Statistical Analyses of Populations by Electronic Computer

Tohoku Univ., Sci. Rep., 2nd ser. (Geol.), v. 43, no. 1, p. 41–58,
6 figs., October 20, 1971

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ABSTRACT

This paper describes the method of computation and program of the electronic computer for statistical analyses of populations. The program includes an analysis for population structure by "Motomura's Plane" and principal factor analysis for correlative analysis.

INTRODUCTION

In the field of paleontology, one is confronted with the assemblages of the remains of dead bodies (thanatocoenoses) preserved in sediments. In this field, the ecological relationships among the individuals when they were alive are no longer observed. Attempts have been made, therefore, to reconstruct the ecological relationship using statistical approaches. In such studies, the statistics invariably sought are the correlation coefficients, and the number of calculations to obtain the correlation coefficient increases exponentially with the number of populations (assemblages) to be compared and with the numbers of constituents making up the population. In analyses of the assemblages of the microfossils which involve large numbers of species and individuals, the calculation soon gets a formidable number to be handled by a desk calculator. Accordingly, one finds the use of a large-sized computor more and more frequently in micropaleontological and sedimentological literatures, particularly those applying the factor analysis to the records.

In this paper the method of computation and a program of electronic computer are described. The program was made by the authors and used for analyses of assemblages of benthonic and planktonic Foraminifera in sedimentary rocks (Niitsuma, 1971), planktonic Foraminifera in deep sea cores (Oba, 1967, 1970), and benthonic Foraminifera in Recent sea bottom (Ujiié *et al.*, 1969).

ACKNOWLEDGMENTS

Thanks are due to Professors Kotora Hatai, Kiyoshi Asano, Chinami Watari, and Dr. Hisao Nakagawa of the Tohoku University for the important discussion and for reading the manuscript. Acknowledgments are also due to Dr. Taro Kanaya for his valuable suggestions and for reviewing the manuscript.

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STATISTICAL TREATMENT IN ANALYSIS OF POPULATIONS

It is a common method of statistical expression of population to describe by the frequency of each constituent of the population in a given area or a sample. Such data could be analyzed by two major methods; i) analysis of structure of the population and ii) correlative analysis of similarity and difference of frequencies of the constituents with those of the other populations. The program described in this paper includes an analysis for population structure by "Motomura's Plane" (Niitsuma, 1968) and principal factor analysis (Harman, 1967) for correlative analysis.

1) ANALYSIS BY "MOTOMURA'S PLANE"

The structure of a population was first studied statistically by Motomura (1932). He concluded that the frequencies of the constituents of a faunal living in a marsh satisfied the relation of a geometric progression, defined by

$$\log y + ax = b$$

where, x is the rank of frequency of a constituent in a population, y is the frequency of the constituent, and a and b are constants which characterize the population. For easier treatment, the equation is rewritten as

$$\log y + a(x-1) = \log B$$

where, B is the constant corresponding to the frequency of the most frequent constituent in a population. This equation expresses that the frequency of each constituent in the population satisfies the relation of geometric progression with the value of a constant ratio, $1/10^a$.

The sum of the frequencies of all the constituents in the population can be written as

$$\sum_{x=1}^{\infty} y = \frac{B}{1-\frac{1}{10^a}}$$

If the frequency of the constituent, y is expressed in percentage (i.e. the sum of y of all constituents in a population would be 100), the equation becomes,

$$B = 100\left(1-\frac{1}{10^a}\right)$$

The equation gives a curve, Motomura's Line (Niitsuma, 1968), on a plane with the coordinate axes of a and B which is called Motomura's Plane (Niitsuma, 1968). A given population fulfilling the relation suggested by Motomura (1932) can be plotted as a point on Motomura's Line. In case of a population which does not fulfill Motomura's Law, the point of the population would not be plotted along the Motomura's Line on the Motomura's Plane though the constants, a and B, would be calculated by the least square method using the frequencies of only some higher ranked constituents. For example, if the population satisfies the relation of logarithmic normal distribution (Preston, 1948), the point for the population would be located on Motomura's Line (Fig. 1). According to Nobuhara *et al.* (1954) and Whittaker (1965), the coactions among the constituents are stronger or the environment for the constituents is less suitable in the population which satisfies Motomura's Law than in that which satisfies the relation of logarithmic normal distribution.

The analysis by Motomura's Plane is a method in which the relation of frequencies and rank of frequency of the constituents in each population is projected as a point and

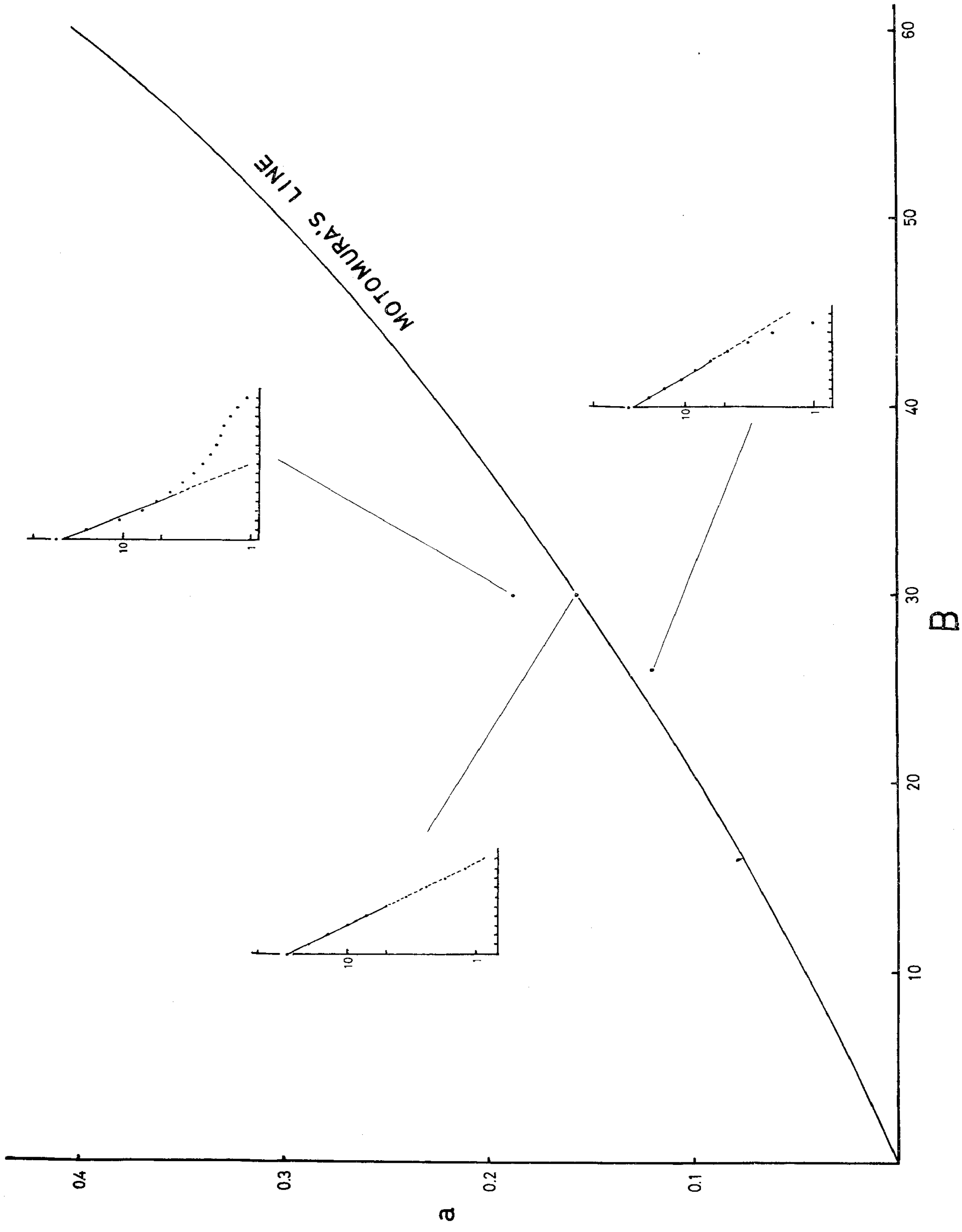

Fig. 1 Motomura's Plane and Motomura's Line.

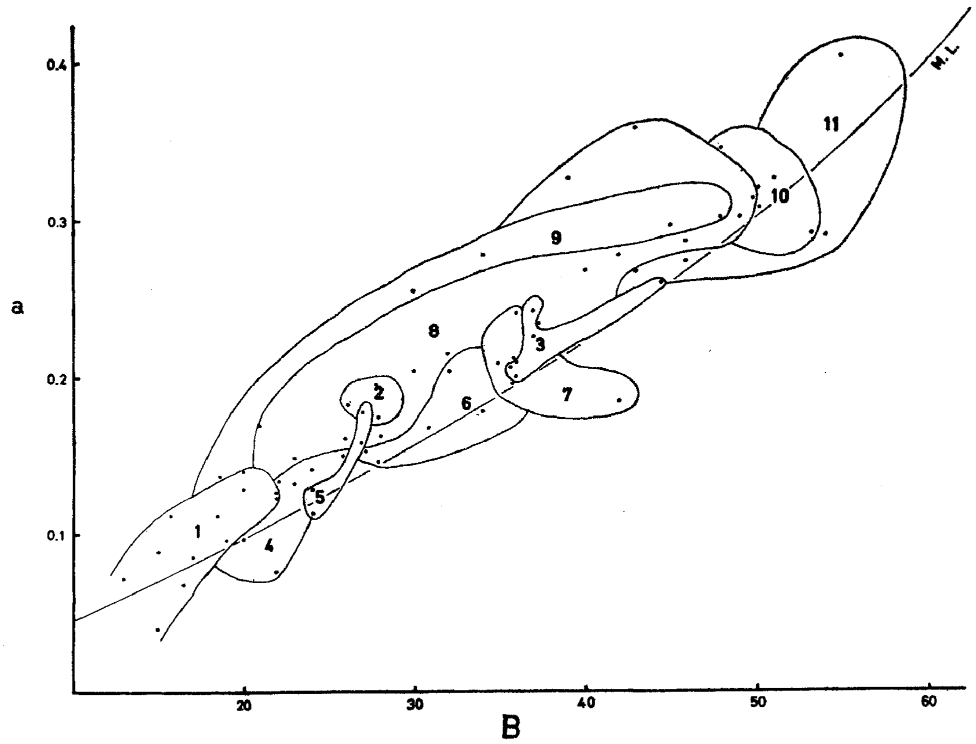

Fig. 2 Projections of benthonic foraminiferal faunas in Todos Santos Bay, Baja California, on Motomura's Plane.
M.L.: Motomura's Line
1: *Bolivina-Bulimina* fauna I, 2: *Bolivina-Bulimina* fauna II, 3: *Bolivina* fauna, 4: complex fauna, 5: *Reophax* fauna, 6: *Cassidulina* fauna I, 7: *Eggerella* fauna, 8: *Rotalia-Discorbis* fauna, 9: *Trochammina* fauna, 10: *Proteonina* fauna, 11: *Cassidulina* fauna II

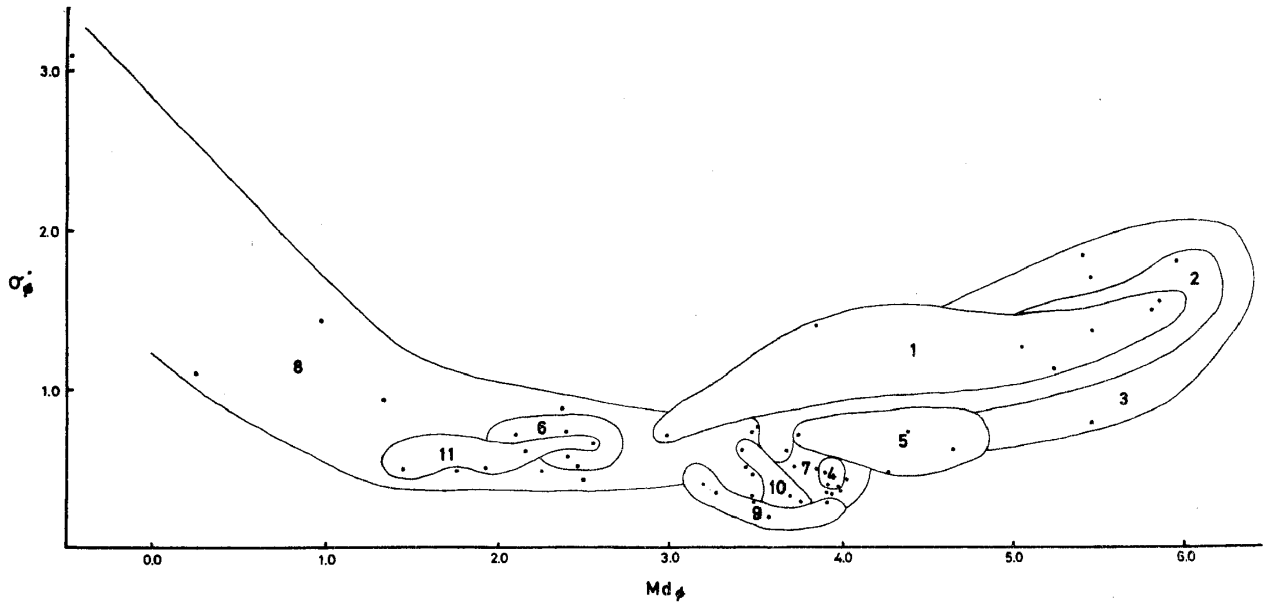

Fig. 3 The relation between benthonic foraminiferal faunas and grain-size distribution of the bottom sediments.
Mdϕ: median diameter, $\sigma'\phi$: standard deviation
1–11: benthonic foraminiferal faunas the same as in Fig. 2.

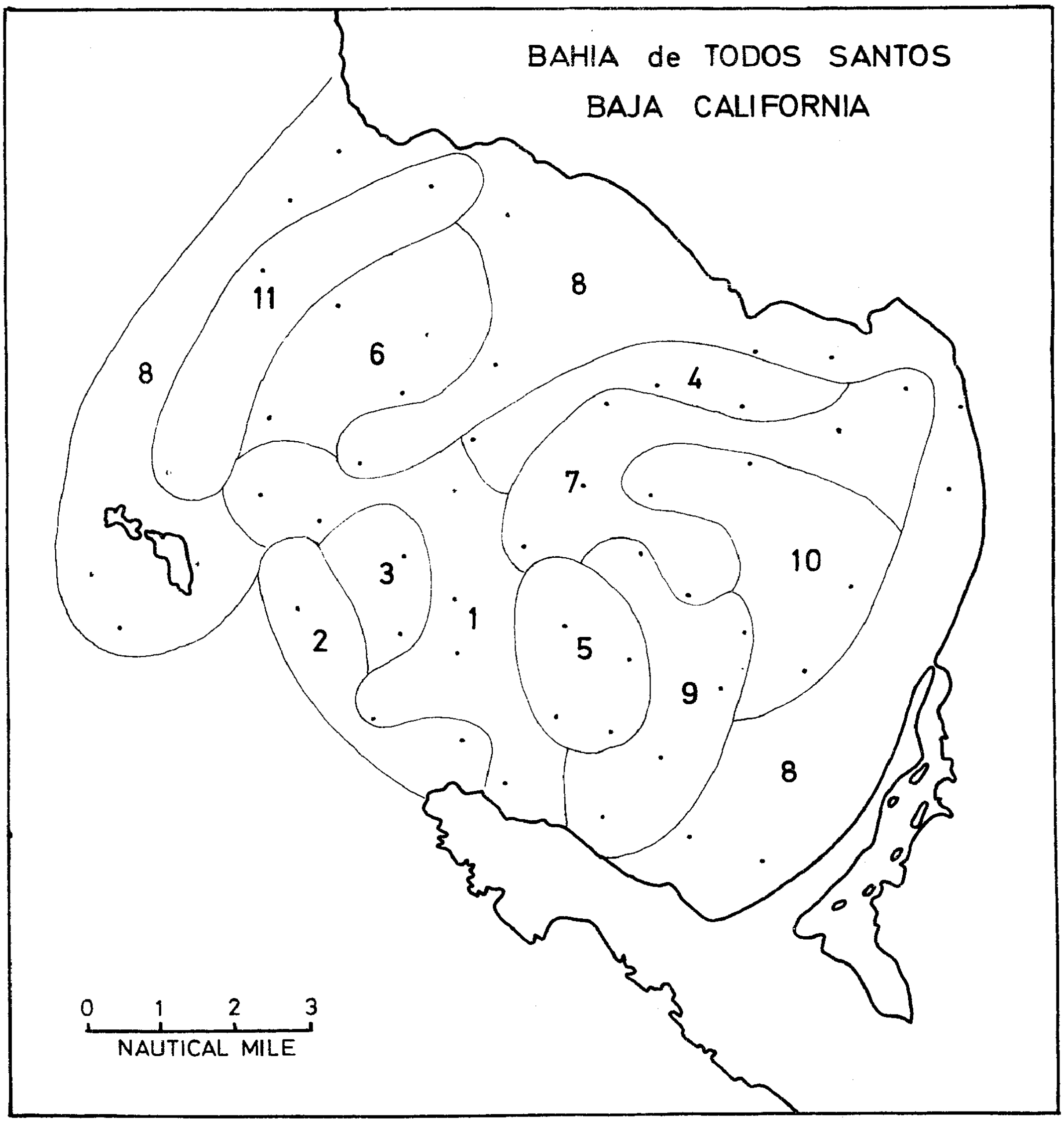

Fig. 4 The geographical distributions of benthonic foraminiferal faunas in Todos Santos Bay, Baja California.
1–11: benthonic foraminiferal faunas the same as in Fig. 2.

the position of the point on a plane demonstrating the character of the structure of each population. By this analysis, the benthonic foraminiferal faunas in Todos Santos Bay described by Walton (1955) could be divided into eleven faunas corresponding to the sedimentary environments (Figs. 2, 3 and 4; Niitsuma, 1968). The method has been applied to analyses of fossil foraminiferal faunas and has been proved to be effective in the interpretation of paleoecology and paleoenvironment (Niitsuma, 1968, 1971).

2) PRINCIPAL FACTOR ANALYSIS

Factor analysis which is one of correlation analyses has been developed mainly by psychologists since the early part of this century and the results have been compiled by Harman (1967). Factor analysis has been utilized in paleontological and sedimentological studies by many authors (e.g. Imbrie *et al.*, 1962; Hattori, 1967; Oba, 1967, 1969, 1970).

In the present paper the terminology of Harman (1967) is followed.

Factor analysis is based on the idea that the frequency of each constituent of each population can be described by a linear model as follows:

$$z_{ji} = a_{j1} F_{1i} + a_{j2} F_{2i} + \cdots + a_{jm} F_{mi} + u_j U_{ji}$$

where, z_{ji} is a normalized frequency of a constituent, i, in a population j. a_{j1}, a_{j2}, . . a_{jm} are loadings of the common factors among the treated populations F_{j1}, F_{j2}, . . F_{mi}, and u_j is a loading of unique factor U_{ji} in population j. Every factor is a normalized orthogonal vector. In factor analysis the loads of factors, a_{j1}, a_{j2}, . . a_{jm} are computed from the values of z_{ji}.

Correlation coefficent r_{jk} is described by the equation:

$$r_{jk} = \frac{\sum_{i=1}^{N} z_j z_k}{N} = a_{j1} a_{k1} + a_{j2} a_{k2} + \cdots + a_{jm} a_{km}$$

where, N is the number of constituents. This equation can be described in matrix form as

$$R = AA'$$

$$\begin{pmatrix} h_1^2 & r_{12} & \cdots & r_{1n} \\ r_{21} & h_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ r_{nl} & \cdots & & h_n^2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{nl} & \cdots & & a_{nn} \end{pmatrix} \times \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{ln} & \cdots & & a_{nn} \end{pmatrix}$$

where, R is a matrix of correlation coefficents r_{jk}, A is a matrix consisting of loadings of common factors a_{ij}, and A' is the transpose matrix of A. The diagonal entries of R, h_1^2. . h_n^2, are called as communality and are described as

$$\begin{aligned} h_j^2 &= a_{j1}^2 + a_{j2}^2 + \cdots + a_{jn}^2 \\ &= r_{jj} \end{aligned}$$

Communality is equal to a self-correlation coefficient, and the sum of the variances for common factors. The whole variance of the population in this case consists of those for common factors and unique factor and the sum of the values is 1.00. The value of communality, however, could not be calculated mathematically and the values should be estimated in the factor analysis.

If an eigenvalue and an eigenvector belonging to the eigenvalue of matrix R are expressed as λ and x, by definition

$$R x = \lambda x$$

Because R is a real symmetric matrix, shown as

$$\begin{aligned} r_{jk} &= a_{j1} a_{k1} + a_{j2} a_{k2} + \cdots + a_{jn} a_{kn} \\ &= a_{k1} a_{j1} + a_{k2} a_{j2} + \cdots + a_{kn} a_{jn} \\ &= r_{kj} \end{aligned}$$

all eigenvalues are non-negative and normalized orthogonal eigenvectors x_j corresponding to each eigenvalue can be calculated.

If a matrix P is defined by

$$P = [x_1, x_2, \cdots x_n]$$

R must be an orthogonal matrix, because $x_1 \ldots x_n$ are orthogonal. Therefore, R can be diagonalized by P:

$$PRP' = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

If D is defined by

$$D = \begin{pmatrix} \sqrt{\lambda_1} & & & 0 \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_n} \end{pmatrix}$$

the next equation should be satisfied,

$$PRP' = DD'$$

and

$$R = P'DD'P$$

If the relation of $P'D = A$ is defined as,

$$R = AA'$$

Therefore, the loadings of the factors can be computed by using the eigenvalue and eigenvector as follow:

$$A = \sqrt{\lambda}\, x$$

PROGRAM FOR ELECTRONIC COMPUTER

The statistical analyses for the populations have been carried out by the electronic computer NEAC 2200 Model 500 in the Computer Center of the Tohoku University. The language for the program is FORTRAN L which is almost equal to FORTRAN IV.

The data for the computation are the frequencies of the constituents in the populations and the names of the constituents and populations, and the following computations are carried out by this program. The constants, a and B, for the analysis by Motomura's Plane are computed in the following steps; i) the frequencies of the constituents in a population are rewritten in percentage, ii) higher ranked constituents are chosen, and iii) the constants are computed by the least square method:

$$\log B = \frac{\begin{vmatrix} \Sigma \log y & \Sigma (x-1) \\ \Sigma (x-1) \log y & \Sigma (x-1)^2 \end{vmatrix}}{\begin{vmatrix} n & \Sigma (x-1) \\ \Sigma (x-1) & \Sigma (x-1)^2 \end{vmatrix}}$$

$$-a = \frac{\begin{vmatrix} n & \Sigma \log y \\ \Sigma (x-1) & \Sigma (x-1) \log y \end{vmatrix}}{\begin{vmatrix} n & \Sigma (x-1) \\ \Sigma (x-1) & \Sigma (x-1)^2 \end{vmatrix}}$$

where, n is the number of high ranked constituents in the population and in this program the value of n is chosen to be six, regarding the statistical reliability of the frequencies and description of the structural difference of each population.

For the factor analysis, the data consisting of frequencies of constituents in the populations are normalized and the correlation coefficient matrix R is computed. The authors estimate the diagonal entries of R (communalities, h^2) by

$$h_j^2 = 1 - \frac{1}{r_{jj}'}$$

basing on the relation (Harman, 1967);

$$h_j^2 \geqq 1 - \frac{1}{r_{jj}'}$$

where, r_{jj}' is a diagonal entry of the inverse matrix R^{-1} of R. The eigenvalues and normalized eigenvectors of R are computed by Jacobi's method. This part of the program has been made by referring to the program HDIAG made by T. Shimizu of the Computation Centre of the University of Tokyo. The loadings matrix of the common factors A is computed by

$$A = \sqrt{\lambda}\, x$$

On a single set of data, the principal factor analysis could be carried out not only on populations but also on their constituents by this program as well as by the reserved control cards.

The following items are written by this program; the constants for the analysis of Motomura's Plane, a and B, the name and frequencies in percentage and logarithm of six higher ranked constituents, correlation coefficent matrix, the eigenvalues, the eigenvectors of the matrix, and the loadings of six higher ranked common factors.

The capacity of memories needed in the present program is about 40000 words in the case of the size of the data matrix, 70×70.

Flow chart of this computation and program of electronic computer are shown in Figs. 5 and 6.

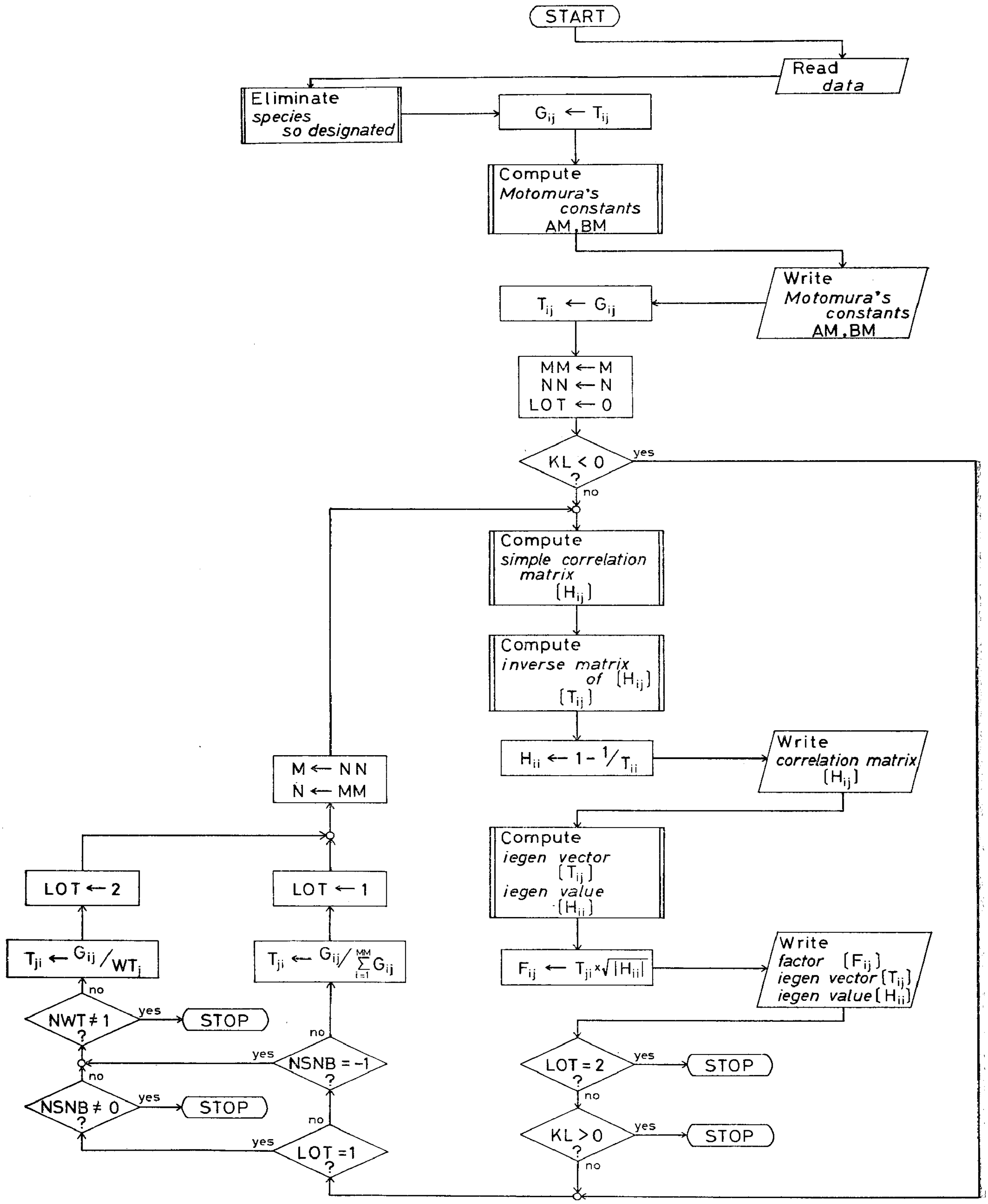

Fig. 5 Flow chart of the program for analyses of populations.

```
C
C       *** PROGRAM FOR FACTOR ANALYSIS *** (A)
C
C     DATA DECK......COM,KK,KL,NSNB,NWT,(M,N),NSTSP,NST(J),NSP(I),MATR,
C                      T(I,J),WT(J),LSP(L),(A BLANK CARD).
C
C         1.COM......10A8
C               A CARD WRITEN COMENTS ABOUT DATA, CAN USE ALL COLUMNS.
C             ( IF WITHOUT COMENT, NEED A BRANK CARD )
C         2.KK.......I5(5,0,-5)
C               KK=5 : GO TO END OF PROGRAM, WRITE CORRELATION MATRIX
C               KK=0 : GO TO END OF PROGRAM, NOT WRITE CORRELATION MATRIX
C               KK=-5: WRITE ONLY CORRELATION MATRIX, NOT GO TO END OF
C                      PROGRAM.
C         3.KL.......I5(5,0,-5)
C               KL=5 : CORRELATION MATRIX BETWEEN STATIONS ONLY
C               KL=0 : CORRELATION MATRIX BETWEEN STATIONS AND BETWEEN
C                      SPECIES
C               KL=-5: CORRELATION MATRIX BETWEEN SPECIES ONLY
C         4 NSNB.....I5(1,0,-1)
C               NSNB=1 : DO  SAME NUMBER OF SPECIMEN ONLY (...NWT=1 OR 0)
C               NSNB=-1: DO  SAME WEIGHT OF SAMPLE ONLY(...NWT=1)
C               NSNB=0 : DO  BOTH(...NWT=1)
C         5.NWT......I5(1,0)  (WEIGHT OF SAMPLE IN A STATION)
C               NWT=1 : WITH DATA
C               NWT=0 : WITHOUT DATA.
C               (IF (NWT=0)......NSNB=1)
C         6.M,N......2I5(M,N)
C               M : SPECIES NUMBER (MAXIMAM  70)
C               N : STATION NUMBER (MAXIMAM  70)
C         7.NSTSP....I5(-1,0,1,2)
C              NSTSP=-1:WITHOUT SPECIES NAME DATA (STATION NAME ONLY)
C              NSTSP=0 :WITHOUT BOTH,SPECIES AND STATIONS NAME DATA
C              NSTSP=1 :WITHOUT STATIONS NAME DATA (SPECIES NAME ONLY)
C              NSTSP=2 :BOTH SPECIES AND STATIONS NAME DATA
C         8.NST(J)...10A7(STATIONS NAME DATA)
C         9.NSP(I)...10A7(SPECIES NAME DATA)
C        10.MATR.....I5(1,-1)
C               MATR=1 : DATA MATRIX ((T(I,J),I=1,M),J=1,N)
C                          T(1,1),T(2,1),...,T(M,1),T(1,2),.....,T(M,N)
C               MATR=-1: DATA MATRIX ((T(I,J),J=1,N),I=1,M)
C                          T(1,1),T(1,2),...,T(1,N),T(2,1),.....,T(M,N)
C        11.T(I,J)...14F5.1(DATA MATRIX)
C        12.WT(J)....7F10.3(WEIGHT OF SAMPLE)
C                   (IF NWT=0 (WITHOUT DATA OF WEIGHT) , NEED NO CARD)
C        13.LSP(L)...I5 (ELIMINATE SPECIES NUMBER(I))
C                   ELIMINATE TWO OR MORE SPECIES, NEED SAME LEAVES OF
C                   CARD. (IF ELIMINATE NO SPECIES, NEED NO CARD)
C        14.A BLANK CARD (DENOTED END OF DATA DECK)
C
         DIMENSION F(6,70),G(70,70),H(70,70),T(140,140),DIV(70),IQ(70),X(70
        1),NST(70),NSP(70),WT(70),COM(10),LSP(70)
         READ(5,501)COM
         READ(5,502)KK,KL,NSNB,NWT,M,N,NSTSP
         NSTSP=NSTSP+2
         GO TO (101,102,103,101),NSTSP
     101 READ(5,503)(NST(J),J=1,N)
         IF(NSTSP.EQ.1) GO TO 102
     103 READ(5,503)(NSP(I),I=1,M)
     102 CONTINUE
         READ(5,504)MATR
         IF(MATR.EQ.1) GO TO 110
         READ(5,505)((T(I,J),J=1,N),I=1,M)
         GO TO 111
     110 READ(5,505)((T(I,J),I=1,M),J=1,N)
     111 CONTINUE
         IF(NWT.EQ.0) GO TO 115
         READ(5,506) (WT(J),J=1,N)
     115 CONTINUE
     501 FORMAT(10A8)
```

Fig. 6a

```
  502 FORMAT(I5/I5/I5/I5/2I5/I5)
  503 FORMAT(10A7)
  504 FORMAT(I5)
  505 FORMAT(14F5.1)
  506 FORMAT(7F10.3)
      LLOT=0
      L=0
  130 CONTINUE
      L=L+1
      READ(5,520)LSP(L)
  520 FORMAT(I5)
      IF(LSP(L).EQ.0)GO TO 131
      IF(L.EQ.1) GO TO 120
      DO 121 LL=2,L
      IF(LSP(LL-1).GE.LSP(LL)) GO TO 122
      LSP(L)=LSP(L)-1
  122 CONTINUE
  121 CONTINUE
  120 CONTINUE
      LLSP=LSP(L)
      NNSP=NSP(LLSP)
      DO 123 J=1,N
      G(M,J)=T(LLSP,J)
  123 CONTINUE
      MMI1=M-1
      DO 124 I=LLSP,MMI1
      IPUL1=I+1
      DO 125 J=1,N
      T(I,J)=T(IPUL1,J)
  125 CONTINUE
      NSP(I)=NSP(IPUL1)
  124 CONTINUE
      DO 126 J=1,N
      T(M,J)=G(M,J)
  126 CONTINUE
      NSP(N)=NNSP
      LLOT=LLOT+1
      GO TO 130
  131 CONTINUE
      M=M-LLOT
C     MOTOMURA
      DIMENSION TMAXM(6,70),IMAXM(6,70),GMAXM(6,70),NSPM(6,70)
      DO 1003 J=1,N
      TSUM=0.0
      DO 1004 I=1,M
      TSUM=TSUM+T(I,J)
 1004 CONTINUE
      DO 1005 I=1,M
      T(I,J)=(100.0*(T(I,J)))/TSUM
 1005 CONTINUE
 1003 CONTINUE
      DO 1001 J=1,N
      KM=0
 1006 TMAX=0.0
      DO 1002 I=1,M
      TIJ=T(I,J)
      IF(TIJ.LE.TMAX) GO TO 1002
      TMAX=TIJ
      IMAX=I
 1002 CONTINUE
      KM=KM+1
      TMAXM(KM,J)=TMAX
      IMAXM(KM,J)=IMAX
      NSPM(KM,J)=NSP(IMAX)
      T(IMAX,J)=0.0
      IF(KM.LT.6) GO TO 1006
 1001 CONTINUE
      DO 1007 KM=1,6
      DO 1008 J=1,N
      GMAXM(KM,J)=ALOG10(TMAXM(KM,J))
```

Fig. 6b

```
 1008 CONTINUE
 1007 CONTINUE
      KP=1
      DO 1009 J=1,N
      YSUM=0.0
      XSUM=0.0
      XYSUM=0.0
      X2SUM=0.0
      DO 1010 KM=1,6
      YSUM=YSUM+GMAXM(KM,J)
      XSUM=XSUM+FLOAT(KM)
      XYSUM=XYSUM+(FLOAT(KM))*GMAXM(KM,J)
      X2SUM=X2SUM+(FLOAT(KM))**2
 1010 CONTINUE
      BM=(YSUM*X2SUM-XSUM*XYSUM)/(6.0*X2SUM-XSUM*XSUM)
      AM=(6.0*XYSUM-YSUM*XSUM)/(6.0*X2SUM-XSUM*XSUM)
      BM=BM+AM
      BM=10.0**BM
      IF(KP.GT.5) KP=1
      IF(KP.EQ.1) GO TO 1017
      WRITE(6,1011) NST(J)
      GO TO 1018
 1017 WRITE(6,1016) NST(J)
 1018 KP=KP+1
      WRITE(6,1025) (NSPM(KM,J),KM=1,6)
      WRITE(6,1012) (TMAXM(KM,J),KM=1,6)
      WRITE(6,1013) (GMAXM(KM,J),KM=1,6)
      WRITE(6,1014) AM
      WRITE(6,1015) BM
 1009 CONTINUE
 1011 FORMAT(1H0//////1H ,12HSTATION NAME,8X,A7)
 1012 FORMAT(1H ,5X,10HPERCENTAGE,10X,6F10.2)
 1013 FORMAT(1H ,5X,15HLOG. PERCENTAGE,5X,6F10.4)
 1014 FORMAT(1H ,10X,11HMOTOMURAS A,14X,F10.4)
 1015 FORMAT(1H ,10X,19HMOTOMURAS B-PERCENT,6X,F10.4)
 1016 FORMAT(1H1,12HSTATION NAME,8X,A7)
 1025 FORMAT(1H ,5X,12HSPECIES NAME,8X,6(3X,A7))
      DO 1021 I=1,M
      DO 1021 J=1,N
 1021 T(I,J)=G(I,J)
C     MOTOMURA OWARI
      DO 127 J=1,N
      DO 128 I=1,M
  128 G(I,J)=T(I,J)
  127 CONTINUE
      MM=M
      NN=N
      IF(KL.LT.0) GO TO 990
      LOT=0
  989 CONTINUE
      HM=FLOAT(M)
      DO 1 J=1,N
      X(J)=0.0
      DO 2 I=1,M
      X(J)=X(J)+T(I,J)/HM
    2 CONTINUE
      DIV(J)=0.0
      DO 3 I=1,M
      DIV(J)=DIV(J)+(T(I,J)-X(J))**2
    3 CONTINUE
      DIV(J)=SQRT(DIV(J)/HM)
    1 CONTINUE
      DO 4 J=1,N
      DO 5 I=1,M
      T(I,J)=(T(I,J)-X(J))/DIV(J)
    5 CONTINUE
    4 CONTINUE
      DO 6 K=1,N
      DO 7 L=1,N
      H(K,L)=0.0
```

Fig. 6c

```
      DO 8 I=1,M
      H(K,L)=H(K,L)+T(I,K)*T(I,L)/HM
    8 CONTINUE
    7 CONTINUE
    6 CONTINUE
      M=2*N
      DO 11 I=1,N
      DO 12 J=1,N
      T(I,J)=H(I,J)
      L=J+N
      T(I,L)=0.0
      IF(I.EQ.J)T(I,L)=1.0
   12 CONTINUE
   11 CONTINUE
      DO 13 K=1,N
      DO 18 L=K,N
      AMAX=0.0
      IF(ABS(AMAX).GE.ABS(T(L,K))) GO TO 18
      IROW=L
      AMAX=T(L,K)
   18 CONTINUE
      DO 19 L=K,M
      B=T(IROW,L)
      T(IROW,L)=T(K,L)
      T(K,L)=B
   19 CONTINUE
      W=T(K,K)
      KPL1=K+1
      DO 14 J=KPL1,M
      T(K,J)=T(K,J)/W
   14 CONTINUE
      DO 15 I=1,N
      IF(I.EQ.K) GO TO 15
      W=T(I,K)
      DO 16 J=KPL1,M
      T(I,J)=T(I,J)-W*T(K,J)
   16 CONTINUE
   15 CONTINUE
   13 CONTINUE
      DO 17 K=1,N
      L=K+N
      H(K,K)=1.0-1.0/T(K,L)
   17 CONTINUE
      IF(KK.EQ.0) GO TO 988
      NPAG=0
      LP1=1
  399 CONTINUF
      LP40=LP1+39
      LPN=LP40-N
      IF(LPN)352,352,351
  351 LP40=N
  352 KP1=1
  398 CONTINUE
      KP15=KP1+14
      KPN=KP15-N
      NPAG=NPAG+1
      IF(LOT.EQ.0) GO TO 353
      DO 354 K=1,N
  354 NST(K)=NSP(K)
      WRITE(6,471)NPAG
  471 FORMAT(1H1,49HSIMPLE CORRELATION MATRIX BETWEEN SPECIES    PAGE(,I4
     1,1H))
      IF(LCT.EQ.2) GO TO 355
      WRITE(6,473)
  473 FORMAT(1H0,5X,32H*** SAME NUMBER OF SPECIMENS ***)
      GO TO 356
  355 WRITE(6,472)
  472 FORMAT(1H0,5X,29H*** SAME WEIGHT OF SAMPLE ***)
      GO TO 356
  353 WRITE(6,474)NPAG
```

Fig. 6d

```
 474 FORMAT(1H1,50HSIMPLE CORRELATION MATRIX BETWEEN STATIONS    PAGE(,I
    14,1H)//)
 356 CONTINUE
     WRITE(6,475)(COM(I),I=1,5)
 475 FORMAT(1H+,70X,2H( ,5A8,5H--- ))
     IF(KPN)315,315,320
 320 KP15=N
     GO TO(314,313,312,311,310,309,308,307,306,305,304,303,302,301),KPN
 314 WRITE(6,414)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 313 WRITE(6,413)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 312 WRITE(6,412)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 311 WRITE(6,411)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 310 WRITE(6,410)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 309 WRITE(6,409)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 308 WRITE(6,408)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 307 WRITE(6,407)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 306 WRITE(6,406)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 305 WRITE(6,405)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 304 WRITE(6,404)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 303 WRITE(6,403)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 302 WRITE(6,402)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 301 WRITE(6,401)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
     GO TO 400
 414 FORMAT(1H0,9X,5A8,2X,5A8,2X,4A8//(1H ,5(A7,1X,5F8.4,2X,5F8.4,2X,4F
    18.4/1H )))
 413 FORMAT(1H0,9X,5A8,2X,5A8,2X,3A8//(1H ,5(A7,1X,5F8.4,2X,5F8.4,2X,3F
    18.4/1H )))
 412 FORMAT(1H0,9X,5A8,2X,5A8,2X,2A8//(1H ,5(A7,1X,5F8.4,2X,5F8.4,2X,2F
    18.4/1H )))
 411 FORMAT(1H0,9X,5A8,2X,5A8,2X,1A8//(1H ,5(A7,1X,5F8.4,2X,5F8.4,2X,1F
    18.4/1H )))
 410 FORMAT(1H0,9X,5A8,2X,5A8//(1H ,5(A7,1X,5F8.4,2X,5F8.4/1H )))
 409 FORMAT(1H0,9X,5A8,2X,4A8//(1H ,5(A7,1X,5F8.4,2X,4F8.4/1H )))
 408 FORMAT(1H0,9X,5A8,2X,3A8//(1H ,5(A7,1X,5F8.4,2X,3F8.4/1H )))
 407 FORMAT(1H0,9X,5A8,2X,2A8//(1H ,5(A7,1X,5F8.4,2X,2F8.4/1H )))
 406 FORMAT(1H0,9X,5A8,2X,1A8//(1H ,5(A7,1X,5F8.4,2X,1F8.4/1H )))
 405 FORMAT(1H0,9X,5A8//(1H ,5(A7,1X,5F8.4/1H )))
 404 FORMAT(1H0,9X,4A8//(1H ,5(A7,1X,4F8.4/1H )))
 403 FORMAT(1H0,9X,3A8//(1H ,5(A7,1X,3F8.4/1H )))
 402 FORMAT(1H0,9X,2A8//(1H ,5(A7,1X,2F8.4/1H )))
 401 FORMAT(1H0,9X,1A8//(1H ,5(A7,1X,1F8.4/1H )))
 315 WRITE(6,415)(NST(K),K=KP1,KP15),(NST(L),(H(K,L),K=KP1,KP15),L=LP1,
    1LP40)
```

Fig. 6e

```
  415 FORMAT(1H0,9X,5A8,2X,5A8,2X,4A8,A7//(1H ,5(A7,1X,5F8.4,2X,5F8.4,2X
     1,5F8.4/1H )))
      KP1=KP1+15
      IF(KP1.LE.N) GO TO 398
  400 CONTINUE
      LP1=LP1+40
      IF(LP1.LE.N) GO TO 399
      IF(KK.LT.0) GO TO 987
  988 CONTINUE
      IEGEN=0
      IF(IEGEN.NE.0) GO TO 725
      DO724 I=1,N
      DO723 J=1,N
      IF(I.NE.J) GO TO 722
      T(I,J)=1.0
      GO TO723
  722 T(I,J)=0.0
  723 CONTINUE
  724 CONTINUE
  725 NR=0
      IF(N.LE.1) GO TO 1000
C     SCAN FOR LARGEST OFF-DIAGONAL ELEMENT IN EACH ROW
C     X(I) CONTAINS LARGEST ELEMENT IN ITH ROW
C     IQ(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
      NMI1=N-1
      DO730 I=1,NMI1
      X(I)=0.0
      IPL1=I+1
      DO720 J=IPL1,N
      IF(X(I).GT.ABS(H(I,J))) GO TO720
      X(I)=ABS(H(I,J))
      IQ(I)=J
  720 CONTINUE
  730 CONTINUE
C     SET INDICATOR FOR SHUT-OFF,RAP=2**-23,NR=NO. OF ROTATIONS
      RAP=1.192093E-7
      HDTEST=1.0E38
C     FIND MAXIMUM OF X(I) S FOR PIVOT ELEMENT AND TEST FOR END OF PRO
C     BLEM
  740 DO770 I=1,NMI1
      IF(I.EQ.1) GO TO760
      IF(XMAX.GE.X(I)) GO TO770
  760 XMAX=X(I)
      IPIV=I
      JPIV=IQ(I)
  770 CONTINUE
C     IS MAX.X(I) EQUAL TO ZERO,IF LESS THAN HDTEST,REVISE HDTEST
      IF(XMAX.EQ.0.0) GO TO 1000
      IF(HDTEST.LE.0.0) GO TO790
      IF(XMAX.GT.HDTEST) GO TO 748
  790 HDIMIN=ABS(H(1,1))
      DO 710 I=2,N
      IF(HDIMIN.LE.ABS(H(I,I))) GO TO 710
      HDIMIN=ABS(H(I,I))
  710 CONTINUE
       HDTEST=HDIMIN*RAP
C     RETURN IF MAX.H(I,J)LESS THAN(2**-23)ABS(H(K,K)-MIN)
      IF(HDTEST.GE.XMAX) GO TO 1000
  748 NR=NR+1
C     COMPUTE TANGENT,SINE AND COSINE,H(I,I),H(J,J)
      TANG=SIGN(2.0,(H(IPIV,IPIV)-H(JPIV,JPIV)))*H(IPIV,JPIV)/(ABS(H(
     1IPIV,IPIV)-H(JPIV,JPIV))+SQRT((H(IPIV,IPIV)-H(JPIV,JPIV))**2+4.0
     2*H(IPIV,JPIV)**2))
      COSINE=1.0/SQRT(1.0+TANG**2)
      SINE=TANG*COSINE
      HII=H(IPIV,IPIV)
      H(IPIV,IPIV)=COSINE**2*(HII+TANG*(2.0*H(IPIV,JPIV)+TANG*H(JPIV,
     1JPIV)))
      H(JPIV,JPIV)=COSINE**2*(H(JPIV,JPIV)-TANG*(2.0*H(IPIV,JPIV)-TANG
     1*HII))
```

Fig. 6f

```
      H(IPIV,JPIV)=0.0
      IF(H(IPIV,IPIV).GE.H(JPIV,JPIV)) GO TO 753
      HTEMP=H(IPIV,IPIV)
      H(IPIV,IPIV)=H(JPIV,JPIV)
      H(JPIV,JPIV)=HTEMP
      HTEMP=SIGN(1.0,-SINE)*COSINE
      COSINE=ABS(SINE)
      SINE=HTEMP
  753 CONTINUE
      DO 750 I=1,NMI1
      IF(I-IPIV)711,750,700
  711 IF(IQ(I).EQ.IPIV) GO TO 741
      IF(IQ(I).NE.JPIV) GO TO 750
  741 HTEMP=H(I,IPIV)
      TEMP=H(I,JPIV)
      H(I,IPIV)=0.0
      H(I,JPIV)=0.0
      IPL1=I+1
      X(I)=0.0
      DO 771 J=IPL1,N
      IF(X(I).GT.ABS(H(I,J))) GO TO 771
      X(I)=ABS(H(I,J))
      IQ(I)=J
  771 CONTINUE
      H(I,IPIV)=HTEMP
      H(I,JPIV)=TEMP
      GO TO 750
  700 IF(I.GE.JPIV) GO TO 750
      IF(IQ(I).NE.JPIV) GO TO 750
      K=IQ(I)
      HTEMP=H(I,K)
      H(I,K)=0.0
      IPL1=I+1
      X(I)=0.0
      DO 769 J=IPL1,N
      IF(X(I).GT.ABS(H(I,J))) GO TO 769
      X(I)=ABS(H(I,J))
      IQ(I)=J
  769 CONTINUE
      H(I,K)=HTEMP
  750 X(IPIV)=0.0
      X(JPIV)=0.0
      DO 772 I=1,N
      IF(I-IPIV) 773,772,774
  773 HTEMP=H(I,IPIV)
      H(I,IPIV)=COSINE*HTEMP+SINE*H(I,JPIV)
      IF(X(I).GE.ABS(H(I,IPIV))) GO TO 775
      X(I)=ABS(H(I,IPIV))
      IQ(I)=IPIV
  775 H(I,JPIV)=-SINE*HTEMP+COSINE*H(I,JPIV)
      IF(X(I)-ABS(H(I,JPIV))) 778,772,772
  774 IF(I-JPIV) 776,772,777
  776 HTEMP=H(IPIV,I)
      H(IPIV,I)=COSINE*HTEMP+SINE*H(I,JPIV)
      IF(X(IPIV).GE.ABS(H(IPIV,I))) GO TO 779
      X(IPIV)=ABS(H(IPIV,I))
      IQ(IPIV)=I
  779 H(I,JPIV)=-SINE*HTEMP+COSINE*H(I,JPIV)
      IF(X(I).GE.ABS(H(I,JPIV))) GO TO 772
  778 X(I)=ABS(H(I,JPIV))
      IQ(I)=JPIV
      GO TO 772
  777 HTEMP=H(IPIV,I)
      H(IPIV,I)=COSINE*HTEMP+SINE*H(JPIV,I)
      IF(X(IPIV).GE.ABS(H(IPIV,I))) GO TO 780
      X(IPIV)=ABS(H(IPIV,I))
      IQ(IPIV)=I
  780 H(JPIV,I)=-SINE*HTEMP+ COSINE*H(JPIV,I)
      IF(X(JPIV).GE.ABS(H(JPIV,I))) GO TO 772
      X(JPIV)=ABS(H(JPIV,I))
```

Fig. 6g

```
      IQ(JPIV)=I
  772 CONTINUE
      IF(IEGEN.NE.0) GO TO740
      DO 781 I=1,N
      HTEMP=T(I,IPIV)
      T(I,IPIV)=COSINE*HTEMP+SINE*T(I,JPIV)
  781 T(I,JPIV)=-SINE*HTEMP+COSINE*T(I,JPIV)
      GO TO740
 1000 CONTINUE
      DO751 I=1,6
      DO752 J=1,N
  752 F(I,J)=T(J,I)*SQRT(ABS(H(I,I)))
  751 CONTINUE
      WRITE(6,601)COM
  601 FORMAT(1H1,2H( ,10A8,2H ))
      WRITE(6,602)NR
  602 FORMAT(1H0,4X,3HNR=,I5)
      WRITE(6,603)(NST(J),(F(I,J),I=1,6),J=1,N)
  603 FORMAT(1H0,16X,8HFACTOR 1,5X,8HFACTOR 2,5X,8HFACTOR 3,5X,8HFACTOR
     14,5X,8HFACTOR 5,5X,8HFACTOR 6//(1H ,5(A8,2X,6F13.5/1H )))
      WRITE(6,604) IEGEN,(H(I,I),I=1,N)
  604 FORMAT(1H0/1H0,7H IEGEN=,I4/1H0,6HH(I,I),4X,6F13.5/1H0/(10X,6F13.5
     1/1H ))
      WRITE(6,601)COM
      WRITE(6,605)
  605 FORMAT(1H0,12HIEGEN VECTOR)
      WRITE(6,603)(NST(J),(T(J,I),I=1,6),J=1,N)
  987 CONTINUE
      IF(LOT.EQ.2) STOP
      IF(KL.GT.0 ) STOP
  990 CONTINUE
      IF(LOT.EQ.1) GO TO 942
      IF(NSNB.EQ.-1) GO TO 944
      X(J)=0.0
      DO 931 J=1,NN
      DO 932 I=1,MM
  932 X(J)=X(J)+G(I,J)
      DO 935 I=1,MM
  935 T(J,I)=G(I,J)/X(J)
  931 CONTINUE
      LOT=1
      GO TO 943
  942 CONTINUE
      IF(NSNB.NE.0) STOP
  944 CONTINUE
      IF(NWT.EQ.0) STOP
      DO 933 J=1,NN
      DO 934 I=1,MM
  934 T(J,I)=G(I,J)/WT(J)
  933 CONTINUE
      LOT=2
  943 CONTINUE
      N=MM
      M=NN
      GO TO 989
      END
```

Fig. 6h

Fig. 6 Program of electronic computer for analyses of populations.

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