nabla)\boldsymbol{u}_0 + y_1(\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u}_0 = -\nabla y_1 + 2gy_1\nabla(\frac{1}{r}) + \sqrt{\frac{2h}{\xi_0}}\, \boldsymbol{i}_0 \times \boldsymbol{b}_0, \tag{47b}$$

$$\nabla \times \boldsymbol{b}_1 = \frac{\beta_0}{2}\sqrt{\frac{2h}{\xi_0}}\, \boldsymbol{i}_0, \tag{47c}$$

$$\nabla \cdot \boldsymbol{b}_1 = 0, \tag{47d}$$

$$\nabla \times \{(\boldsymbol{u}_0 - \frac{\boldsymbol{i}_0}{y_0}) \times \boldsymbol{b}_1 + (\boldsymbol{u}_1 - \frac{\boldsymbol{i}_1}{y_0}) \times \boldsymbol{b}_0 + \frac{y_1}{y_0}\frac{\boldsymbol{i}_0 \times \boldsymbol{b}_0}{y_0}\} = 0. \tag{47e}$$

The toroidal component of $\boldsymbol{i}_0 \times \boldsymbol{b}_0$ drives the toroidal flow

$$u_{1\phi} = \sqrt{\frac{2h}{\xi_0}}\, \frac{b_0}{r}(1 - \frac{1}{r}) \sin\theta, \tag{48a}$$

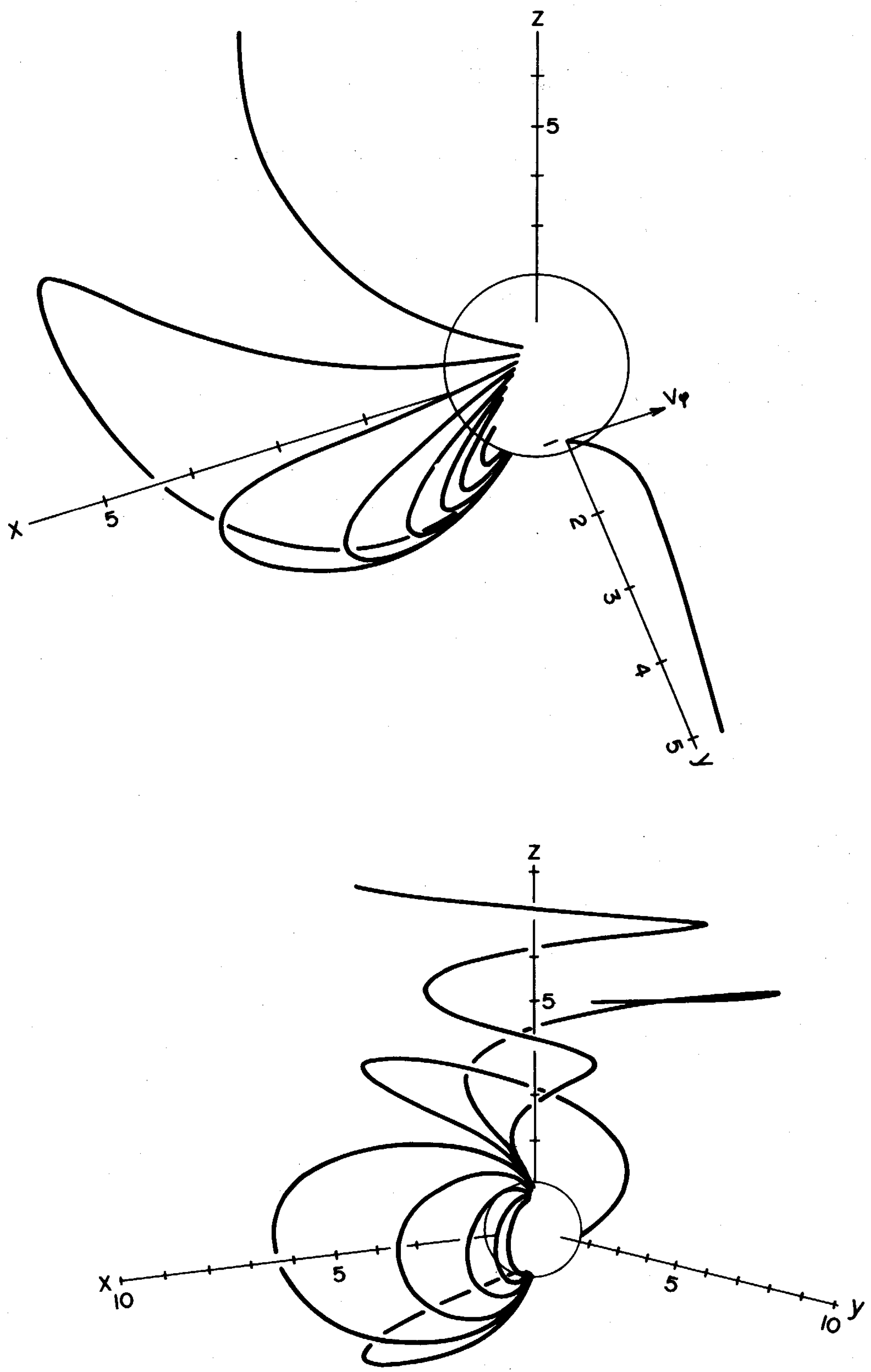
and for other components of the velocity $\boldsymbol{u}_1$ and the density y_1 we have

$$u_{1r} = u_{1\theta} = 0, \quad y_1 = 0 \,. \tag{48b}$$

The magnetic field is modified by $\boldsymbol{i}_0$ as

$$b_{1\phi} = \begin{cases} 0 & : \theta = 0 \\ -\frac{\beta_0}{2}\sqrt{\frac{2h}{\xi_0}}\, u_0(1)\frac{\cos\theta}{r\sin\theta} & : 0 < \theta < \pi \\ 0 & : \theta = \pi \end{cases}, \tag{49a}$$

$$b_{1r} = b_{1\theta} = 0 \,. \tag{49b}$$



(a)

(b)

Fig. 3-a and b. Magnetic field structure for a case of weak electromagnetic interaction. Field lines are calculated within the first order approximation (i.e. $b_0+b_1\varepsilon$) by setting arbitrarily $\varepsilon(\beta_0/2)\sqrt{2h/\xi_0}u_0(1)=0.05$ (i.e. $\varepsilon=1.68\times10^{-7}$ for solar parameters and therefore the effect of inertia is very exaggerated). The first-order toroidal velocity, u_1, on the equatorial plane is also shown graphically.

This magnetic field requires the poloidal surface current;

$$i_{surf} = \frac{u_0(1)\cos\theta}{\sin\theta}, \tag{50}$$

which closes the electric current circuit on the inner boundary.

The magnetic field configuration, $b_0+b_1\varepsilon$, together with toroidal velocity $u_{1\phi}$ on the equatorial plane are shown in Fig. 3. It is quite interesting that this result have some resemblances to the numerical calculation of Kuo-Petravic et al.[5] As will be shown in section 4, the limiting process, $\varepsilon\dot{e}\to 0$, magnifies the equatorial current region (region II in Fig. 4) which in most actual cases can be regarded as a thin sheet, and as a result the polar region (region I in Fig. 4) converges to the polar axis. In this situation, the common features in our results to that of Kuo-Petravic et al.[5] are as follows; (1) in the euatorial region, protons flow outward across the closed magnetic lines of force, (2) corresponding to this, electrons are emitted from the polar region, (3) quantitative behaviours of the torqidal flow $u_{1\phi}$ in both cases are similar to each other and (4) magnetic field lines also have similar configurations in the equatorial region. We can expect that these features appear when the frozen-in condition is violated due to the inertial effects. The greatest difference between our situation and that of Kuo-Petravic is in the space-charge distribution; though the plasma is charge-separated in Kuo-Petravic's case, electrons are distributed everywhere in our case so as almost to compensate the positive charges of flowing protons.

§4. Discussion and Conclusion

We first discuss about the effects of the electron masses. The violation of flux-freezing dealt with in the above discussion is only for protons. Electrons are still frozen into the magnetic flux, since their masses are neglected compared with proton masses (i.e. $\delta\to 0$). If we suppose that $\delta=1$, however, the inertial term $i\times b+\xi\eta e$ in the generalized Ohm's law disappears and instead the term $\nabla\cdot(ui+iu)$ becomes important. This term may play an important role in the electron-positron plasma, for example, and in the equatorial current sheet where the scale length expected to be very small. In order to examine the effects of this term in the electron-proton plasma (in which $\delta<<1$), we must take the finiteness of electron masses into account at least up to the first order.

The lowest order equations in the limit of weak electromagnetic coupling with the effects of finite δ are

$$\nabla\cdot(y_0 u_0) = 0, \tag{51a}$$

$$y_0(u_0\cdot\nabla)u_0 + \delta(i_0\cdot\nabla)\frac{i_0}{y_0} = -\nabla y_0 + 2gy_0\nabla(\frac{1}{r}), \tag{51b}$$

$$\nabla \times \boldsymbol{b}_0 = 0, \tag{51c}$$

$$\nabla \cdot \boldsymbol{b}_0 = 0, \tag{51d}$$

$$\nabla \times \boldsymbol{e}_0 = 0, \tag{51e}$$

$$\nabla \cdot \boldsymbol{e}_0 = \eta_0, \tag{51f}$$

$$\boldsymbol{e}_0 + \frac{1-\delta}{2y_0}\nabla y_0 - \delta\{\nabla \cdot (\boldsymbol{u}_0\boldsymbol{i}_0 + \boldsymbol{i}_0\boldsymbol{u}_0 - \frac{\boldsymbol{i}_0\boldsymbol{i}_0}{y_0})\} = 0. \tag{51g}$$

In this case we can choose $\boldsymbol{i}_0=0$ instead of $\boldsymbol{i}_0=y_0\boldsymbol{u}_0$, for the same choice of the lowest order solutions for the plasma flow and the magnetic field as in section 3-3. In other words, quasi-neutral plasma can flow across the field lines due to the inclusion of electron inertia as well as proton inertia.

The first order correction to the current density is shown to have only ϕ-component which is given by

$$i_{1\phi} = \frac{1}{\delta}\sqrt{\frac{2h}{\xi_0}}\ \frac{u_0(1)}{2u_0(r)}\ \frac{1}{r^3}\ (1 - \frac{1}{r})\ \sin\theta. \tag{52}$$

This means that the ϕ-component of the motional field, $[\boldsymbol{V}\times\boldsymbol{B}/c]_\phi$, in the generalized Ohm's law is balanced by that of the inertial term which serves as the effective electric field, i.e.,

$$[\boldsymbol{E}_{eff}]_\phi = -\frac{m_e m_p}{\rho e^2}[\nabla \cdot (\boldsymbol{V}\boldsymbol{j} + \boldsymbol{j}\boldsymbol{V} - \frac{m_p}{\rho e}\boldsymbol{j}\boldsymbol{j})]_\phi. \tag{53}$$

This fact gives the rough estimate of the ratio between the poloidal components in a closed magnetic field structure near the equatorial plane,

$$\frac{B_\theta}{B_r} \sim \frac{(c/L_r)(c/L_c)}{\omega_p^2}, \tag{54}$$

where L_c is the thickness of the current sheet, L_r is the characteristic length in the radial direction and ω_p is the electron plasma frequency. Assuming $B_\theta/B_r \sim L_c/L_r$, we have from Eq. (54)

$$\frac{L_c}{L_r} \sim \frac{c/L_r}{\omega_p}. \tag{55}$$

We can see from this relation that the thickness of the current sheet is magnified extremely (i.e. $L_c/L_r \to \infty$) in the limit of weak electromagnetic coupling, $\varepsilon e \to 0$, although in actual thermal winds the thickness seems to be very small. From this consideration it is understood naturally that, since Kuo-Petravic et al.[5] have introduced artificially large rest masses for plasma particles, the current sheet is largely magnified in their calculation. Also in a relativistic region the current sheet is expected to expand owing to the large inertia as shown by them.

As a summary, we synthesize in Fig. 4 the results for the three extreme situations to infer the global structure of actual stellar magnetospheres including the inertial effects of the stellar winds. There are three typical regions in this figure corresponding to the three extreme situations.

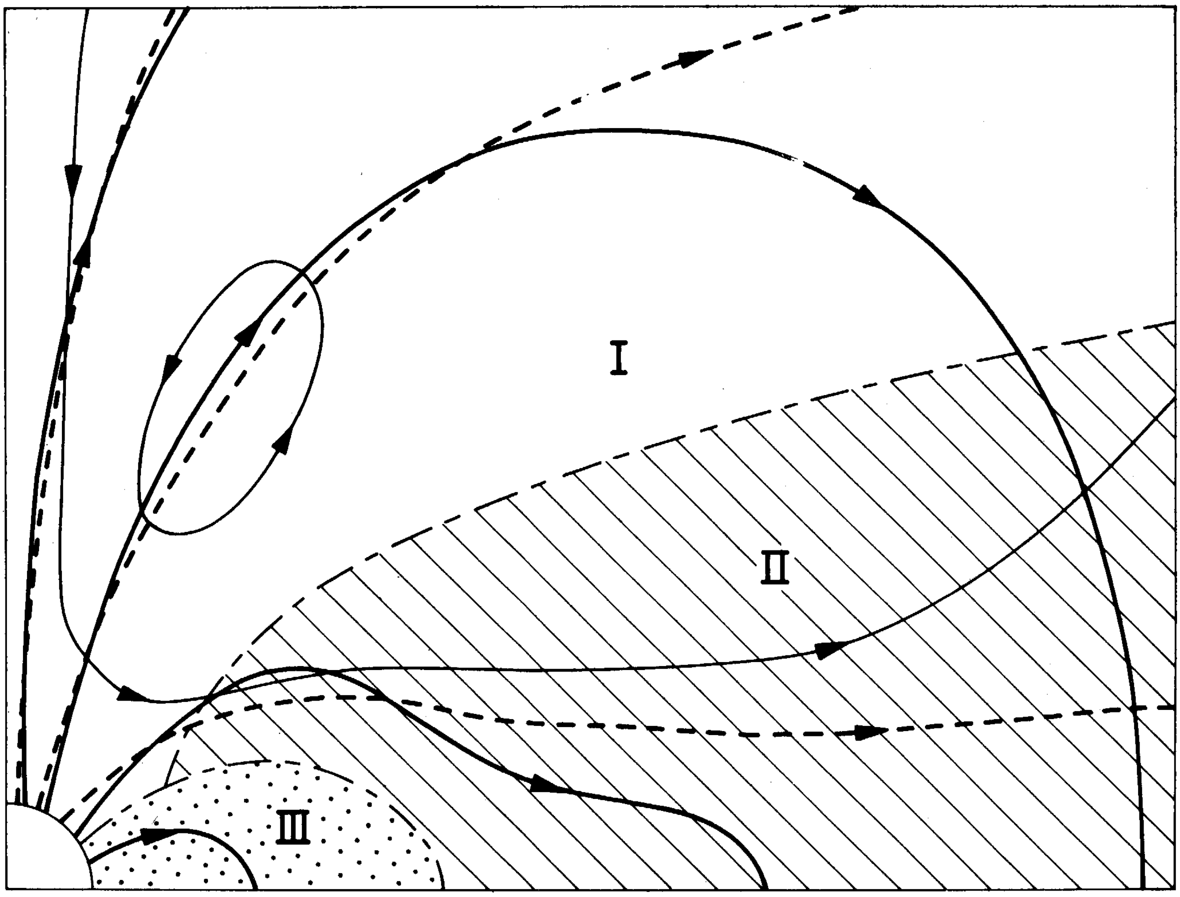


Fig. 4. A schematic picture of the global structure of stellar magnetospheres (drawn only in a poloidal plane). The thick solid curves represent magnetic lines of force, the dashed curves the stream lines of plasma flow and the thin solid curves the stream lines of the current density. This picture is synthesized from the considerations in the three extreme situations and has correspondingly three regions denoted by I, II (hatched region) and III (dotted region).

In region I, where the solutions are well represented by those obtained in the limit of weak magnetic field, the frozen-in condition is almost satisfied and the stellar wind is bent toward the equatorial plane due to the interaction between the flow and the magnetic field (see also Fig. 2). In this region, unless the rotation of the inertial force vanishes, the poloidal loop current is produced associated with a little violation of flux-freezing. However, no accurate calculation can presently be done for this current density.

In region II, the violation of flux-freezing is serious. The wind of protons blows across the closed field lines resulting in the convection current which is smoothly connected with the loop current in region I. This type of solutions are obtained in the limit of weak electromagnetic coupling by neglecting the electron mass entirely. Since $L_c/L_r \to \infty$ as $\varepsilon e \to 0$, region II occupies in this limit the whole space except the polar axis to which region I converges. If we include also the electron mass, a quasi-neutral plasma can flow across the closed magnetic lines of force. In the magnetospheres with powerful thermal wind like the heliomagnetosphere, region I occupies the most part of them and region II is pressed to a thin current sheet. Nevertheless according to our picture, all field lines in region I are closed in region II however elongated they may be. Moreover, since $L_c/L_r \sim r$ provided that $\rho \sim r^{-2}$, region II becomes dominant over region I at large distances from the star. However, this distance is about 1 pc for solar parameters.

Region III forms a so-called dead zone or a corotation zone. In this region, the effect of inertia is not so important and the magnetic field is almost the dipole one. The plasma almost corotate with a central star, although, due to the inertial effect, there is a small poloidal flow across the lines of force in the outer part of this region. Therefore, the boundaries among these three regions are not the sharp ones but vague ones. All physical quantities vary smoothly across them.

To conclude, we have a magnetospheric model in which the magnetic lines of force take a closed structure as a whole and the stellar wind blows across the lines of force owing to the inertial effects of the plasma more or less stretching them and forming a current sheet at about the equatorial plane.

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