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# *Deformation Rate Analysis : A New Method for In Situ Stress Estimation from Inelastic Deformation of Rock Samples under Uni-Axial Compressions*

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*Abstract* : We study the effect of previously applied stresses on inelastic deformation of rock specimens under cyclic uni-axial compression tests to develop a new method for estimating *in situ* stresses from core samples of bore-holes. The strain difference function is defined as a function of axial stress, the values of which are obtained by subtracting the axial strains observed in the first loading from those in the second one of two successive loading cycles on a rock specimen. The function is found to show a sharp gradient change at an axial stress nearly equal to the peak value of the previous stress applied artificially to the specimen. We propose a new method of deformation rate analysis (DRA) for *in situ* stress estimation from gradient changes of strain difference functions. The method is applied to the core samples retrieved from three depths of about 73, 100 and 143 m at two sites; one in Iwate Prefecture and the other in Ibaraki Prefecture, Japan. The results are as follows: 1) A sharp gradient change is found for a strain difference function to suggest the value of previous stress (probably *in situ* stress). 2) The previous stresses estimated for horizontal directions show an azimuthal dependence consistent with that required theoretically for a stress field. 3) The vertical previous stresses are nearly equal in value to the overburden pressures at the respective depths calculated from the densities of core samples. We conclude from these results that the values of previous stresses estimated by DRA indicate not the relative values but the absolute ones of the normal components of the *in situ* stress field. The DRA is thus found to be practically effective for *in situ* stress estimation.

## 1. Introduction

Rocks are known to have the property that they memorize the peak values of previously applied stresses, or previous stresses. The Kaiser effect, which is observed for the activity of acoustic emissions (AE) in a rock specimen under compression, is well-known as one of the phenomena showing the rock property of stress memory. Some efforts have been made to utilize this property for estimating *in situ* stresses.

The Kaiser effect is used to express the characteristic behavior that acoustic emissions in a rock specimen under a compression test are activated at applied stresses larger than the peak stress previously applied to the specimen. Kanagawa *et al.* (1977)

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proposed a method for estimating *in situ* stresses by utilizing the Kaiser effect of rock specimens under uni-axial compressions. In their method, the change in AE activity is measured with an increase in the applied stress. The value of the applied stress at which the AE rate becomes rather abruptly higher is taken as the value of the *in situ* stress. They applied their method to the core samples recovered from the downholes in which *in situ* stresses had been measured directly by a stress relief method. They found that the stresses estimated by their method agree well with the results by the direct measurement method both in value and in orientation. The method using the Kaiser effect is usually called AE method. Yoshikawa and Mogi (1981, 1982) developed an AE method to estimate *in situ* stresses at some sites. Although there are no independent data of *in situ* stress measurements to be compared, the estimated stresses are consistent in orientation with those inferred from other geophysical data.

Horibe and Kobayashi (1958) carried out an experiment to estimate previous stresses from the pressure-strain relations observed for rock specimens under cyclic loading tests of hydrostatic pressure. They successively increased the peak value of the applied pressure with the number of loading cycles. They found a relatively clear change in gradient of the pressure-strain curve near the peak pressure of the preceding cycle. Further, they applied their method to the specimens obtained form boring cores to estimate the values of *in situ* stresses at the boring site. However, the estimated values of *in situ* stresses do not seem very reliable because of the indistinct changes in gradient of the observed pressure-strain curves.

A method called differential strain curve analysis (DSCA) has been developed to determine the principal axes as well as the ratios of the principal values of *in situ* stress by Strickland and Ren (1980), Montgomery and Ren (1981), and Ren and Roegiers (1983). The DSCA is an extension of the differential strain analysis (DSA) proposed by Simmons and his collaborators to examine microcracks in rock samples (Simmoms *et al.*, 1974; Siegfried and Simmons, 1978). The method of DSCA is based on the assumption that a rock core retrieved from its downhole confined conditions will expand proportionally to the pre-existing *in situ* stress field. The expansion of the rock core is composed of elastic expansion and inelastic one due probably to microcracking. Their assumption requires that not only its elastic expansion but also its inelastic expansion should be in proportion to the *in situ* stress field. As was done in the method of Horibe and Kobayashi (1958), hydrostatic pressures are applied to the rock core in their method. In stead of making use of bending points in observed pressure-strain curves, however, they estimate the orientations and the ratios of the *in situ* stress components from measured inelastic strains. Their method is thus seriously dependent on the assumption stated above.

Considering a clear relation between AE rate and dilatancy as found by, *e.g.*, Kusunose *et al.* (1979), Kurita and Fujii (1979), the results by Kanagawa *et al.* (1977) suggest that the inelastic strain rate changes at an applied stress nearly equal in value to the *in situ* stress. The unsatisfactory result by Horibe and Kobayashi (1958) implies that the gradient change may be so small that we can hardly detect it through the conventional technique of strain measurement. Such a small gradient change as to correspond to the Kaiser effect may easily be masked with the nonlinear behavior of stress-strain relations as caused by closure of pre-existing cracks. In order to detect a small change in inelastic strain rate for the purpose of *in situ* stress estimation, the strain measurement should have a high resolution.

In this paper, we present a technique for high resolution measurement to obtain accurate stress-strain relations of rock specimens under uni-axial compression tests and investigate the effect of previously applied stresses on the inelastic behavior of the stress-strain relations. Based on the inelastic behavior of rock specimens, we further propose a new method, which we call deformation rate analysis (DRA), to estimate the peak values of previously applied stresses. We apply the DRA to some core samples to examine the validity of the method for estimating *in situ* stresses.

## 2. Specimens and Strain Measurement

Cyclic uni-axial compression tests are performed on granodiorite and andesite specimens. Both the granodiorite and the andesite sample were produced in Fukushima Prefecture in the northeastern Honshu, Japan. The apparent Young's moduli measured by uni-axial compressions are about 20 GPa for the granodiorite specimens and about 57 GPa for the andesite ones. Our cylindrical specimens are 18.5 to 18.8 mm in diameter and 40 to 45 mm in length (Fig. 1(a)). This size is convenient for us to cut specimens out of a core sample with a diameter of about 65 mm in directions perpendicular to the core axis.

The studies by Kusunose *et al.* (1981) showed that shear cracks are generated in rock specimens even under uni-axial compression (Hirasawa and Yamamoto, 1984; Kuwahara *et al.*, 1985; Satoh *et al.*, 1986). The inelastic deformation due to the shear cracks should contribute to both the axial and the radial strain of the specimens. The inelastic behavior of axial strain is thought to be governed mainly by shear microfracturing and by closure and/or collapse of pre-existing pores. The behavior of the radial strain is more complicated. The development of pre-existing tensile cracks and the creation of new tensile cracks, which are considered to accompany shear cracks (Brace *et al.*, 1966; Kuwahara *et al.*, 1990), also contribute significantly to the inelastic strain. Further, the measurement of radial strain is generally substituted by that of circumferential strain. It is difficult to measure accurately the circumferential strain because of the bending of strain gauges pasted on the cylindrical surface of the specimen with a small diameter. For these reasons, we use in the following analysis only the data of axial strains measured by four strain gauges with a length of 5 mm pasted on the middle of the free surface of a specimen.

According to the results of Kusunose *et al.* (1977, 1979, 1980) and Kurita and Fujii (1979), an increment of inelastic volumetric strain with an event of AEs, that is, the total inelastic volumetric strain divided by the cumulative number of AEs, is estimated at $0.1\times10^{-6}$ strain ($\mu$ strain) in the order of magnitude. In order to have the same resolution as that in the AE method, the resolution of axial-strain measurement should be

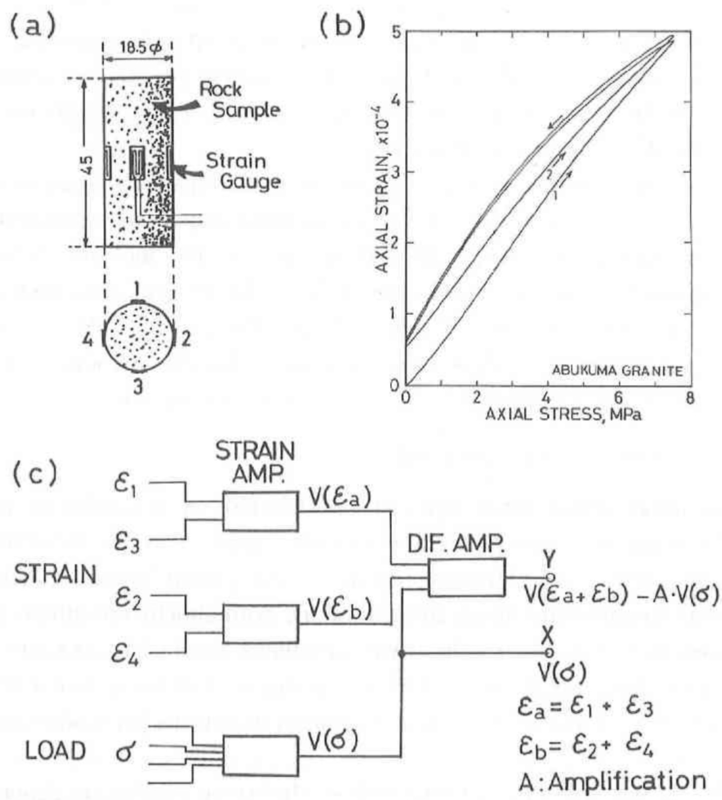


Fig. 1 (a) Schematic illustration for sensor arrangement on a specimen used for uni-axial compression tests. Four strain gauges are pasted at the middle of the free surface of the specimen. (b) A typical example of stress-strain curves obtained for an Abukuma granite specimen by two loading and unloading cycles of uni-axial compression. (c) Block diagram to illustrate our device for high-resolution strain measurement.

better than 0.1 $\mu$ strain to estimate *in situ* stresses from gradient changes in the observed stress-strain curves. Figure 1(b) illustrates the stress-strain relation obtained for a granite specimen by a conventional technique of strain measurement. As seen in the figure, the applied stress of a few Mega-Pascals produces an axial strain of the order of $10^{-4}$. The resolution of 0.1 $\mu$ strain is one thousandth of the amount of the total strain. We are thus required to improve the technique of strain measurement.

The observed axial strain as a function of the applied axial stress may be divided into a linear part and the remaining nonlinear part. The former is generally much larger in value than the latter. We therefore subtract the linear part from the total strain in order to amplify the residual nonlinear part. That is, we take the difference between the output of a strain gauge and that of load cell, as illustrated in Fig. 1(c). It is noted that we do not have to separate rigorously the nonlinear part from the linear part for the present purpose. The residual nonlinear part, in which a fraction of the linear part may possibly be included, will be called the reduced strain hereafter. The relations between applied stress and the reduced strain, or the reduced stress-strain relations, are recorded by an X-Y recorder. This technique enables us to detect a small change in the gradient of stress-strain relation with a resolution better than 0.05 $\mu$ strain, providing that electrical and mechanical noises are reduced in the way as described below.

The noises with periods less than 1 minute are serious in our experiments. These kinds of noises are caused mainly by air current, which may give abrupt changes in temperature of strain gauges and electrical leads. We covered the loading apparatus together with the specimen and the electrical leads with a box made of styrene foam to prevent the air current. High-resolution strain-measurement is disturbed also by irregular motion of the spherical seating on which a specimen is placed. The contacting spherical surfaces of the spherical seating were rubbed against each other with suspension of 1 $\mu$m alumina grit in order that they might smoothly slide on each other.

We apply the axial stress to the specimen through a hand-pump. The stress rate is 1.5 or 3.0 MPa/min. The amount of inelastic strain being sensitive to the rate, we keep the rate as constant as possible by monitoring a stress-time chart. Since one stroke of the hand-pump produces the stress up to about 15 MPa, the stress rate can be controlled to be sufficiently constant for stresses less than about 15 MPa.

## 3. Deformation Rate Analysis

Figure 2 shows the reduced stress-strain curves obtained for a granodiorite specimen

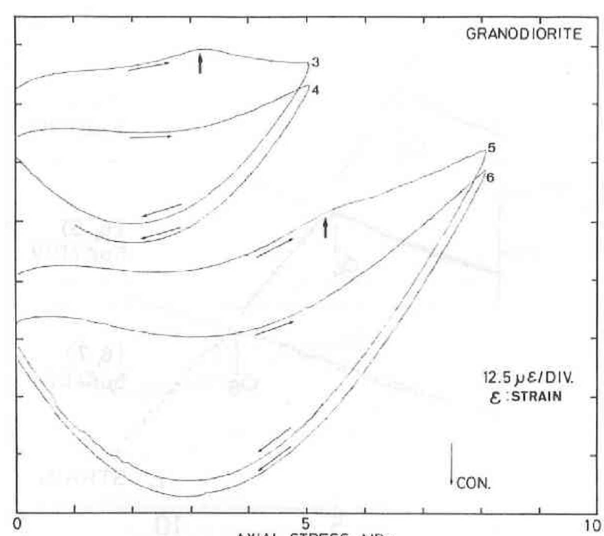


Fig. 2 Examples of the reduced stress-strain curves obtained for a granodiortie specimen by a cyclic uni-axial compression test, where contraction is taken to be the negative direction of the ordinate and the curves of the fifth and the sixth cycle are shifted downward to avoid confusion with those of the third and the fourth cycle. The loading history is shown in Fig. 3(a).

in the third to the sixth loading cycle, where the loading history is schematically illustrated in Fig. 3(a). The peak stress in a cycle was successively increased every two cycles. It is found from Fig. 2 that the amplitude of the reduced strain is of the order of $10^{-5}$. This value is about 10% of the amplitude of the original raw strain as given in Fig. 1(b). The reduced strain curve of a cycle of loading and unloading exhibits a hysteresis loop with a considerable amount of residual strain at the end of unloading. A kind of creep recovery phenomenon is also seen in the figure between the third (or the fifth) unloading and the fourth (or the sixth) loading. The time interval was about 1 minute between the end of unloading and the beginning of the next loading. The most important is that a gradient change as indicated by an arrow in the figure is clearly seen in the third or the fifth loading while no significant gradient change is found either in the fourth or in the sixth loading. This gradient change occurs at an axial stress nearly

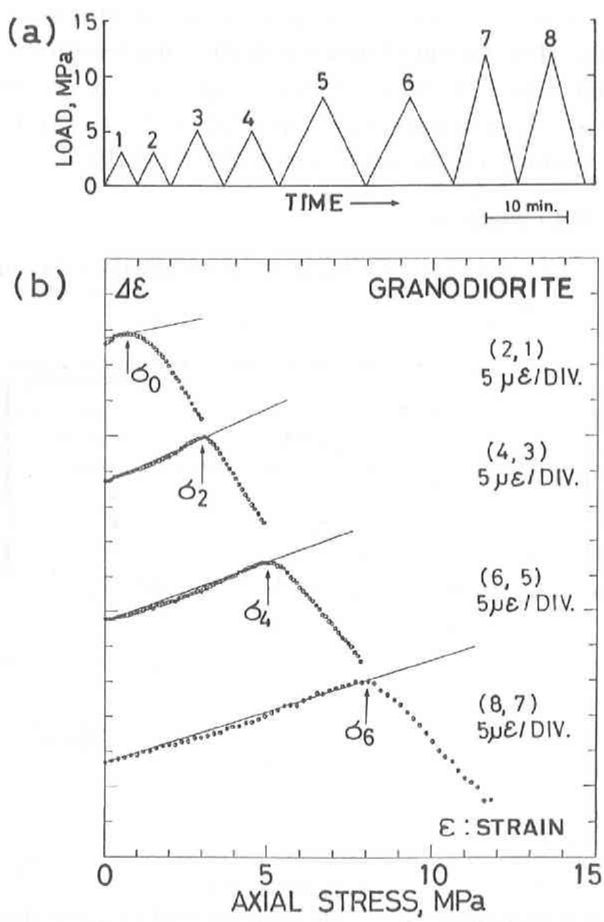


Fig. 3 (a) Schematic illustration for the time history of cyclic loading on a granodiorite specimen. (b) The obtained strain difference functions $\Delta\varepsilon_{j,i}$, where $(j, i)$ given in the figure indicates the subscripts of $\Delta\varepsilon_{j,i}$. The peak value of previous stress in the preceding $k$-th cycle is shown by $\sigma_k$. The meaning of $\sigma_0$ is explained in the text.

equal to the peak stress of the preceding loading cycle.

We expect that the gradient change stated above should be detected more easily and more precisely by taking the difference in reduced strain between the third (the fifth) and the fourth (the sixth) loading cycle. Let us introduce the strain difference function $\Delta\varepsilon_{j,i}(\sigma)$ in a general form as defined by

$$\Delta\varepsilon_{j,i}(\sigma)=\varepsilon_j(\sigma)-\varepsilon_i(\sigma);\ j>i, \tag{1}$$

where $\sigma$ is the applied axial stress and $\varepsilon_i(\sigma)$ the reduced axial strain for the $i$-th loading.

Figure 3(b) shows the curves of the strain difference functions obtained from the reduced stress-strain curves given in Fig. 2. The peak values ($\sigma_i$; $i \geqq 2$) of previously applied stresses are indicated by arrows in Fig. 3(b), where the meaning of $\sigma_0$ will be discussed later. We see from this figure that the function $\Delta\varepsilon_{i+1,i}(\sigma)$ is approximated by a straight line with a positive gradient at stresses less than the previous peak stress and that it sharply bends down near the previous peak stress. The negative gradient at applied stresses higher than the previous peak stress indicates that the rock specimen is easier to deform in the first loading than in the second one of two successive loading cycles. This is suggestive of the Kaiser effect. The specimen is considered ready to enlarge pre-existing cracks and/or to create new cracks only at its first experience of high applied stresses. On the contrary, the positive gradient means that the specimen is more compliant to the applied stress in the second loading than in the first one. This is due probably to the increase in crack denstiy at high stresses of the first loading.

Figures 4 and 5 show another example of the reduced stress-strain curves and the

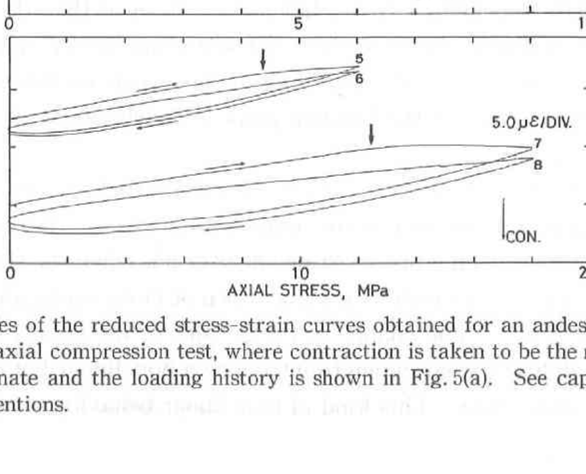


Fig. 4 Examples of the reduced stress-strain curves obtained for an andesite specimen by a cyclic uni-axial compression test, where contraction is taken to be the negative direction of the ordinate and the loading history is shown in Fig. 5(a). See caption to Fig. 2 for other conventions.

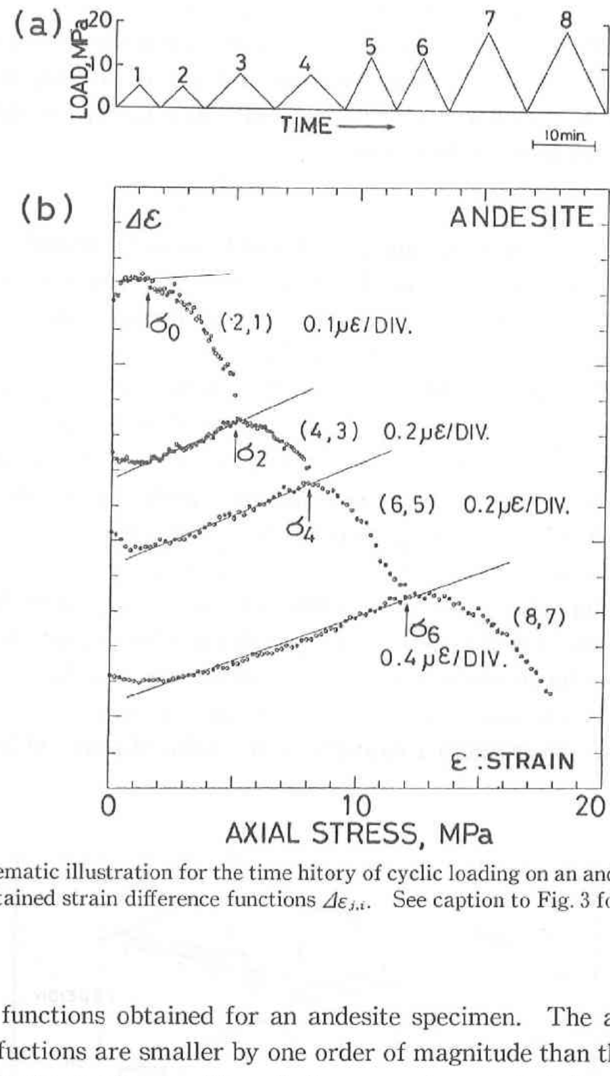


Fig. 5 (a) Schematic illustration for the time hitory of cyclic loading on an andesite specimen. (b) The obtained strain difference functions $\Delta\varepsilon_{j,i}$. See caption to Fig. 3 for other conventions.

strain difference functions obtained for an andesite specimen. The amplitudes of the strain difference fuctions are smaller by one order of magnitude than those given in Fig. 3 for the granodiorite specimen. Nevertheless, we observe that the strain difference functions are very similar in shape to those for the granodiorite specimen. It is thus confirmed that we can detect clearly a bending point of a strain difference function and that the value of axial stress at the bending point is nearly equal to the peak value of previously applied stress.

The mechanical behavior of pre-existing cracks in a rock specimen causes, more or less, non-linear strains with respect to the applied axial stress. For instance, frictional sliding is expected to occur on a pre-existing shear crack when the shear stress exceeds a critical value. An isolated tensile crack may open or close elastically according to the change in the axial stress. The change in the density of the tensile crack changes the effective elastic moduli of the specimen to introduce a non-linear but elastic response to the axial stress (Walsh, 1965). This kind of non-linear behavior in strain is considered to be mostly reversible during many cycles of loading, as far as the pre-existing cracks do not change their size (Holcomb and Stevens, 1980; Stevens and Holcomb, 1980; Kuwahara *et al.*, 1990). The reversible components of strain is cancelled by the operation of (1). The axial stress applied to a rock specimen may enlarge some of pre-existing cracks and create new cracks. Considering what the Kaiser effect implies, this should happen especially when the applied stress exceeds the peak value of previous stress. The strain resulting from this is irreversible for two successive cycles and not cancelled in the strain difference function defined by (1).

From the above consideration it is clear that the use of the strain difference function has the advantage of emphasizing the irreversible component of the measured non-linear strain by eliminating the reversible component. This was practically confirmed by the two examples of our experiments on a granodiorite and an andesite specimen as shown in Figs. 2 and 3 and Figs. 4 and 5. Using the strain difference function we can detect more easily a bending point of stress-strain curve to estimate the peak value of previously applied stresses. The method described above will be called deformation rate analysis (DRA) hereafter.

As found from Fig. 3 for a granodiorite specimen or Fig. 5 for an andesite one, the curve of strain difference function $\Delta\varepsilon_{2,1}$ sharply bends approximately at the axial stress denoted by $\sigma_0$. No artificial stresses had been applied to the specimen before the first loading except for some forces which might have been exerted on it during sample preparation. If the effects of unintended forces can be disregarded, the value of $\sigma_0$ is considered to indicate the value of *in situ* stress to which the rock sample was subjected at the site.

## 4. Application of DRA

### 4.1. *Esashi Samples*

It was suggested in the preceding section that the DRA can be used to estimate *in situ* stresses. In order to examine its applicability, we will study the azimuthal dependence of previous stresses estimated for horizontal directions as well as the depth dependence of previous stresses for the vertical direction. Granodiorite core samples

Table 1. Average Physical Constants of Esashi Core Samples.

| Depth m | | Density g/cm³ | Porosity % | Vp[1] km/s | Vs[1] km/s | E[2] GPa |
|---|---|---|---|---|---|---|
| 30~ 34 | DRY | 2.69±0.02 | 1.2±0.6 | 5.30±0.29 | 3.28±0.16 | 68.8 |
| | WET | 2.70±0.02 | | 5.75±0.26 | 3.39±0.15 | 76.6 |
| 73~ 74 | DRY | 2.69±0.00 | 0.9±0.1 | 5.65±0.10 | 3.44±0.01 | 76.8 |
| | WET | 2.70±0.00 | | 5.90±0.04 | 3.52±0.04 | 81.9 |
| 98~102 | DRY | 2.71±0.00 | 0.8±0.1 | 5.57±0.05 | 3.40±0.01 | 75.4 |
| | WET | 2.72±0.00 | | 5.93±0.12 | 3.45±0.06 | 80.6 |

[1] P- and S-wave velocities.

[2] Young's modulus calculated from the elastic wave velocities and the density.

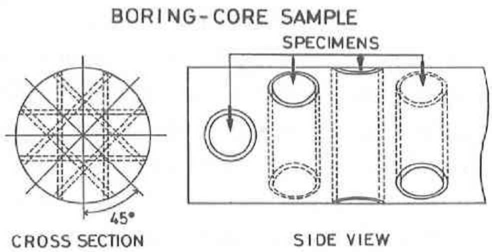


Fig. 6 Illustration for our cutting out horizontal specimens from a core sample. The specimens of four horizontal directions at 45° interval are used to estimate horizontal previous stresses.

were obtained from depths of about 73 m and about 100 m at Esashi, Iwate Prefecture, northeastern Honshu, Japan. The orientation of the core samples was not made at the site. The physical constants are shown in Table 1, where the dynamic Young's modulus was calculated from the elastic wave velocities and the density.

The specimens used for the measurement of horizontal stresses were cut in four horizontal directions at an interval of 45° from a core sample, as illustrated in Fig. 6. The number of specimens prepared for each azimuth is three for the 73 m depth and four for the 100 m depth. Three specimens were also prepared for the vertical stress measurement at each depth. Table 2 shows the apparent Young's moduli of these specimens, which are obtained as average values in the range of axial stresses from zero to 4 MPa. The values are about 75 GPa on an average and smaller than the dynamic ones in a dry state only by about 3%. This suggests for these specimens to show small amounts of inelastic strains at low stresses of compressional loading.

Two cycles of uni-axial loading were performed on each of the specimens to obtain

Table 2. Apparent Young's Moduli of Esashi Specimens.

| Specimens | Young's Modulus[1] in GPa | | | | |
|---|---|---|---|---|---|
| | Azimuth[2] | | | | Vertical direction |
| | 0° | 45° | 90° | 135° | |
| Depth: 73 m | | | | | |
| No. 1 | 73.1 | 72.8 | 77.2 | 74.3 | 71.8 |
| No. 2 | 78.0 | 79.0 | 77.8 | 77.3 | 74.7 |
| No. 3 | 77.7 | 76.4 | 75.5 | 75.5 | 72.1 |
| Average | 76.3±2.7 | 76.1±3.1 | 76.8±1.2 | 75.7±1.5 | 72.9±1.6 |
| Depth: 100 m | | | | | |
| No. 1 | 70.0 | 69.4 | 74.4 | 73.5 | 76.2 |
| No. 2 | 70.7 | 71.8 | 72.6 | 74.2 | 69.9 |
| No. 3 | 70.8 | 74.1 | 72.0 | 73.8 | 68.4 |
| No. 4 | 78.2 | 70.9 | 71.9 | 76.4 | |
| Average | 72.4±3.9 | 71.6±2.0 | 72.7±1.2 | 74.7±1.3 | 71.5±4.1 |

1) The values were obtained from the amounts of axial strains as the applied uni-axial stress was increased from zero to 4 MPa.

2) The reference direction in azimuth is arbitrarily defined for each depth.

the strain difference functions, $\Delta\varepsilon_{2,1}$. The peak values of applied stresses were about 6 MPa for the specimens of 73 m depth and 8 MPa with some exceptions of about 6 MPa for the specimens of 100 m depth. The stress rate was kept constant at about 1.5 MPa/min for all the loading cycles. The measurement of $\Delta\varepsilon_{2,1}$ for each specimen was repeated about one week after the first measurement. All the results obtained for $\Delta\varepsilon_{2,1}$ are shown in Fig. 7 for 73 m depth and in Fig. 8 for 100 m depth.

Almost all the observed curves of strain difference function have a linear segment with a positive gradient at low axial stresses, as seen in Figs. 7 and 8, in which the solid straight lines (dashed ones for ambiguous data) are drawn for the sake of reference. There exist, however, some exceptional cases in which the observed curve has a linear segment with a negative gradient. Particular examples for this are seen on the curves A2 and D1 in the left column of Fig. 8. The negative gradient indicates that the apparent compliance of the rock specimen was higher in the first loading than in the second one of two successive loading cycles. As demonstrated by the differential strain curve analysis of, *e.g.*, Strickland and Ren (1980), the rock specimen before the first loading was likely to contain a considerable number of pre-existing open cracks, many of which had probably been produced during the sample removal from the bore-hole at site. These open cracks were closed by loading of the first cycle. It is reasonable to consider that all of them were not completely re-opened at the end of unloading in the first cycle. If the number of the unopen cracks is larger than that of new cracks created

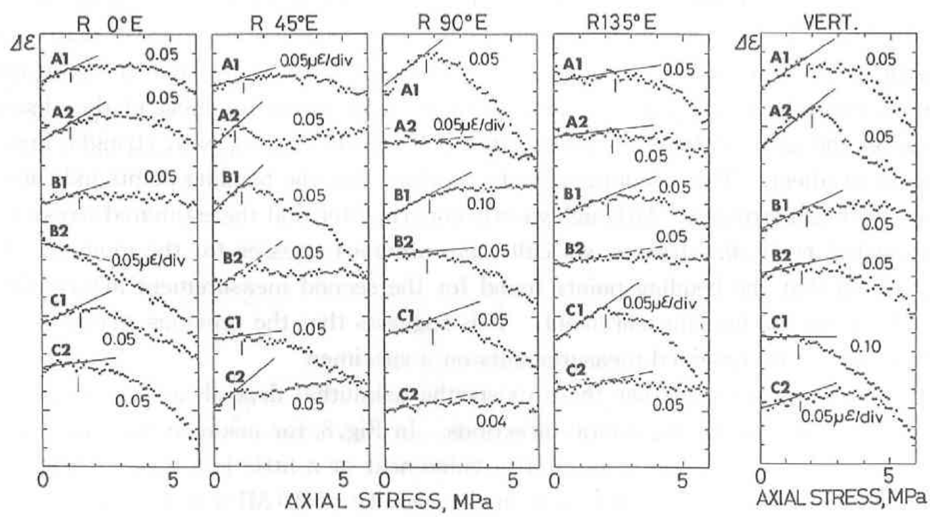


Fig. 7 The strain difference functions $\Delta\varepsilon_{2,1}$ obtained for the specimens retrieved from the depth of about 73 m at Esashi. The curves in the right-hand side are for vertical specimens. The curves in each column are obtained for the horizontal specimens of the same azimuth. A, B and C in each column indicate the results obtained by using different specimens, while A2, for example, denotes the result for specimen A in the second measurement carried out about one week after the first measurement. The solid and dashed lines are drawn for the sake of reference, where the dashed lines indicate ambiguous cases. The bending points we read are indicated by tick marks.

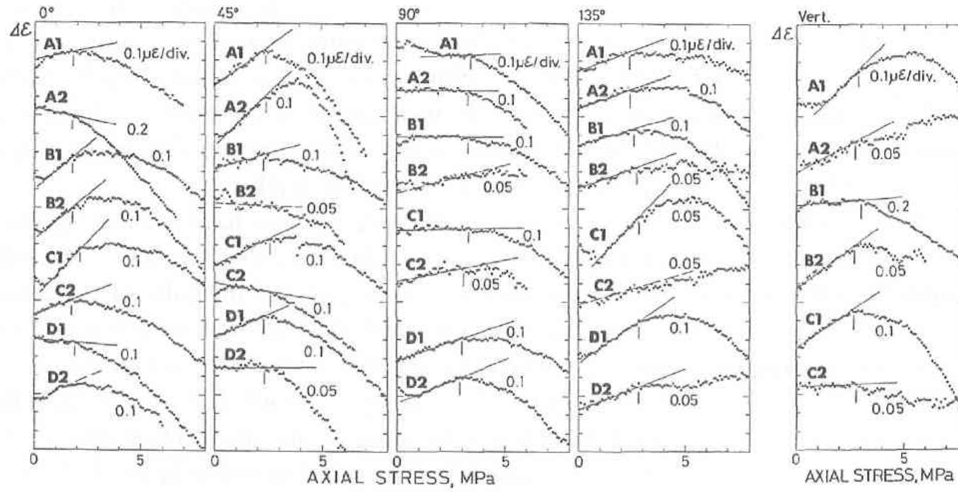


Fig. 8 The strain difference function $\Delta\varepsilon_{2,1}$ obtained for the specimens retrieved from the depth of about 100 m at Esashi. See caption to Fig. 7 for other conventions.

in the first loading, the strain difference function should have a negative gradient, as found for the exceptional cases in Figs. 7 and 8.

Most of curves in Figs. 7 and 8 are seen to have fairly clear bending points. There is, of course, some arbitrariness in reading them. To illustrate our judgement, the clear bending points we read are indicated by tick marks. In the cases of curves with dashed straight lines, however, we avoided reading them. It is found that sharp gradient changes take place nearly at the same value of axial stress for most of the observed curves in the same column. This is true also for the curves with straight lines of negative gradient. This encourages us to consider that the bending points indicate the values of *in situ* stresses. Although we will confirm later that the estimated stresses can be regarded as *in situ* stresses, we call them previous stresses for the moment. It is emphasized that the bending points found for the second measurement are consistent with those for the first measurement. This suggests that the previous stresses can be estimated even by repeated measurements on a specimen.

We will examine whether there exists the azimuthal dependence of the previous stresses $\sigma_0$ observed for horizontal directions. In Fig. 8, for instance, we can estimate the previous stress for 0° in azimuth at a value near or a little less than 2.0 MPa. No gradient changes are found, however, in the vicinity of 2.0 MPa in the case of 90° in azimuth. The previous stress for 90° is thus estimated at about 3.0 MPa. The estimated previous stress seems to depend on the azimuth. Let us introduce a Cartesian coordinate system $(x, y, z)$, in which the $z$-axis is taken to be vertical downward and the $x$-axis coincides with the reference azimuth used in our measurement. As described before, the core sample was not oriented. The reference azimuth is arbitrarily defined. Since the principal axes of stresses do not necessarily coincide with $(x, y, z)$, there exist six independent components of stresses; $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$, $\sigma_{xy}$, $\sigma_{yz}$, $\sigma_{xz}$. Rotating $(x, y, z)$ around the $z$-axis by $\theta$ clockwise, we have another coordinate system $(x', y', z)$. From the theory of elasticity, we have

$$\sigma_{x'x'}=\frac{1}{2}(\sigma_{xx}+\sigma_{yy})+\frac{1}{2}(\sigma_{xx}-\sigma_{yy})\cos 2\theta+\sigma_{xy}\sin 2\theta, \tag{2}$$

$$\sigma_{x'y'}=\frac{1}{2}(\sigma_{yy}-\sigma_{xx})\sin 2\theta+\sigma_{xy}\cos 2\theta. \tag{3}$$

It is possible to determine $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ from three independent data of $\sigma_{x'x'}$ by the use of (2). From the knowledge of $\sigma_{xx}$ and $\sigma_{yy}$, we find the directions of $\sigma_{x'y'}=0$ by (3). The directions are the azimuths of the maximum and minimum compressions that are exerted on vertical planes. They agree with the axes of principal stresses only if the $z$-axis coincides with one of the principal axes.

If the previous stresses estimated for core samples reflect properly a locally uniform *in situ* stress field, they should satisfy the $\theta$-dependence given by (2). Figures 9(a) and (b) exhibit the azimuthal distributions of the previous stresses together with the vertical ones estimated for 73 m depth and for 100 m depth, respectively. The solid curves are obtained by least squares fitting of the observed data to the theoretical relation (2). As seen in the figures, the azimuthal dependence of the observed data satisfactorily agree with the theory. This confirms that the estimated previous stresses are regarded as $\sigma_{x'x'}$ components of *in situ* stresses, as far as the relative magnitudes are concerned.

The maximum and minimum horizontal compressions are thus estimated at $2.19\pm0.07$ and $0.94\pm0.07$ MPa for 73 m depth, and at $3.19\pm0.05$ and $1.86\pm0.05$ MPa for 100 m depth, as listed in Table 5. The vertical stresses are $1.72\pm0.18$ and $2.80\pm0.13$ MPa for the depths of 73 m and 100 m, respectively. These values do not differ significantly from the overburden pressures at the corresponding depths, as will be discussed later. The ratios of the maximum shear stress to the average stress in horizontal directions are

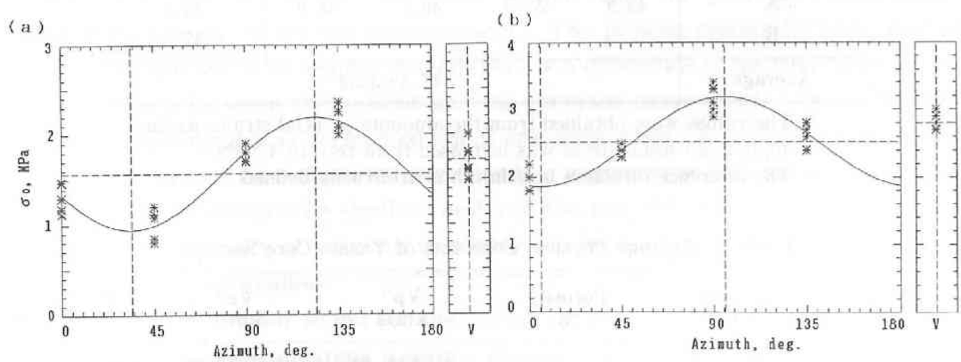


Fig. 9 The azimuthal dependence of horizontal previous stresses together with vertical previous stress estimated for Esashi. The solid curve expresses the sinusoidal function of azimuth obtained from the plotted data by a least squares method. (a) Results for 73 m depth. (b) Results for 100 m depth.

calculated to be about 0.40 and 0.26. Tanaka (1987) has examined the relations between the maximum shear stress and the average stress obtained by *in situ* stress measurements in Japan. According to his result, almost all the data of the ratio are less than 0.6.

### 4.2. *Yasato Samples*

Granodiorite core samples were obtained from a depth of about 143 m at Yasato, Ibaraki Prefecture, Kanto, Japan. The apparent Young's moduli of rock specimens are measured by uni-axial compressions to range from 42 to 51 GPa, as shown in Table 3. The dynamic Young's moduli of the core samples obtained from the same bore-hole are calculated to be about 65 GPa and about 52 GPa on an average from velocity and density measurements in wet and dry states, respectively, as shown in Table 4. The difference of the dynamic Young's modulus in dry state from the apparent one is very big, amounting to about 10% on an average. This is contrasted with the small difference of about 3% found previously for the Esashi samples in the preceding section. The density of the core samples is about 2.65 g/cm$^3$ in a wet state on an average. Assuming that the mean density of overburden rocks is 2.6 g/cm$^3$, the overburden pressure is calculated to be 3.6 MPa at the depth of 143 m.

Four cycles of loading with a peak value of 7 MPa were successively applied to each of two vertical specimens A and B, where the loading rate was 1.5 MPa/min. Three strain difference functions $\Delta\varepsilon_{2,1}$, $\Delta\varepsilon_{3,2}$ and $\Delta\varepsilon_{4,3}$ were thus obtained and are shown in Fig. 10. The data of $\Delta\varepsilon_{2,1}$ at axial stresses higher than 2.5 MPa are omitted from the figure

Table 3. Apparent Young's Moduli of Yasato Specimens.

| Specimens | Apparent Young's Modulus[1] in GPa | | | | |
|---|---|---|---|---|---|
| | Azimuth[2] | | | | Vertical direction |
| | 0° | 45° | 90° | 135° | |
| A | 43.9 | 51.3 | 49.7 | 48.9 | 42.2 |
| B | | | | | 47.5 |
| Average | 47.3±3.52 | | | | |

[1] The values were obtained from the amounts of axial strains as the applied uni-axial stress was increased from zero to 4 MPa.
[2] The reference direction in azimuth is arbitrarily defined.

Table 4. Average Physical Constants of Yasato Core Samples.

| | Density g/cm$^3$ | Porosity % | Vp[1] km/s | Vs[1] km/s | E[2] GPa |
|---|---|---|---|---|---|
| Dry | 2.64±0.01 | 0.8±0.04 | 4.74±0.04 | 2.85±0.06 | 52.3 |
| Wet | 2.65±0.01 | | 5.58±0.09 | 3.08±0.09 | 64.5 |

[1] P- and S-wave velocities.
[2] Young's modulus calculated from the elastic wave velocities and the density.

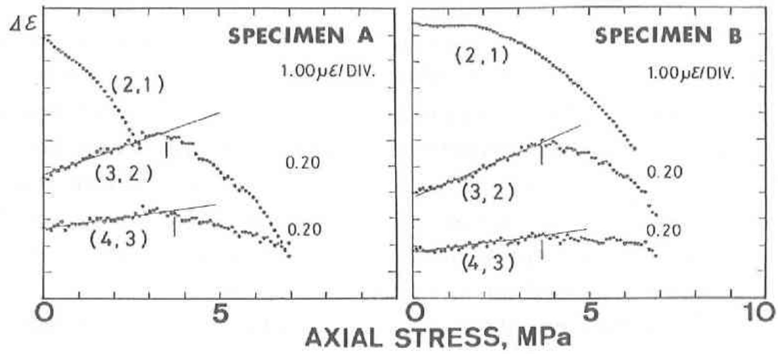


Fig. 10 The strain difference functions $\Delta\varepsilon_{j,i}$ obtained for two vertical specimens A and B retrieved from the depth of about 143 m at Yasato. Four cycles of loading were applied to each specimen, where $\Delta\varepsilon_{j,i}$ are distinguished by ($j$, $i$) given in the figure.

for specimen A, because its amplitude variation is too large to show. It is apparent from the figure that the two curves of $\Delta\varepsilon_{2,1}$ have negative gradients and that their amplitude variations are much larger than those of $\Delta\varepsilon_{3,2}$ and $\Delta\varepsilon_{4,3}$ of those of $\Delta\varepsilon_{2,1}$ in Figs. 7 and 8. We found in the preceding section that the vertical stress estimated at Esashi is close in value to the overburden pressure. It may be generally accepted that the vertical stress should not differ so much from the overburden pressure. In the present case, however, it is impossible to find any bending point of $\Delta\varepsilon_{2,1}$ around the overburden pressure of 3.6 MPa.

In contrast to the curves of $\Delta\varepsilon_{2,1}$, the curves of $\Delta\varepsilon_{3,2}$ and $\Delta\varepsilon_{4,3}$ are seen to have positive gradients at low applied stresses and sharp gradient changes in the vicinity of the overburden pressure. It is noted that there is no clear bending point other than the one near the overburden pressure in a curve of $\Delta\varepsilon_{3,2}$ or $\Delta\varepsilon_{4,3}$. The average values of vertical previous stresses estimated from the bending points are 3.6 MPa for specimen A and 3.7 MPa for specimen B. These values are very close to the overburden pressure of 3.6 MPa. We found previously for Esashi samples that the previous stress could be estimated even from the second measurement. The present results for $\Delta\varepsilon_{3,2}$ and $\Delta\varepsilon_{4,3}$ are similar to the previous results, though there is a difference in the time interval of the repeated loading between the present and the previous experiment. As stated before, the second experiment was performed about one week after the first one in the previous case. In the present case, however, four loading cycles were successively carried out. In spite of this difference, the similar results of the two different kinds of experiments suggest that the memory of previous (probably *in situ*) stresses is so stable as to remain after a few cycles of loading.

The peculiar behavior found for the curves of $\Delta\varepsilon_{2,1}$ seems to be related to the big difference between the apparent Young's modulus and the dynamic one of the rock specimen. This is related also to the large magnitude and the negative gradient of $\Delta\varepsilon_{2,1}$. We speculate that a large number of cracks were produced in Yasato samples during and after coring and stabilized to the state of stress free. We may call this the memory of zero-stress by analogy of the memory of previous stresses. The mechanical behavior of the stabilized cracks in the first loading may be different from that in the second loading to produce a considerable amount of inelastic strains irreversible between the first and the second loading cycle. The memory of zero-stress, however, is much less stable than that of previous *in situ* stresses, because the former was found to have been destroyed by the first loading. This may be the reason why we can detect previous stresses not in $\Delta\varepsilon_{2,1}$ but in $\Delta\varepsilon_{3,2}$ and $\Delta\varepsilon_{4,3}$. It is concluded that there are some cases in which the data obtained in the first loading are seriously contaminated by the effect of zero-stress memory and should not be used for *in situ* stress estimation.

Figure 11 exhibits the strain difference functions obtained for horizontal specimens. The curves of $\Delta\varepsilon_{2,1}$ and seen to be quite similar to those of Fig. 10 and not used for stress

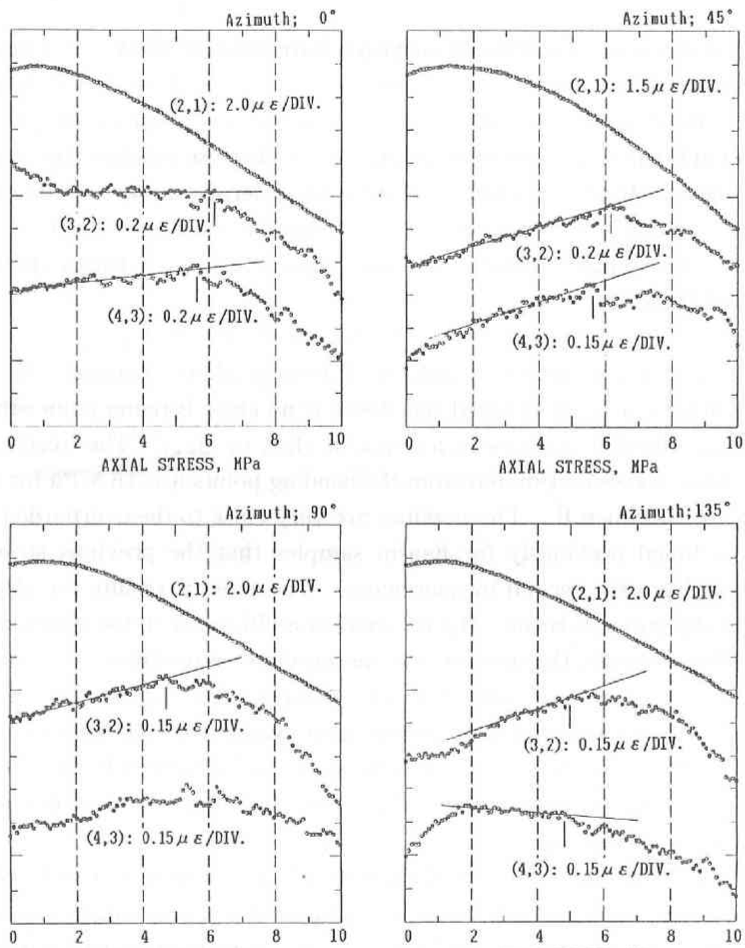


Fig. 11 The strain difference functions $\Delta\varepsilon_{j,i}$ obtained for the horizontal specimens retrieved from the depth of about 143 m at Yasato. See caption to Fig. 10 for other conventions.

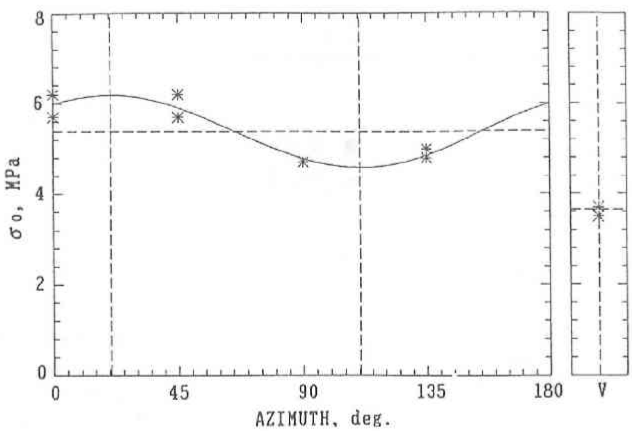


Fig. 12 The azimuthal dependence of horizontal previous stresses together with vertical previous stress estimated for Yasato. The solid curve expresses the sinusoidal function of azimuth obtained from the plotted data by a least squares method.

estimation. The clear bending points are found in most of the other curves. The azimuthal distribution of the horizontal previous stresses thus read is shown in Fig. 12. The observed values are found to be expressed well by a sinusoidal function of azimuth as given by (2). The maximum and the minimum compression in horizontal directions are determined to be 6.19±0.19 and 4.58±0.19 MPa by a least squares method. Since the estimated vertical stress is lower than the estimated minimum horizontal stress (*cf*. Table 5), the stress field is of predominantly reverse fault type. Ikeda and Tsukahara (1986) and Tsukahara and Ikeda (1987) carried out *in situ* stress measurements by a hydrauric fracturing technique at depths more than 400 m at Tsukuba and Ishige, Ibaraki Prefecture. The sites are located at distances of about 15 to 20 km from Yasato. Their results show the stress field of reverse fault type for both the sites. Their results are thus consistent with ours.

Figure 13 shows the depth dependence of the vertical stresses estimated for Yasato and Esashi. The observed data are found to lie approximately on a single straight line. The density variation with depth is not known either for Yasato or for Esashi. The core samples from the two sites are of the same rock type of granodiorite. The difference in average value of measured densities between them is about 2%. We therefore expect

Table 5. The Maximum and Minimum Horizontal Compressions and the Vertical Stresses Estimated for Esashi and for Yasato.

| | | Esashi | | Yasato |
|---|---|---|---|---|
| Depth | m | 73～74 | 100～101 | 143～144 |
| $\sigma_{hmax}$ | MPa | 2.19±0.07 | 3.19±0.05 | 6.19±0.19 |
| $\sigma_{hmin}$ | MPa | 0.94±0.07 | 1.86±0.05 | 4.58±0.19 |
| $\sigma_{vert}$ | MPa | 1.72±0.18 | 2.80±0.13 | 3.65±0.10 |

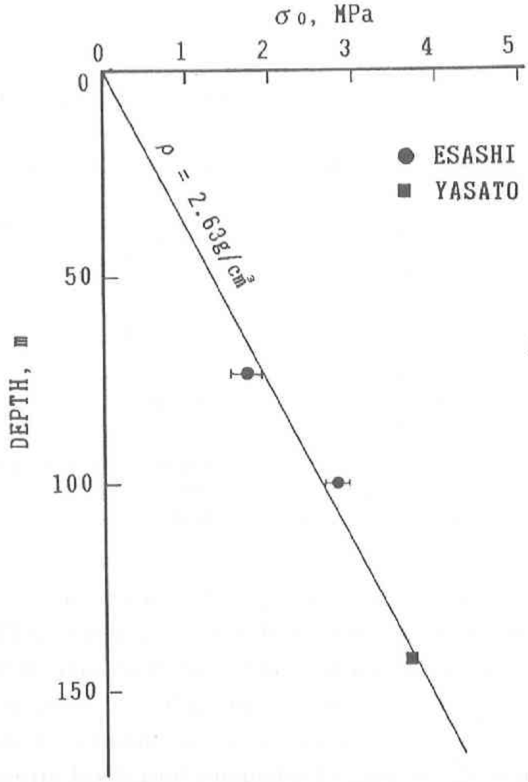


Fig. 13 The depth dependence of the vertical previous stresses estimated for Esashi and Yasato. The gradient of the regression line given in the figure corresponds to an average density of 2.63 g/cm³.

that the overburden pressures at the same depth of the two sites should not differ so much from each other. The gradient of the straight line in the figure corresponds to the average density of $2.63 \pm 0.11$ g/cm³. This value is consistent with the measured densities of core samples. It is concluded that the vertical stresses obtained at both the sites are nearly equal to the respective overburden pressures. This suggests that the estimated values of previous stresses express not the relative values but the absolute values of *in situ* stresses.

## 5. Summary and Conclusion

In order to develop a new method for estimating *in situ* stresses from core samples of bore-holes, we examined the effect of previously applied stresses on the inelastic behavior of rock specimens under uni-axial compression tests. We first improved the technique of strain measurement to attain a high resolution better than $0.05\ \mu$ strain. We next defined the strain difference function as given by (1), which emphasizes the irreversible component of inelastic strains between two successive loading cycles. The detailed analysis of the observed strain difference functions gives the following results in the case of the previous stress applied artificially to a rock specimen: The observed function is approximated by a straight line (with a positive gradient in most cases) at low axial stresses up to the peak value of the previously applied stress. Further, the function bends down to show a sharp gradient change at an axial stress nearly equal to the previous peak value. It is thus concluded that the peak value of previously applied stress can be estimated from the bending points of strain difference functions. This method of stress estimation is called deformation rate analysis (DRA).

The DRA was applied to the rock core samples retrieved from bore-holes at Esashi in Iwate Prefecture and Yasato in Ibaraki Prefecture, Japan, to estimate the *in situ* stress fields. Our findings are as follows: 1) A sharp gradient change is found for a strain difference function with respect to applied axial stress to suggest the value of previous stress (probably *in situ* stress). 2) In some cases, inelastic strains in the first loading show a peculiar behavior very different from those in later cycles of loading. Nevertheless, clear bending point are generally found for the curves of strain difference functions derived from the data of later cycles. 3) The previous stresses obtained for horizontal rock specimens show an azimuthal dependence, which is well explained by the azimuthal dependence expected from the theory of elasticity in general for the normal stress components of a stress field. 4) We estimated the maximum and minimum horizontal compressions and the vertical stress from Esashi samples as ($2.19 \pm 0.07$, $0.94 \pm 0.07$, $1.72 \pm 0.18$ MPa) at 73 m depth and ($3.19 \pm 0.05$, $1.86 \pm 0.05$, $2.80 \pm 0.13$ MPa) at 100 m depth. Since the core samples were not oriented, the stress directions are not related to the geographical azimuth. 5) The maximum and mimimum horizontal compressions and the vertical stress are found for Yasato samples at 143 m depth to be $6.19 \pm 0.19$, $4.58 \pm 0.19$, $3.65 \pm 0.10$ MPa. 6) The stress fields estimated above suggest that earthquakes are predominantly strike-slip faulting for Esashi and predominantly reverse faulting for Yasato. The result for Yasato is consistent with the previous results of *in situ* stress measurements by a hydrauric fracturing technique (Ikeda and Tsukahara, 1986; Tsukahara and Ikeda, 1987). 7) The vertical stresses estimated for Esashi and Yasato are nearly equal in value to the overburden pressures of the respective depths calculated from the densities.

We conclude mainly from (3) and (6) that the horizontal stresses estimated by DRA reflect properly the *in situ* stress components normal to vertical planes. Strictly speaking, this is true only for the relative magnitudes and the orientations of stresses. We further conclude from (7) that the values of estimated stresses are considered to express not the relative magnitudes but the absolute magnitudes of *in situ* stress components. It is thus found that rocks have the property of memorizing the *in situ* stresses subjected to themselves and that we can read the memory on the curves of strain difference functions obtained by uni-axial compression tests of core samples. The result of (2) suggests that a core sample has memorized the stress-free state since it was removed from a bore-hole. The zero-stress memory is much less stable than the *in situ* stress memory. This is understood from the experimental fact that the zero-stress memory was destroyed by the first loading while the *in situ* stress memory remained in later loading cycles. Although the stability of the stress memory is one of important issues unsolved in the present study, the DRA is found to be very effective for the practical purpose of *in situ* stress estimation.

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