Fracturing in the Solid Earth

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Fracturing in the Solid Earth

Hiroyuki Nagahama*

ABSTRACT

In the Solid Earth, fracturing is a pervasive phenomenon: weathering, explosion, impact, faulting, earthquake and so forth. Several empirical studies on fractures have demonstrated a power-law dependence of the cumulative number $N(r)$ of fragments of which sizes are larger than size r, $N(r) \sim r^{-D}$. This is taken as evidence that the fracturing is a scale-invariant process concerning the size distribution. Therefore, fractures can be described from the viewpoint of fractal. This description derives mathematically Gaudin-Schuhmann relation and Charles' relation and is sufficiently in incorporation of the three theories on size reduction: Rittinger's, Kick's and Bond's theories. The fractal dimension (D) provides a measure of the relative importance of large versus small objects and is related to both energy density for fracturing and Weibull's coefficient of uniformity (w) when the "size effect" of tensile strength is taken into consideration. Fracture surface is also a typical example of fractals. The specific surface area S of each fragment is plotted as functions of the mean fragment size $\bar{r}$. Then the surface fractal dimension D' can be defined by $S \sim \bar{r}^{D'-3}$. The D'-value for fractures increases as the energy of fracturing increased. This indicates that the surface fractal dimension D' can be a measure of fracture intensity. By analyzing the self-affinity of fracture trace curve, however, the fracture trace actually seems to be self-affine but not self-similar. Similarly, the growth pattern of various fractures, such as faults, pull-apart basins, landslides, crater morphologies and stream patterns, over a wide range of size scales is not necessarily isometric (self-similar). Therefore, the scaling law should be represented as $C_Y = C_X^{\beta}$, where C_Y and C_X are scale ratios on Y and X between fractures in scale. Moreover, this relation can also be extended to the relation between individual and system of the fracture and the relationship between the displacement and thickness of the ductile shear zone.

Key words: Fracture, fractal, Self-affinity, Scaling law, Solid Earth

CONTENTS

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1. INTRODUCTION

Fracturing is a pervasive phenomenon : weathering, explosion, impact, faulting, earthquake and so forth. Takeuchi and Mizutani (1968), Hartmann (1969), Turcotte (1986, 1989) and Mizutani (1989) have discussed fractures in a wide range of size scales. A variety of statistical power-law relations have often been used to correlate data on the size distribution of fragments (*ex.*, Gaudin, 1926 ; Schuhmann, 1940 ; Hawkins, 1960 ; Hartmann and Hartmann, 1968 ; Hartmann, 1969 ; O'Keefe and Ahrens, 1985). A power-law relation between number and size is by definition a fractal (Mandelbrot, 1982). This is taken as evidence that fracturing is a scale invariant process concerning the size distribution.

Since Rittinger (1867) proposed his theory on size reduction, a number of theories on size reduction have been developed empirically and theoretically. Among them, Rittinger's, Kick's and Bond's theories have widely been referred to. However, many discrepancies between these theories and actual size reductions have arisen.

Therefore, Section 3 describes fracturings from the viewpoint of fractal and material strength. This description derives a variety of simple power-law relations on fracturing and is sufficiently in incorporation of the three theories mentioned above. Then the present author discusses theoretically the size distributions of fractures in the Solid Earth.

When a piece of material is fractured, the fracture surface is rough and irregular. The shape of the fracture surface is affected by the material's microstructure such as grains whose characteristic length is large relative to the atomic scale, as well as by macrostructure such as the size, the shape of a specimen, and the notch from which the fracture begins (Mandelbrot *et al.*, 1984).

Most fracture surfaces have been found to be fractal with the surface fractal dimension D' greater than 2 (Avnir *et al.*, 1983 ; Mandelbrot *et al.*, 1984 ; Cahn, 1989). In this case, surface fractal dimension D' is shown to be a measure of roughness and irregularity of the fracture surface. By the fracture experiments of glass, Nii *et al.* (1985) pointed out that the fractal dimension of the fracture surface can be used to estimate the energy of fracturing, and can be a useful parameter of the nature and process of fragmentation. The surface fractal dimension D' can represent not only the degree of irregularity of the fracture surface but also a measure of the intensity of fracturing.

By the research of San Andreas fault, Aviles *et al.* (1987), Okubo and Aki (1987), and Power *et al.* (1987) insisted that the shape of seismic faults is also the fractals. On the other hand, Dieterich (1978), and Okubo and Dieterich (1984) pointed out that the fault roughness influences the physical properties of fault on the frictional instabilities by the frictional slip experiments. Therefore, quantitative considerations of scaling law on the roughness and irregularity of fracture in the Solid Earth and its measurement are also important subjects of structural geology and seismology.

In Section 4, the present author will explain the relation between the energy density of fracturing and fracture surface. Then, for testing the suitability of fractals for describing the scaling of fracture surface in the Solid Earth, he will examine several natural surfaces in the rock over a wide range of size scales. These results have significant implications about the scaling of both surface topography and mechanical properties of fractures.

Many quatitative studies of scaling of fractures, namely, fault, joint, crack and so forth, have been conducted by Allègre *et al.* (1982), Mandelbrot *et al.* (1984), Brown and Scholz (1985), Scholz and Aviles (1986), Power *et al.* (1987), Aviles *et al.* (1987) and Power *et al.* (1988), by considering the roughness of fracture surfaces as geometric parameters. However, there are few studies on both growing processes and scaling laws on fracture sizes such as displacement, thickness or length (*ex.*, Ranalli, 1977; Scholz, 1982; Watterson, 1986; Hull, 1988). Quantitative considerations of scaling laws on fracture sizes in the Solid Earth and their measurements are important subjects of structural geology and seismology. One of the effective approaches is to examine empirically the relationships among the fracture sizes in the Solid Earth on greatly different scales (Hull, 1988).

Therefore, in Section 5, the present author explains the relation between the growth pattern and a power-law size distribution of fractures and the growth pattern of fractures on the Solid Earth is allometric. Then we will find the scaling property of fractures.

By these scaling law on size distributions, shape and growth pattern of fracturing, we can compare the mechanical properties of fractures in the Solid Earth of great different scales. To take an instance, we can discern the mechanical properties of large fractures on a regional scale on analogy of fractures on the microscopic scale by the aid of these scaling laws. Therefore, the purpose of this paper is to examine the extent of similarity concerning the size distribution, shape and growth pattern of fractures in the Solid Earth.

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3. FRACTURING AND FRACTAL DISTRIBUTION

a. FRACTALS

In order to understand the scale invariant process or a phenomenon in a wide range of sizes, Mandelbrot (1977, 1982) has introduced a fractal concept. A fractal distribution $N(r)$, such as fragments (*ex.*, Hartmann, 1969) is defined as :

$$N(r) \sim r^{-D} \qquad (1)$$

where r, $N(r)$ and D denote a characteristic linear dimension, the cumulative number of objects larger than r and the fractal dimension, respectively. No characteristic length scale enters into the definition (1). If scale invariance extends over a sufficient range of length scales, the fractal distribution provides a useful description of the applicable statistical distribution. The fractal dimension D provides a measure of the relative importance of large versus small objects (Mandelbrot, 1977, 1982 ; Turcotte, 1989).

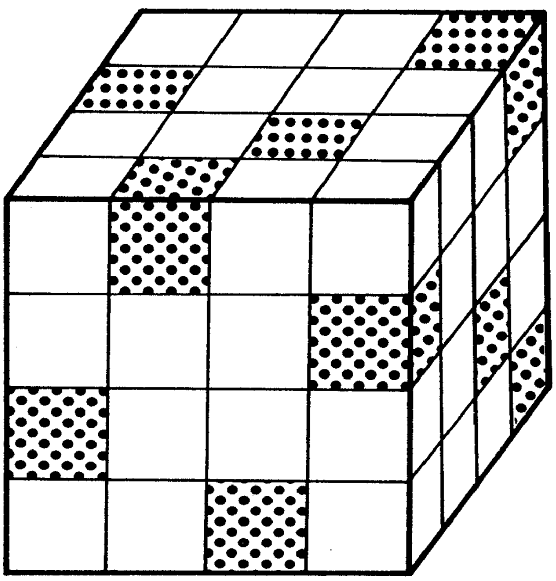

Fig. 1. A collection of 3-dimensional hypercubes with various sizes (hatched ones), which are arranged randomly. In this case, $J=64$, $I=16$, $D=3\log 16/\log 64=2.0$.

b. FRACTURING

Fracturing is an irreversible phenomenon. It breaks rocks into a big collection of pieces with no apparent size scale, a fractal distribution. In order to explain how dose such a fractal distribution result in a fracturing, the present author proposes a fracturing "cascade" model as follows.

Suppose a hypercube with sides R_0 and a huge strain energy. In the first step, divide it into J subcubes with sides $R_1 = R_0 J^{-1/3} (J>8)$, and choose randomly I subcubes to hatch (Fig. 1). In the second step, divide each of I subcubes into J sub-subcubes with side $R_2 = R_1 J^{-1/3} = R_0 J^{-2/3}$, and choose randomly I sub-subcubes to hatch. Repeating the same procedure, the hypercubes with side $R_n = R_0 J^{-n/3}$ will be newly hatched in the n-th step. Let the hypercubes be fragments and suppose that the energy is dissipated by fracturing the I hypercubes and not in the others $(J-I)$ in each step. Then, the total energy for fracturing has decreased from those in the J fragments of size R_n into those in $I(<J)$ fragments with size R_{n+1}. This phenomenological model is similar to that of Matsushita (1985), Sammis *et al.* (1986) and Turcotte (1986, 1989) for fracturing and resembles the β-model on the energy transfer cascade in a fully developed turbulent flow by Novikov and Stewart (1964) and Frisch *et al.* (1978). The set of these energy-dissipated regions has fractal dimension :

$$D = 3\log I/\log J \qquad 0 \leq D \leq 3. \qquad (2)$$

In the volume of side $R = R_0$, the total number of the hypercubes with side $r=R_n$ is $(R_0/R_n)^3=(R/r)^3$ and the total number of the hypercubes in the dissipated regions in $(R/r)^D$. If we measure the dissipated region with the scale r, the volume V_r of the dissipated region is $(R/r)^D r^3$. Accordingly, the occupation ratio P_r of the volume V_r of the dissipated region versus the whole volume $V=R^3$ is given by

$$P_r=V_r/V=(R/r)^{D-3}=(R/r)^{-\mu}, \quad (3)$$

$$\mu=3-D. \quad (4)$$

Now, let E_r be the average dissipated energy per unit volume in the dissipated region, then

$$E_T V=E_r V_r \quad (5)$$

where E_T is the energy in the whole region. In $(R/r)^D$ hypercubes of side r, E_r is given by

$$E_r=E_T P_r^{-1}\sim r^{-\mu}. \quad (6)$$

The relation (6) represents that E_r is the r-dependence and is equivalent to Walker-Lewis' relation given by

$$E=Cr^{-n+1} \quad (7)$$

(Charles, 1957; Schuhmann, 1960; Tartaron, 1963) where E is the energy input of fracturing per unit volume of material and C is the constant. Comparison of (6) and (7) gives

$$\mu=n-1. \quad (8)$$

In this model, the dissipated energy is not equal to zero in only D-dimensional set of hypercubes. This assumption indicates that they are uniform in the D-dimensional set and are filled uniformly with the whole body when $\mu=0$ $(D=3)$.

When the dissipated region has D-dimensional fractal set, the cumulative number $N(r)$ of fragments (hypercubes) of which size are larger than r is given by the relation (1). The incremental number $dN(r)$ is related to the incremental mass $dM(r)$ of fragments as

$$dN(r)\sim r^{-3}dM(r). \quad (9)$$

From (1) and (9), we can get

$$M(r)\sim r^{-D+3} \quad (10)$$

where $M(r)$ is the cumulative mass of fragments with a radius (volume$^{1/3}$) less than r. This relation (10) is equivalent to Gaudin-Schuhmann relation given by

$$\frac{M(r)}{M_T}=\left[\frac{r}{\sigma}\right]^h \quad (11)$$

where M_T is the total mass, σ is related to the mean size of fragments, and h is constant.

From (10) and (11),

$$h=3-D. \quad (12)$$

Therefore, combining (4), (8) and (12) gives Charles' relation

$$h-n+1=0 \qquad \text{(Charles, 1957).}$$

When the h-value (or n-value) is large, there are a larger number of coarse fragments and less dispersion and vice versa. From the relation (4) and (12), we can infer that the h-value (or n-value) is related to the fractal dimension (D) of the set of the dissipated regions.

c. ENERGY-SIZE REDUCTION RELATIONSHIPS IN FRACTURING

(1) Three theories on energy-size reduction

The present status of the theory on size reduction of material is extremely unsatisfactory. The basic underlying concept has not been known well as yet.

Rittinger (1867) postulated that the energy for size reduction would be proportional to the area of new surfaces created by fracturing. Rittinger's hypothesis can be stated mathematically as follows:

$$S=\frac{\text{surface area}}{\text{weight}}=\frac{\pi r^2}{(\pi/6)r^3\cdot\delta}=\frac{6}{r\cdot\delta} \quad (13)$$

S: specific surface area, r: particle size, δ: specific gravity.

Therefore, S is inversely proportional to the particle size. When particles are reduced in size from r_1 to r_2, the energy input of fracturing per unit volume E is given by

$$E = C_r(1/r_2 - 1/r_1) \quad (14)$$

where C_r is a constant.

Kick (1885) proposed, instead of Rittinger's hypothesis, that the energy for fracturing depends only upon the reduction ratio, and that equivalent amounts of energy should result in the equivalent ratio of size reduction. An original mathematical treatment of Kick's theory is introduced by Bond and Wang (1950) as follows:

Let p be the energy in $hp-hr$ per ton required to reduce one ton of rocks of size r to size $r/2$. This represents one reduction step, and f steps are required for reduction to size r/n. The reduction ratio is given by $n = 2^f$. Taking the logarithm of it gives $f = \log n/\log 2$. The total input energy is the sum of that for each step. In the first step, size r is reduced to $r/2$, and in the second step size $r/2$ is reduced to $r/4$. In the final step, $r/2^{f-1}$ is reduced to $r/2^f$. The number of particles per ton increases 2^3 times at each step. Therefore, the total energy E for fracturing is

$$E = p + 2^3 p(2^{-1})^3 + (2^3)^2 p(2^{-2})^3 + (2^3)^3 p(2^{-3})^3 + \cdots\cdots + (2^3)^{f-1} p(2^{1-f})^3 = fp = (\log n/\log 2)p. \quad (15)$$

The relation (15) depends on the reduction ratio $n = 2^f$, and is independent of particle size. When the size is reduced from r_1 to r_2, the energy input of fracturing per unit volume E is given by

$$E = C_k \log n = C_k \log(r_1/r_2) \quad (16)$$

where C_k is a constant. Moreover, the relation (16) can also be derived from the consideration based on the principle of similarity as applied to the fracturing process (Andreases, 1957).

Bond (1952) has proposed that since neither Kick's nor Rittinger's hypotheses seem correct for fracturing a compromise between the two would be more applicable. Bond's theory is introduced as follows: The strain energy in a cube is proportional to its volume r^3. The strain energy effectively follows the newly formed crack surface, with area of r^2. Then the proportionate energy absorbed is $r^{2.5}$ which is intermediate between r^2 and r^3. Variance of particle number of similar shape in a unit volume is as much as $1/r^3$, so that the energy input per unit volume should be proportional to $r^{2.5}/r^3$ or $1/\sqrt{r}$. Therefore, when the size of particles is reduced from r_1 to r_2, the energy input of fracturing per unit volume E is given by

$$E = C_b(1/\sqrt{r_2} - 1/\sqrt{r_1}) \quad (17)$$

where C_b is a constant.

(2) Fractal theory on energy-size reduction

In the brittle materials with cleavages, as a variety of minerals, new fractures are created as the input energy increased. In this case, Rittinger's theory is suitable. On the other hand, when the elastic energy stored in very hard materials arrives at the maximum permissible dose, the materials are broken into fragments similar in shape. In this case, Kick's theory is suitable. But these two types of fracturing cooperate in the actual fracturing. In this case, it is concluded from the relation (6) that the energy for the size reduction is proportioned to r^D where D is the fractal dimension of the set of the dissipated hypercubes. Thus,

Table 1. Fractal dimension D and three theories on energy-size relation.

Theory on energy-size reduction	D	n
Rittinger's theory	2.0	2.0
Bond's theory	2.5	1.5
Kick's theory	3.0	1.0

in the size reduction of particle from size r_1 to r_2, the energy input of fracturing per unit volume E is given by

$$E = C_f(r_2^{D-3} - r_1^{D-3}) \qquad (18)$$

where C_f is a constant. The three old theories mentioned above are sufficiently general to be incorporated into this theory. The energy relation (18) is the same as Kick's theory when $D = 3.0$ ($n = 1.0$), Bond's theory when $D = 2.5$ ($n = 1.5$) and Rittinger's theory when $D = 2.0$ ($n = 2.0$) (Table 1).

d. THE SIGNIFICANCE OF FRACTAL DIMENSION D

According to the summary on fractal of fragments by Turcotte (1986, 1989), the D-values in a wide range of size scales vary considerably but most lie in the range $2 < D < 3$. When the fractal dimension is less than 3, the volume (mass) of small fragments is negligible. When the fractal dimension is greater than 2, the surface area of small fragments dominates. Volume (mass) is conserved through fracturing while surface area increases. The creation of surface area by fracturing requires energy. Thus Turcotte (1989) infers that the fractal dimension will increase with an increase of the energy density available for fracturing. The relation (18) indicates the Turcotte's hypothesis. By the fracture experiment of tuff (Matsui *et al.*, 1982), the fractal dimension (D) of the fragments seems to become steep with an increase of the energy density (Fig. 2).

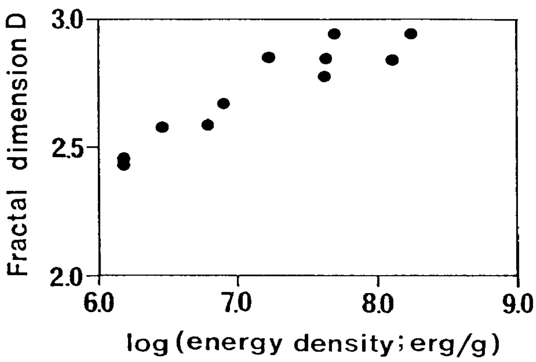

Fig. 2. Fractal dimension D of the fragment size distribution vs. the energy density. (Data from Matsui *et al.*, 1982).

By the renormalization group studies, Turcotte (1986) shows that the fractal dimension (D) is a measure of the fracture resistance of materials and predicts fragile materials to have a smaller fractal dimension. The present paper demonstrates that D-value is related to the fractal dimension of the assemblage of the energy-dissipated hypercubes. But from a viewpoint of fractal, D-value dose not give us a specific value concerning the fracture mechanism. Thus in this section the present author will discuss the relationship between D-value and the fracture mechanism from a viewpoint of the material strength.

Majima and Oka (1969) and Oka and Majima (1969) assumed that the fracture is only possible when the tensile stress reaches the tensile strength of the rock and proposed that the energy for size reduction is proportioned to the $3(1-2/w)$th order of particle size r, where w is a coefficient of uniformity when "size effect" of tensile strength is taken into consideration. When particles are reduced from size r_1 to size r_2, the energy input of fracturing per unit volume E can be given by

$$E = C_m(r_2^{-6/w} - r_1^{-6/w}) \qquad (19)$$

where C_m is a constant relating to tensile strength, Young's modulus of the rock, a standard size and the number of particles of standard size (Majima and Oka, 1969; Oka and Majima, 1969). Thereafter, from the experimental study, Kanda *et al.* (1969, 1970) proposed that E can be given by

$$E = C_y(r_2^{-k/w} - r_1^{-k/w}) \qquad (20)$$

where C_y and k are constant. In this relation (20), the k-value is determined to be 5 by the disk splitting experiment and is determined to be 6 by the sphere

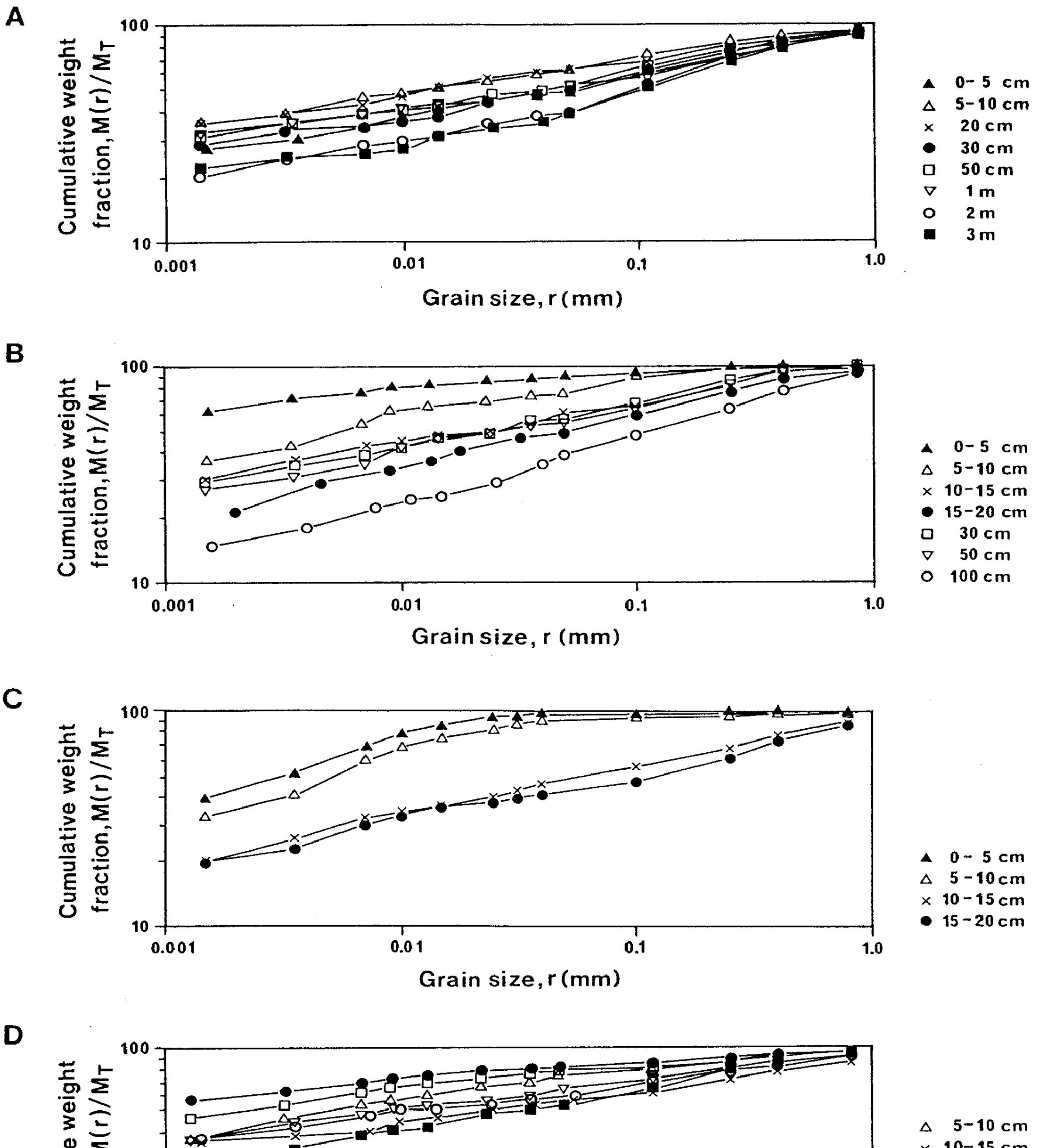

Fig. 3. Size distribution of fault fragments. Symbol marks show the distances from the fault plane (slip plane) (Data from Fukushima, 1984 MS). A : Sodezawa Fault, B : Kizugawa Fault, C : Hananoki Fault, D : Atotsugawa Fault.

splitting experiment. The relation (20) holds for a single particle. Such an approach can be extended to the comminution of particles by using statistical methods (Schuhmann, 1960). In fact, the relation (20) permits an accurate energy calculation for selected comminution experiments (Majima and Oka, 1969; Oka and Majima, 1969). Therefore, combining (18) and (20) gives

$$D=3-k/w. \qquad (21)$$

The relation (21) indicates that the D-value is related to the Weibull's coefficient of uniformity (w) when "size effect" of tensile strength is taken into consideration. When the volume of material is constant, the relative dispersion of material strength decreases as w-value increases (Weibull, 1939a; 1939b; Mogi, 1962a). Therefore, it is concluded from the relation (21) that the fractal dimension (D) increases as the relative dispersion of the strength decreases.

e. SIZE DISTRIBUTIONS OF FRAGMENTS IN THE SOLID EARTH

(1) Size distribution of fault gouge

Generation of fault gouge may be related to displacement of a shear fracture or a fault. Fracturing or milling of fault fragments reduces the size rapidly. Then the translation and rotation of particles as rigid bodies without much milling result in a decrease in the rate of change of medium size. Therefore, the evolution of fault gouge can be characterized with the aid of cumulative frequency curves.

Specimens from four active fault zones were used to characterize the fault gouges (Fukushima, 1984MS): (A) the Sodezawa Fault, Nagano Pref., Chubu District, Japan; (B) the Kizugawa Fault, Kyoto Pref., Kinki District, Japan; (C) the Hananoki Fault, Mie Pref., Kinki District, Japan; (D) the Atotsugawa Fault, Gifu Pref., Chubu District, Japan. Each curve is constructed by the correlation between the cumulative percentage of grains (by volume) and the grain size of fault gouge on a double logarithmic scale (Fig. 3). Here these size distributions are straight lines at some angle to the abscissa. By these results, it is concluded that the size distribution of fault gouge can be represented by the Gaudin-Schuhmann relation (11). Therefore, the slopes of the curves are equal to $(3-D)$-values of the size distributions of fault gouges. The D-values of fault gouge lie in the range from 0 to about 2.6. They may increase as the distance from the fault plane (slip plane) decreases. In other words, the

Table 2. Power-law relation of fracture size.

r	P	Source	Detail
Fault length	0.52	Kodama (1976)	Normal fault (Experiments)
	1.30	Yamaguchi & Hase (1983)	Lineament
	1.70, 1.95	Ohno & Kojima (1988)	Lineament
Fault width	0.39-0.94	Ogata (1976)	Fracture thickness
	0.39-0.94	Ogata & Honsho (1981)	Fracture thickness
	0.60, 1.40	Ohno & Kojima (1988)	Fracture thickness
Joint length	0.30, 0.80	Segall & Pollard (1983)	Joint
Crack width	0.78-1.50	Watanabe (1979)	Micro-crack
Displacement	0.30-1.40	Kakimi & Kodama (1974)	Normal-slip fault
	0.30-1.40	Kakimi (1980)	Normal-slip fault

D-value of fault gouge increases as the faulting process proceeds. Therefore, it is concluded that the D-value of fault gouge can be a useful parameter of the nature and process of faulting, such as the intensity of shear fracture or the activity of faults.

(2) Size distributions of fractures in the Solid Earth

The previous workers (Table 2) have detected distributions of fracture size, such as length, width and displacement on the ground surface. All of these distributions take power-law distributions as:

$$N(r) \sim r^{-P} \tag{22}$$

where r and $N(r)$ denote the fracture size and the cumulative number of fractures larger than r, respectively, and P is constant. The relation similar to (22) also represents the frequency-magnitude relation for earthquake (Aki, 1981), where the square root of the area of fault break corresponds to r in the relation (22). This relation has been traced to the size distribution of seismic faults by Wesnousky *et al.* (1983). It is notable that the power-law form of relation (22) holds over a wide range of fracture sizes and for any sampling size. The power-law frequency distribution (22) is similar to the fractal distribution (1).

Ranalli (1976, 1980) has derived the distributions of fault lengths from the stochastic model of faulting. His model is afforded by the Kolmogorov-type breakage process. However, the size distribution derived from Kolmogorov-type breakage process is not the power-law distribution but log-normal distribution (Kolmogorov, 1941; Aithchison and Brown, 1957). In the section (3−b), we can find that a pair of fracture sizes forms a power-law distribution. This derivation is simplest one from self-similarity assumption which is different from Ranalli's (1976) assumption. This suggests that relation (22) indicates a self-similarity in frequency-magnitude and is apparently valid for various fracture scales from microcrack to large fault; not only the fault results over the long geologic past but also seismic fault. In other words, fracturing in the Solid Earth is also regarded as a scale-invariant process concerning the size distributions.

According to the power law relation of fracture sizes (Table 2), the most P-values lie in the range $0<P<2$. Because the relation (22) is detected on the ground surface which is regarded as a cross-section through self-similar hypercubes shown in Fig. 1, the P-value is equal to the fractal dimension of the cross-section of the fragments hypercubes and lies in the range $0<P<2$. Watanabe (1979) discovered that the P-value about crack width trends to decrease as the rock porosity increases. Kakimi (1980) pointed out that P-values about fracture displacement appear to be related to the tectonic conditions in the fault region, such as the homogeneity of mechanical structure, stability of stress distribution and sharpness of regional deformation. These facts suggested that the P-value actually reflects the rock properties and tectonic conditions which are related to the structural uniformity (w) discussed in Section 3−d.

(3) Gudenberg-Richter relation

The frequency-magnitude relation for earthquakes relation is also similar to relation (22). Under many circumstances, the number of earthquakes $N(M)$ with a magnitude greater than M satisfies the empirical relation (Gudenberg and Richter, 1954)

$$\log N(M) = -bM + a \tag{23}$$

where a and b are constants. The b-value is widely used as a measure of seismicity. Aki (1981) showed that the relation (23) is equivalent to the definition of fractal distribution (1). Turcotte (1989) introduces that as follow :

The moment of an earthquake is defined by :

$$M_0 = \xi \eta A \qquad (24)$$

where ξ is the shear modulus, A is the area of the fault break and η is the mean displacement on the fault break. The moment of the earthquake can be related to its magnitude by

$$\log M_0 = gM + h' \qquad (25)$$

where g and h' are constants. Kanamori and Anderson (1975) have established a theoretical basis for taking $g=1.5$. These authors have also shown that it is a good approximation to take

$$M_0 \sim r^3 \qquad (26)$$

where $r = A^{1/2}$ is the linear dimension of the fault break. Combining (23), (25), and (26) gives,

$$N(r) \sim r^{-2b} \qquad (27)$$

This relation (27) is equivalent to the relations (1) and (22). Combining (1), (21), and (27) gives

$$D = 3 - k/w = 2b \qquad (28)$$

The relation (28) indicates that the fractal dimension (D) of regional or world-wide seismic activity is simply twice the b-value as a measure of seismicity and is related to the energy density for fracturing and the Weibull's coefficient of uniformity (w) in the Solid Earth. By the results of laboratory experiments of the fracturing of various materials, Mogi (1962b, 1967) pointed out that the b-value increases with both the increasing degree of heterogeneity and increasing density of cracks in the medium. The relation (28) is not contradictory to Mogi's (1962b, 1967) results. Therefore, the b-value gives some information about mechanical structures of a seismic region, such as the degree of the structural nonuniformity of the earth's crust.

4. FRACTURING AND FRACTAL SURFACE

a. FRACTAL SURFACE AND ENERGY DENSITY FOR FRACTURING

Fractals are the concern of a new geometry (Mandelbrot, 1982), whose primary objective was to describe the great variety of natural structures that are irregular, rough, having irregularities of various sizes that bear a special scaling relationship to one another (Mandelbrot *et al.*, 1984). Crushed fragments are sieved into several fractions and have the irregular and rough fracture surfaces. Fracture surface is also a typical example of fractals (Mandelbrot, 1982 ; Nii *et al.*, 1985 ; Brown and Scholz, 1985). The specific surface area S of each fraction is plotted as functions of the mean particle diameter $\bar{r}$ (Nii *et al.*, 1985). The surface fractal dimension D' is defined by

$$S \sim \bar{r}^{D'-3}$$

(Avnir *et al.*, 1983). (29)

Fractal geometry characterizes the scaling structure of a fracture surface by D'-value that can range from 2, when the surface is smooth, up to 3 (Avnir *et al.*, 1983 ; Mandelbrot *et al.*, 1984). This surface ($2 < D' < 3$) is called fractal surface.

By the fracture experiments of the glass, Nii *et al.* (1985) pointed out that the surface fractal dimension of fractured glass increases with the impact energy of fracturing (Fig. 4) and is a measure of the intensity of fracture. This indicates that the surface fractal dimension D' of the fracture surface can be used for estimating the energy density of fracturing, and can be a useful parameter of the nature and process of fragmen-

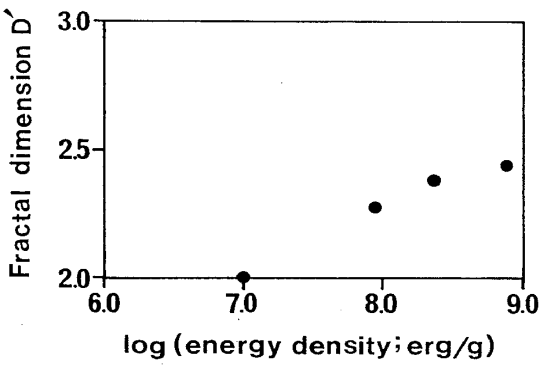

Fig. 4. Surface fractal dimension D' as a function of the energy density (Data from Nii *et al.*, 1985).

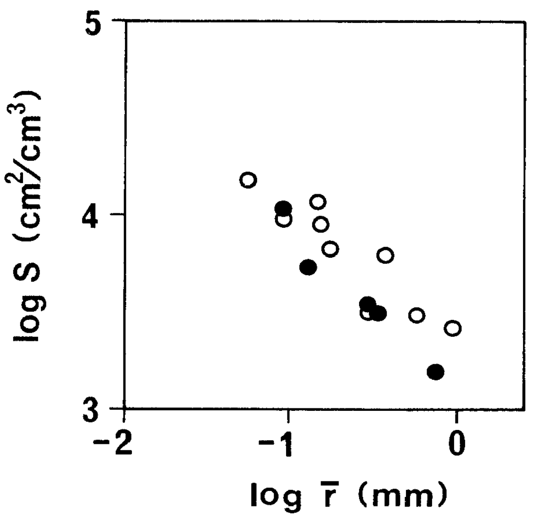

Fig. 5. The specific surface area S (cm^2/cm^3) as function of mean particle diameter $\bar{r}$ (mm). ○ : Straight Creek tunnel. ● : Henderson Mine tunnel and Haulage tunnel (Data from Brekke and Howard, 1973).

tation. Therefore in the next subsection, the present author will examine the surface fractal dimension of fault fragments.

b. FRACTAL SURFACE OF FAULT FRAGMENTS

The empirical relations between the mean diameter and the surface area of fault fragments have been summarized by Kanaori *et al.* (1980) and Kanaori *et al.* (1982). These studies indicate that the specific surface area S is plotted as functions of the mean particle diameter $\bar{r}$ for fault fragments (Fig. 5) and S increases as $\bar{r}$ decreases. The power-law relations (29) can express these empirical relations and surface fractal dimensions of fault fragments can be obtained from the slopes of these relations. The surface fractal dimensions of fault fragments are summarized in Table 3. According to Table 3, the D'-values of fault fragments lie in the range from 2.47 to 2.74. These results indicate that the

Table 3. Surface fractal dimensions D' of fault fragments. MTL: Median Tectonic Line.

Fracture		D'-value	Source
Atotsugawa F.	(Amodani)	2.57	Kanaori *et al.* (1982)
	(Makawa)	2.50	Kanaori *et al.* (1982)
		2.70	Kanaori *et al.* (1980)
MTL	(Yoshinogawa)	2.74	Kanaori *et al.* (1980)
	(Kinogawa)	2.62	Kanaori *et al.* (1980)
Neodani F.		2.66	Kanaori *et al.* (1980)
		2.72	Kanaori *et al.* (1980)
Straight Creek tunnel		2.47	Brekke & Howard (1973)
Nast tunnel, Henderson Mine tunnel, and Haulage tunnel		2.47	Brekke & Howard (1973)
Weathered rocks		2.60	Nossin & Levelt (1967)
Same particle sizes		2.00	

D′-value increases with increasing intensity of faulting. Therefore, the *D′*-value can be a useful parameter of the nature and process of faulting.

c. SELF-AFFINITY OF FRACTURE AND FAULT GEOMETRY

Since the geometrical similarity of fractures (fault, joint, crack and so forth) was examined (Tchalenko, 1970 ; Tchalenko and Ambraseys, 1970 ; King, 1983 ; King and Nabelenko, 1985), many quantitative studies of scaling of fractures have been conducted by Allègre *et al.* (1982), Mandelbrot *et al.* (1984), Brown and Scholz (1985), Scholz and Aviles (1986), Power *et al.* (1987), Aviles *et al.* (1987) and Power *et al.* (1988), using the surface roughness of fractures as geometric parameters. They pointed out that the irregularity of fractures can be characterized quantitatively by a fractal dimension or power spectrum and there is a remarkable similarity among the fracture surfaces. However, Power *et al.* (1987) found that the roughness of fault surfaces can not be described by a single parameter and the amplitude of the topography increases in an approximate proportion to the wave length under consideration. Then, they pointed out that the fault surface dose not seem to be self-similar and is strongly anisotropic. Therefore, the vertical variation of fault trace must be scaled differently from any horizontal one.

When given patterns are scaled differently in different directions (or scaled anisotropically), they are called self-affine fractals. For testing the suitability of fractals for describing the scaling of fault trace, the self-affinity of the fault trace will be examined by the scale-independent analysis (Matsushita and Ouchi, 1989) in this subsection.

Measure curve length, *N*, and standard

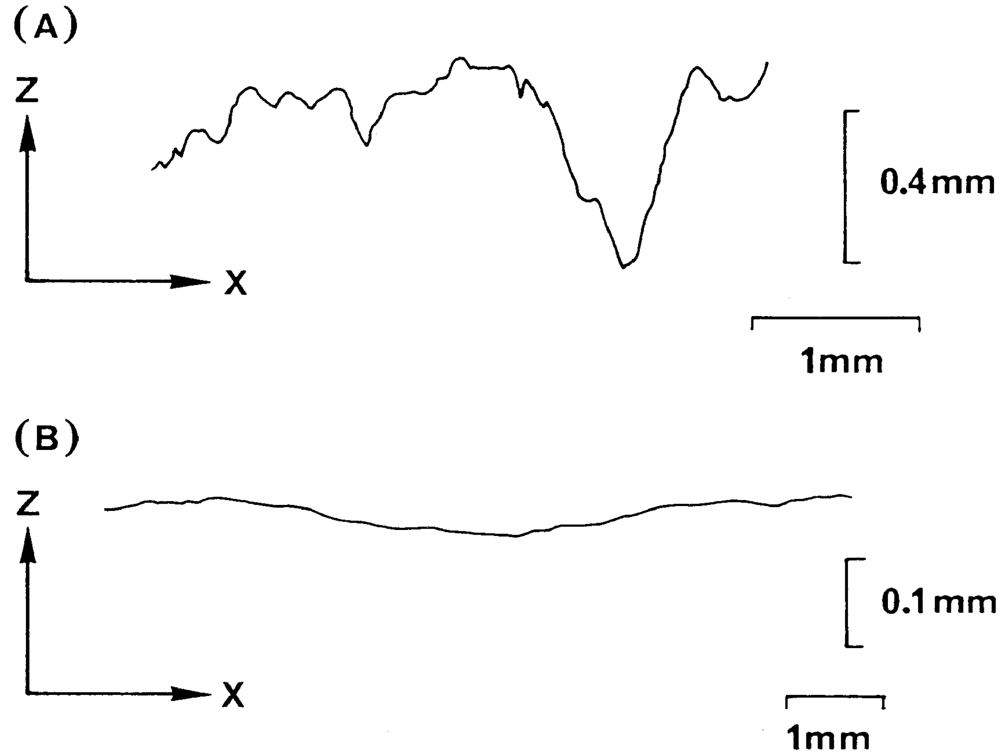

Fig. 6. Fracture surface. A : Westerly Granite, 2.5× vertical exaggeration (after Power *et al.*, 1988), B : Natural joint surface, 10× vertical exaggeration (after Brown and Scholz, 1985).

deviations for two appropriately chosen coordinates, X and Z, of a curve in two dimensions between many arbitrary pairs of points on the curve by using the smallest fixed scale (yard stick) for many pairs of points on the curve and check by log–log plots of X and Z versus N whether they scale as

$$X \sim N^{H_X} \tag{30}$$

$$Z \sim N^{H_Z} \tag{30'}$$

where the exponents H_X and H_Z are in general different. If so, they are then related to each other as

$$Z \sim X^{H} \tag{31}$$

where the exponent H is given by

$$H = H_Z / H_X. \tag{32}$$

We can check the scaling relation between the exponents because H characterizes the self-affinity of curves. Let us apply the method to a real fault trace. Fig. 6A illustrates a fracture trace shown by Power *et al.* (1987). Regarding this curve as being given, we calculated the curve length N and standard deviations of coordinates X and Z (Fig. 6A). We repeated the procedure for many pairs of points. The results are shown in Fig. 7A, where X and Z are plotted against N. As is expected, the standard devia-

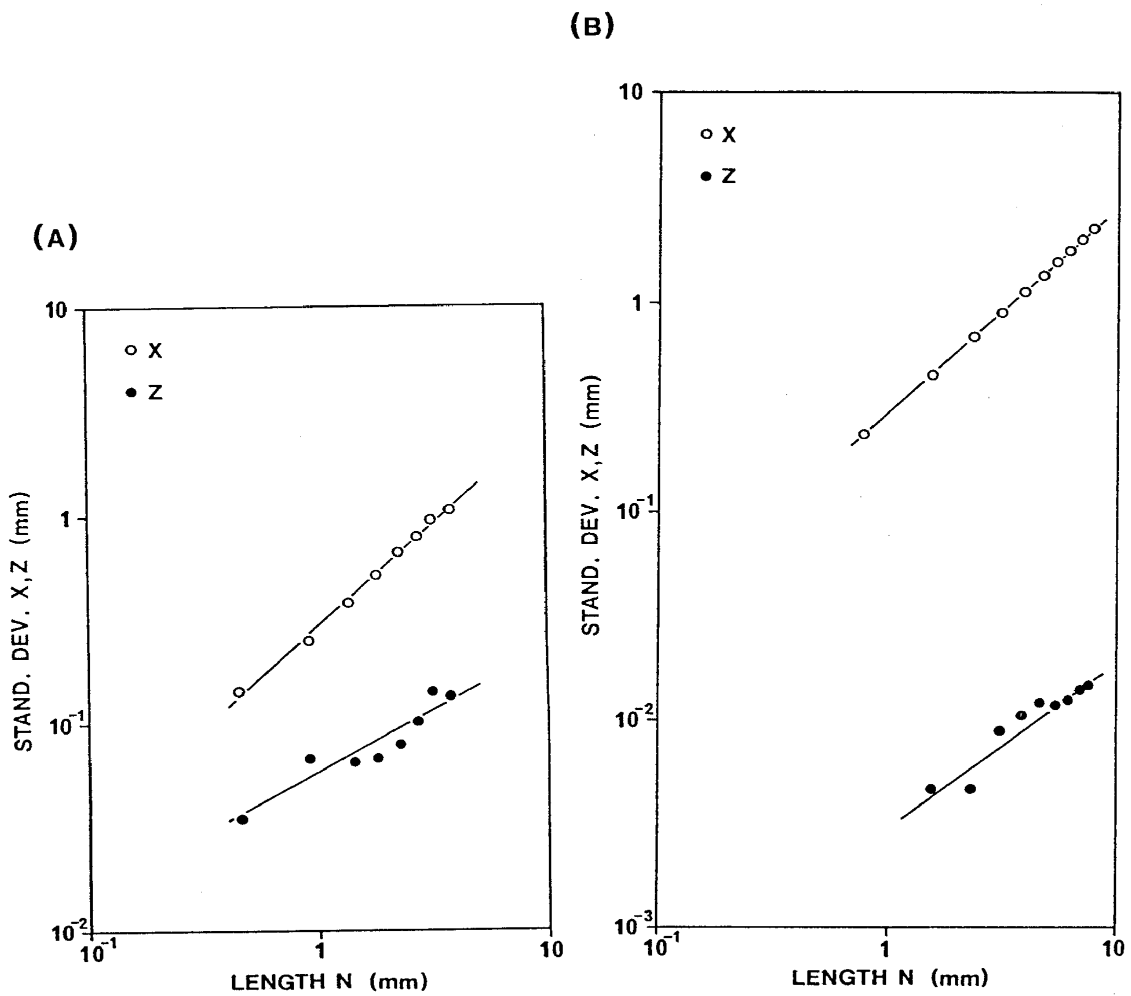

Fig. 7. Dependence of the standard deviation of two coordinates X and Z on the curve length N between many pairs of points on the fault trace shown in Fig. 6. A : Westerly Granite (after Power *et al.*, 1988), B : Natural joint surface (after Brown and Scholz, 1985).

tion of the X coordinates is proportional to the trace length, i.e., $X \sim N$ and $H_X = 1$. On the other hand, that of the Z coordinates shows an approximate dependence of $Z \sim N^{H_Z}$ with $H_Z = 0.57$. The fact that the fracture trace such as the one shown in Fig. 6A is approximately self-affine with $H_X = 1$ and $H_Z < 1$ means that they can be represented by a fractional Brownian motion with $H = H_Z = 0.5$.

One more example is given in Fig. 6B, which was taken from natural joint surface (Fig. 1; Brown and Scholtz, 1985). We again calculated the dependence of the standard deviations of X and Z coordinates of this curve, on the curve length N. The results are shown in Fig. 7B. As is expected, the standard deviation of the X coordinates is proportional to the curve length, i.e., $X \sim N$ and $H_X = 1$. On the other hand, that of the vertical coordinates shows approximate dependence of $Z \sim N^{H_Z}$ with $H_Z \fallingdotseq 0.78$. This fault trace is a self-affine curve with $H_X = 1$ and $H_Z < 1$.

The present author has applied a simple and useful method (Matsushita and Ouchi, 1989) to analyze and check the self-affinity of fault trace curve. These results indicate that the fault trace seems to be self-affine and not self-similar. The relation (30) and (30′) may be useful to investigate the self-affinity of fault trace.

5. ALLOMETRIC GROWTH OF FRACTURE ZONES

a. FRACTAL DISTRIBUTION AND ALLOMETRIC GROWTH

Let $N(X)$ and $N(Y)$ be the cumulative number of two different kinds of size parameters X and Y which are dependent. If $N(X)$ and $N(Y)$ can be represented respectively by,

$$N(X) \sim X^{-D_X} \qquad (33)$$

$$N(Y) \sim Y^{-D_Y} \qquad (33')$$

where D_X and D_Y are constants, the relations (33) and (33′) give the relationship between two size parameters:

$$Y \sim X^{\beta}. \qquad (34)$$

where the exponent β is given by

$$\beta = D_X / D_Y. \qquad (35)$$

The power-law relation (34) has been quite common and used for the description of the growth pattern in biology and paleontology. Huxley and Teisser (1936) first termed it allometric growth, and depicted the growth of one part relative to that of the whole body or a standard part. According to this study, the correlation of the geometric size parameters Y and X of growth pattern could be expressed as a general form by the power-law relation (34). Thereafter the theory of allometric growth has been developed by biologists and paleontologists (*ex.*, Hamai, 1941; Reeve and Huxley, 1945; von Bertalenffy, 1960, 1968; Gould, 1966; Rosen, 1967). Relation (34) can be derived from the theory of allometric growth as follows (*ex.*, Rosen, 1967):

Let $R_X = (1/X)(dX/dt)$ and $R_Y = (1/Y)(dY/dt)$ be the relative growth rates of X and Y, respectively. If R_Y/R_X is a constant, say β, and ΔX and ΔY are the values of (dX/dt) and (dY/dt) integrated over an infinitesimal period of time Δt, then

$$\Delta Y / \Delta X = \beta Y / X. \qquad (36)$$

We approximate equation (36) to continuum and obtain the following equation:

$$dY / dX = \beta Y / X. \qquad (37)$$

Relation (37) means that the ratio of infinitesimal increases of Y and X within an infinitesimal time increment is proportional to size ratio (Y/X). The general solution of this differential equation (37) is expressed by a power-law form as relation (34). Hence, it is concluded that if R_Y/R_X is a constant β then the relationship between Y and X obeys the power-law form as relation (34) and the growth pattern is, by definition, "allometric".

The parameter β of the relation (34) is the ratio between the relative growth rate of one part of the growth pattern and the relative growth rate of the reference part of the growth pattern. Huxley and Teisser (1936) defined isometric as a special case of allometric, where the growth rate of one part is proportional to the reference part, which corresponds to the case of $\beta=1$. In this case, the similarity keeps strictly through the growth process and the growth pattern is strictly self-similar. On the other hand, the growth pattern where $\beta \neq 1$ is called anisometric. In this case, similarity does not hold through the growth process and the growth pattern is self-affine. The relation (34) together with (35) is useful to investigate the self-similarity of growth pattern.

b. GEOMETRIC SIZE PARAMETERS OF FRACTURE

Ogata (1976) proposed the relationship between the thickness $T(m)$ of breccias and the length $L(m)$ of faults in granitic rocks as follows,

$$L=4.8\ T^{0.87}. \qquad (38)$$

Moreover, Ohno and Kojima (1988) mathematically predicted that the relationship between the fault length and thickness may be expressed by relation (34) from the power-law distributions of them. These indicate that the growth pattern of faults is allometric (especially self-affine).

Similarly, the relation (34) has already been found among geometric size parameters of various fractures in the Solid Earth: stream patterns (Hack, 1957), landslides (Fujii, 1969), pull-apart basins (Aydin and Nur, 1982; Ito, 1989), and crater impact morphologies (Baldwin, 1965; Pike, 1967, 1974). These indicate that the growth patterns of these phenomena are also allometric. Woldenberg (1966) and Pike (1967) have already adopted the idea of allometric growth for river systems and crater impact morphologies, respectively. According to various fractures in the Solid Earth it should be emphasized that the idea of allometric growth can be extended to the geometric size parameters of fractures, not only river systems and crater morphologies but also faults, landslides and pull-apart basins.

c. DISPLACEMENT AND GEOMETRIC PARAMETER OF FAULTS

The theory of allometric growth can depict the growth of one part relative to that of a standard part. Thus when we regard the displacement of a fault as a standard part, we can investigate the relationship between the displacement and other geometric size parameters of a fault.

Table 4. Relationships between the displacement $D_d(m)$ and the length $L(m)$ of faults.

Relationship	Source	Details
$D_d=10^{-5.10}L^{1.23}$	(Iida, 1965)	Earthquake fault length (Dip-slip)
$D_d=10^{-4.01}L^{0.92}$	(Iida, 1965)	Earthquake fault length (Strike-slip)
$D_d=0.02\ L^{1.17}$	(Ranalli, 1977)	Fault length (Strike-slip)
$=0.02\ L\log L$		

Table 5. Relationships between the displacement $D_d(m)$ and the thickness $T(m)$ of faults.

Relationship	Source	Details
$D_d=9.6\ T^{1.10}$	(Otsuki, 1978)	Fracture thickness (Experiment)
$D_d=114.8\ T^{1.07}$	(Otsuki, 1978)	Minor fault thickness
$D_d=56.2\ T^{1.01}$	(Otsuki, 1978)	Fault thickness (Strike-slip)
$D_d=51.29\ T^{1.17}$	(Miyata, 1978)	Fault gouge thickness
$D_d=2.11\ T^{1.29}$	(Kojima *et al.*, 1981)	Minor fault thickness
$D_d=31.5\ T^{1.30}$	(Kojima *et al.*, 1981)	Micro-fault thickness
$D_d=4.65\ T^{0.62}$	(Kojima *et al.*, 1981)	Fault breccia thickness
$D_d=94\ T^{0.833}$	(Robertson, 1983)	Fault thickness
$D_d=63\ T^{0.97}$	(Hull, 1988)	Cataclasite thickness

(1) Relationship between size parameters of faults

The empirical relations among thickness T of gouge or breccia, length L, and displacement D_d of faults have been proposed by many workers (Table 4, 5). These studies indicate that the empirical relations can be expressed by the power-law relation (34) and are allometric, not necessarily self-similar. But these data correspond to different faults of various sizes, not a single fault at different positions of ones. Blenkinsop and Rutter (1986) noted that individual faults may exhibit D_d-T relationship and its relation is different to that of the global population. However, it is pointed out that the New Jersey fault shows the same positive correlation of individual faults as they do for the fault population (Hull *et al.*, 1986; Hull, 1988). Hence, these relations (Table 4, 5) represent the average trends of faults.

Ranalli (1980) has already adopted the idea of allometric growth for the length and offset of the brittle strike slip faults. According to relations of faults (Table 4, 5), it should be emphasized that the idea of allometric growth can be extended to the relationship between any two different kinds of fault size parameters, not only L and T, but also D of faults.

(2) D_d-T relationship of ductile shear zones

Figure 8 summarizes the published data on displacement D_d versus thickness T of natural mylonitic zones dominated by intracrystalline plasticity, from different geological environments. The thickness of mylonitic zones is defined here as the sum of these constituents. In almost all cases, the displacement along the mylonitic zones was calculated by using the simple-shear assumption and deflected foliations. A few mylonitic zones have the certain net slip (Mitra, 1979; Sørensen, 1983). All of the data were reported from metamorphic, igneous, or highly consolidated sedimentary rocks.

This empirical relationship between D_d and T of mylonitic zones, which ranges in size from order 10^{-3} (m) to 10^5 (m), can be expressed by the following regression relation,

$$D_d=2.4\ T^{1.04} \qquad (39)$$

with a correlation coefficient $\gamma=0.99$. This result is slightly different from Mitra's (1979) or Hull's (1988) results, due to the additional data for large mylonitic zones. The positive relation is expected, as thickness is actually plotted against $T\bar{s}$ where $\bar{s}$ is the average shear strain, rather than independently measured displacement (Hull, 1988). Because the exponent value of the rela-

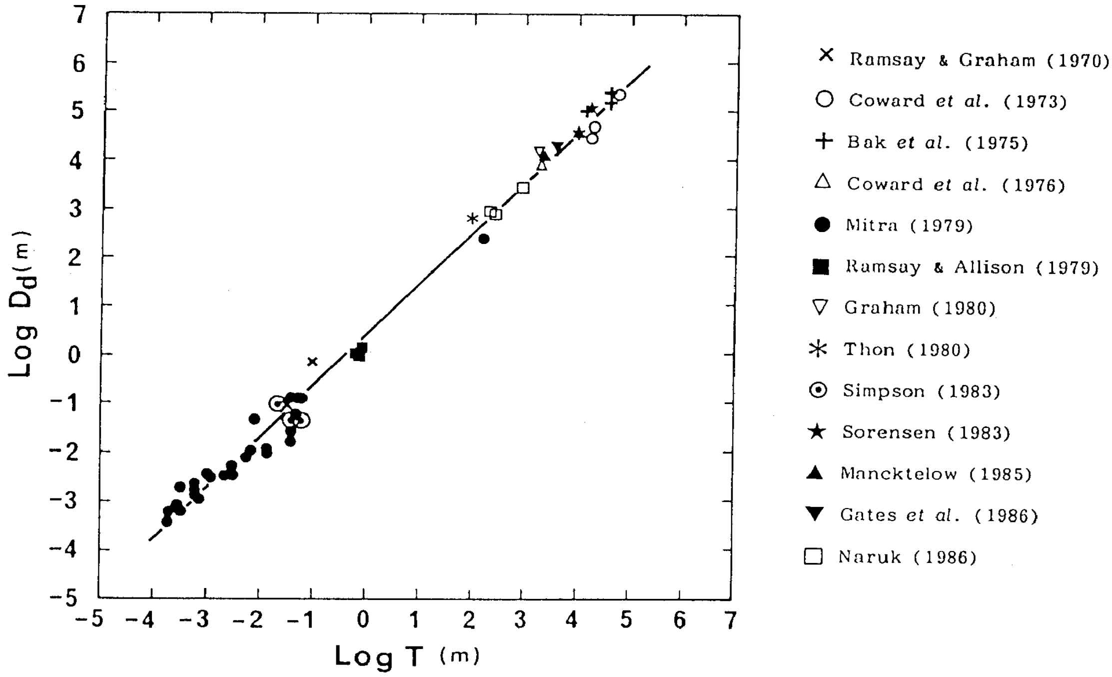

Fig. 8. Correlation between the displacement $D_d(m)$ and the thickness $T(m)$ of mylonitic zones on double logarithmic scale.

tion (39) is nearly equal to 1, $\bar{s}$ is constant irrespective of the size of mylonitic zones.

From pseudotachylite zones cutting the Lewisian gneiss in the Outer Hebrides, Sibson (1975) proposed the relationship between thickness and separation with a correlation coefficient $\gamma=0.94$ as follows,

$$D_d = 7277\ T^{1.73}. \qquad (40)$$

This relation (40) is a significantly better fit than a linear relation, and indicates that the change in displacement for a given change in thickness is larger as the size of pseudotachylite zone increases. That is to say, the thicker the pseudotachylite zone, the higher the tectonic activity proportion of D_d versus T.

The relationship for mylonitic zones and pseudotachylite zones can also be expressed by the power-law form as relation (34). Because relations (39) and (40) deal with an ensemble of mylonitic zones or pseudotachylite zones, the relations (39) and (40) represent average trends of the mylonitic zone and pseudotachylite zone, respectively.

Therefore, it should be emphasized that the idea of allometric growth can be extended to not only the $D_d - T$ relationship of faults, but also $D_d - T$ relationship of ductile shear zones such as mylonitic zones and pseudotachylite zones. In this paper, "ductile" means the strain in which material continuity is maintained or in which discontinuity is very small.

d. INDIVIDUAL AND SYSTEM OF FRACTURES

The relationship between the cumulative size ($r_S = \sum_i r_i$) of a fracture size or fragment size and the largest size (r_{max}) of them can be expressed by

$$r_{max} \sim r_S^{\,1/D} \qquad (41)$$

(see Appendix). Moreover, noting $m \sim r^3$, the relation (41) can be written by

$$m_{\max} \sim m_s^{1/D} \quad (42)$$

where $m_{\max}$ is the largest mass of fragments and $m_s(=\sum_i m_i)$ is the cumulative mass of fragments. Since we can identify $r_{\max}(m_{\max})$ with an individual of a fracture system and $r_s(m_s)$ with the population in the fracture system, the relations (41) and (42) indicate that an individual in a fracture system is related allometrically to system of the fracture. Therefore, the relations (41) and (42) indicate the growth constraints of a fracture in a fracture system.

The power-law distribution has an upper limit of fragment size. Gault *et al.* (1963) determined this limit from a number of laboratory and field experiments. In this study, he developed a relationship between the largest fragment in the ejecta basket and the crater size. This relation is given by

$$m_{\max} = 0.2\ m_s^{0.8}. \quad (43)$$

This relation (43) indicates the size of the largest ejecta fragment increases with crater size. In this result, we can recognize that the growth constraint of a fracture in a fracture system is given by the relation (42).

e. SCALING LAW OF FRACTURES IN THE SOLID EARTH

Kanamori and Anderson (1975) and Scholz (1982) found a linear relationship between the thickness of a fault and the amount of slip during a single seismic event, and proposed a scaling law as follows,

$$C_Y = C_X$$
$$C_Y = Y_i/Y_j,\ C_X = X_i/X_j, \quad (44)$$
$$i, j = 1, 2, \cdots\cdots, n$$

where dimensionless C_Y and C_X are respectively defined as scale ratios on Y and X between faults (i and j) of different size scales. The growth pattern shown by the relation (44) is isometric (self-similar). Scholz (1987) has derived a model based on wear theory which predicts a linear relationship between displacement D_d and gouge thickness T.

On the other hand, Watterson (1986) and Walsh and Watterson (1988) derived the relationship $D_d \sim W^2$ where W is the length of semi-major axis of top loop, on the basis of an "arithmetic" growth model. This "arithmetic" model bases on that the ratio between the amount of slip and fault length is constant for all slip events of a single fault. This condition means that slip always occurs when the shear stress on a fault plane reaches a fixed critical value and ceases when the amount of slip is sufficient to have reduced the stress to another fixed value (Walsh and Watterson, 1988). This is a special case of the anisometric growth, where $\beta = 2$ and can not be expressed by the relation (44).

According to various fractures in the Solid Earth, the growth pattern of fractures over a wide range of size scales is not necessarily isometric (self-similar). Therefore, the scaling law of fractures over a wide range of size scales should be represented as:

$$C_Y = C_X^{\beta}. \quad (45)$$

Moreover, this relation (45) can also be extended to the relation between individual and system of the fracture and the $D_d - T$ relationship for the ductile shear zone.

6. CONCLUSIONS

This study of fractures in the Solid Earth can be summarized as follows:

1) Several empirical studies on fractures have demonstrated a power-law dependence of the cumulative number of fragments of which sizes are larger than size r, $N(r) \sim r^{-D}$. This is taken as evidence that the fracturing is scale

invariant process concerning the size distribution.

2) Fractures can be described from a viewpoint of fractal. This description derives mathematically Gaudin-Schuhmann relation and Charles' relation and is sufficiently general in incorporation of the three theories on size reduction: Rittinger's theory, Kick's theory and Bond's theory.

3) The fractal dimension (D) provides a measure of the relative importance of large versus small objects and is related to energy density for fracturing and the Weibull's coefficient of uniformity (w) when the "size effect" of tensile strength is taken into consideration.

4) The D-values of fault gouge may increase as the faulting process proceeds. Therefore, the D-value of fault gouge can be a useful parameter of the nature and process of faulting, such as the intensity of shear fracture or the activity of faults. Moreover, the P-values of fractures in the Solid Earth may reflect the rock properties and tectonic conditions. These results are in incorporation of Conclusion (3). This may be suitable to the b-value as a measure of seismicity.

5) Fracture surface is also a typical example of fractals. The specific surface area S of each fraction is plotted as function of the mean particle diameter $\bar{r}$ of the fault gouges. Then the surface fractal dimension D' can be defined by $S \sim \bar{r}^{D'-3}$.

6) The D'-value of fault fragments may increase with increasing intensity or activity of faulting. Therefore, the surface fractal dimension D' may be a useful parameter of the nature and process of fault.

7) This author checks the self-affinity of fault trace curve by the scale-independent analysis (Matsushita and Ouchi, 1989). These results indicate that the fault trace seems to be self-affine and not self-similar.

8) The growth pattern of various fractures, such as faults, pull-apart basins, landslides, crater morphologies, and stream patterns, over a wide range of size scales is allometric and not necessarily isometric (self-similar). Therefore, the scaling law should be represented as $C_Y = C_X^{\beta}$. Moreover, this relation can also be extended to the relation between individual and system of the fracture and the $D_a - T$ relationship for the ductile shear zone.

The new theoretical concept introduced in this paper is the fractal concept for fracturing in the Solid Earth. The fractal concept can derive a variety of simple power-law relations for fracturing and is sufficiently in incorporation of the empirical relation for fracturing in the Solid Earth. A question arises why the fracturing in the Solid Earth is scale-invariant process and by what kind of mechanism the scaling exponents (D, D', H, β) are determined. To solve it one has to know details of dynamics of fracturing, which is a very difficult problem open to the fracturing.

7. APPENDIX

Now, let r_s be the cumulative fracture size in a fracture system. Then r_s can be approximated by

$$r_s = \int_{r\min}^{\infty} r d\left[-\frac{C}{r^D}\right] = \int_{r\min}^{\infty} \frac{rDC}{r^{D+1}}$$

$$= DC \int_{r\min}^{\infty} \frac{1}{r^D} = \frac{D}{D-1} C r_{\min}{}^{D-1}$$

where $r_{\min}$ is the smallest size of one. Therefore,

$$C = r_s r_{\min}{}^{1-D}\left[\frac{D-1}{D}\right]. \qquad \text{(a)}$$

Here, let $r_{\max}$ be the largest fracture size of a fracture system. Then,

$$1 = \frac{C}{r_{\max}{}^{D}}. \qquad \text{(b)}$$

Combining (a) and (b), the relationship between r_s and $r_{\max}$ is given by

$$r_{\max} = C^{1/D} = \left\{ r_s r_{\min}^{1-D} \left[\frac{D-1}{D} \right] \right\}^{\frac{1}{D}}$$

or

$$r_{\max} \sim r_s{}^{1/D}.$$

This mathematical derivation is equal to the Beckmann's (1958) allometric growth model for the city pouplation.

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