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Dynamic Features of Expanding Shear Cracks in the Presence of Frictions

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Abstract: The dynamic fracture process on a pre-existing fracture surface is investigated under the prescirbed stress boundary conditions in the presence of the static and the dynamic frictions as modelling the source process of small earthquakes. The solutions show that the final displacement exceeds the value expected from the static solution of the problem. It is demonstrated that, in spite of the excess of the displacement, the backward slip is hardly possible for frictional slidings. It is found that slippage continues after the crack expansion stops, which is considered to be an important feature of the dynamic fracture process. We also study the effects of the viscous friction on the fracture processes to find that the physical quantities such as the rupture velocity, the slip velocity, the final displacement, and the duration of motion are mutually related through the conditions of frictions.

1. Introduction

The source mechanism of earthquakes has been one of the most important problems in seismology. It is generally assumed that seismic radiations are generated by the fracture of material at the focus. Recent observations show that the shear fracture is a likely process at an earthquake focus, and the thoeretical predictions of seismic radiations have mainly been based upon the consideration of shear dislocation. Dislocation models have been of use in predicting the static fields of seismic radiations and have well applied to the observed data of crustal deformations caused by earthquakes (Maruyama, 1964 Steketee, 1958; Chinnery, 1961; 1964; Savage and Hastie, 1966). Dislocation models have also been utilized in predicting the dynamic fields of seismic radiations and in the analyses of the observed seismic waves to estimate the physical parameters of the source (Aki, 1968; Haskel, 1964, 1969; Savage, 1966; Wyss and Brune, 1968). Most models have assumed simple forms of the dislocation time function for the purpose of ready utility, the temporal and spatial variations of the dislocation function being arbitrarily given.

The seismic radiation fields must depend upon the fracture process at the focus, or mathematically upon the dislocation time function, and the actual process is obviously governed by the stress fields and the material properties in the source region. Therefore, the dislocation time function should satisfy certain physical conditions in the source region. In propositions of more elaborate source models, attempts have been made at eliminating the arbitrariness in specifying the dislocation time function, and the seismic source process has become to be discussed in detail in terms of the relation between the spectral parameters of seismic waves and the physical parameters of the source. Brune (1970) has related the time function directly to the effective stress available for the acceleration of the two sides of the fault, and has presented a theoretical representation of the shear wave spectrum. His model has widely been used (Hanks and Wyss, 1972; Trifunac, 1972; Wyss and Hanks, 1972), and has well applied to the field data, however, his derivation of the theoretical representation still appears intuitive. In particular, the effect of the propagation and stoppage of rupture are not accounted for. On the other hand, Sato and Hirasawa (1973) have counted in the rupture propagation in their source model. They have placed restrictions on the dislocation time function so as to satisfy the condition that static equilibrium of the stress is kept at each moment during rupture. Their assumption of static equilibrium of the stress would be applicable to the dynamic process of rupture in some limited cases, but in others, its validity must carefully be exmained, especially in rélation to the rupture velocity.

The spécification and the derivation of the dislocation time function in the previous models seem not to be free from being intuitive, and the assumptions not to be always justified from the physical standpoint. The common lack may be sufficient examinations of the consequent relation between the rupture velocity, the temporal and spatial variations of the slip motion resulting from the physical conditions. In order to study the seismic source process in more detail, it is naturally to be desired that the source time function should be a derivation from the appropriate physical conditions. The possible process of slip motion should be investigated under the physical conditions considered.

From this point of view, the dynamic behaviour of the medium with cracks has been of increasing importance. There have been two ways of treating the rupture problems, the difference being on the fracture criterion. One is based upon the consideration of energy balance at the tip of the crack (Ida, 1972; Kostrov, 1964a, 1964b), and the other upon the consideration of friction as a controlling factor (Burridge, 1973; Burridge and Halliday, 1971; Burridge and Levy, 1974; Knopoff et al., 1973). In the former case, no preferred surface is needed in principle, while in the latter, a pre-existing surface of weaker breakage strength is preferred. In recent years, the importance of the role of friction in thé dynamics of fracture has been pointed out. It is considered that friction is an essential factor in investigating the dynamic process of the seismic source.

In this paper, we study the dynamic behaviour of cracks under given stress conditions in the presence of the static and the dynamic frictions as modelling the source process of rather small earthquakes, and the interrelation between the rupture velocity, the final displacement, and the duration of motion is investigated through the conditions of stresses and frictions. Unfortunately, it is in general very difficult to solve the rupture problem to given conditions of stresses and frictions in a three-dimensional manner, that the two-dimensional antiplane strain shear cracks are treated. We employ the fracture criterion that slip will occur when the total shear stress overcomes the static limiting friction. As concerns the fracture process during an earthquake, it is considered to be more appropriate to study the fracture process on a pre-existing fracture surface, and the fracture criterion employed to be physically reasonable.

According to Burridge, the stress just ahead of the tip of the crack in self-similar solutions is finite under the same situations as is considered here. This suggests applicability of the difference method to the problem. Solutions are obtained numerically by the difference method within the desired accuracy. In order to make the physical meanings clear, only the initial stresses and the frictional stresses are given on the fracture surface and the fracture criterion mentioned above assumed, no other assumptions being made. As for the dynamic friction, the viscous term which increases proportionally to the slip velocity is considered. The influence of this term on the fracture process will be discussed. The velocity-dependent friction has been taken into account only by Weertman (1969).

2. Description of the problem

We consider an unbounded elastic medium, which is supposed to be homogneous and isotropic, with a pre-existing fracture surface. It is considered that across the fracture surface the material is not welded but merely in frictional contact. The fracture surface is taken to be a plane, and we refer to the Cartesian coordinates x, y, z so that the plane $y=0$ corresponds to the fracture surface (Fig. 1).

In the initial state, the medium is held in static equilibrium by the tractions at infinity which create the uniform stress field around the fracture surface. The initial stress has a negative component $\sigma_{yy}{}^0=-p_0$ and positive shearing component $\sigma_{yz}{}^0=\tau_0$, all other shearing components zero. The static limiting friction, the product of the normal stress and the coefficient of static friction, is now supposed to be high enough to

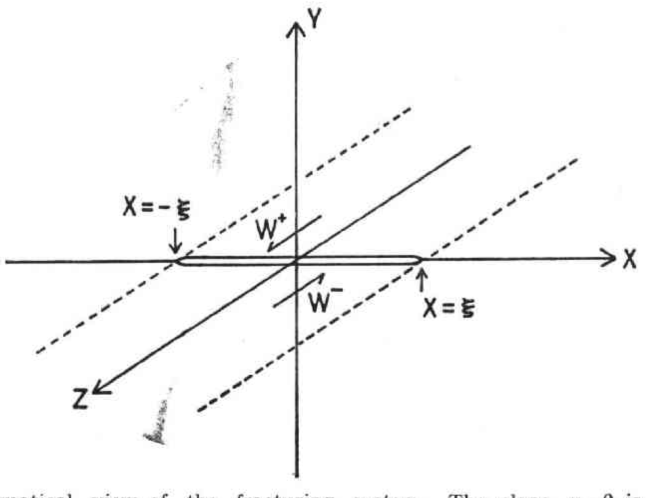

Fig. 1. The schematical view of the fracturing system. The plane $y=0$ is the fracture surface. The displacement is always parallel to the z-axis. ξ indicates the crack edge at time t.

prevent relative slipping across the fracture surface in spite of the non-zero initial shear stress $\sigma_{yz}{}^0$.

Relative slipping will take place when the total shear stress overcomes the static limiting friction. While relative slipping is in progress, the motion is resisted by the dynamic friction, which is taken to be somewhat lower than the initial shear stress. This is a necessary condition, or no motion will take place.

We suppose that at some instance in time, $t=0$, and due to some local irregularity in the static limiting friction, slip across the fracture surface is initiated in the z-direction and along the line $x=0$. Then the zone of slip may spread out symmetrically in the positive and negative x-directions. We shall refer to the zone of slip as "the crack". The coordinates of the crack tips at the positive and negative side shall be denoted as ξ and $-\xi$, respectively, and we refer to 2ξ as "the crack length". During the subsequent motion, slip occurs in a strip $-\xi<x<\xi$, and $y=0$. Here we assume that the extent to which the crack is allowed to propagate is limited by some barrier of high static friction, so that displacement discontinuity takes place only within a range $-L<x<L$, $2L$ being the maximum crack length.

Let u, v, w be the displacement components measured from the static state of the initial stress in the x-, y-, and z-directions, respectively. Owing to the assumption that the initial stress field is uniform around the fracture surface with all shearing components being zero except for the yz-component, the crack is infinitely long in the z-direction and the displacement components u, v are always zero, the only non-zero component w depending upon the coordinates x, y and time t, but not upon the coordinate z. Then the problem is reduced to a two-dimensional one, and only SH motion will take place. We need no longer mention the displacement components u, v and the coordinate z.

We assume that the stress-strain law for infinitesimal linear elasticity holds. Then the equation of motion is

$$\frac{1}{\beta^2}\frac{\partial^2 w}{\partial t^2}=\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2} \qquad \text{for } t>0 \tag{2-1}$$

and initially

$$w=\frac{\partial w}{\partial t}=0 \qquad \text{for } t\leq 0 \tag{2-2}$$

where the shear wave velocity $\beta=\sqrt{\mu/\rho}$, μ being the shear modulus of the medium and ρ its density. The stress components σ_{yz}, σ_{xz} of the total stress field are given by

$$\begin{aligned}\sigma_{yz}&=\sigma_{yz}{}^0+\mu\frac{\partial w}{\partial y}=T_0+\mu\frac{\partial w}{\partial y}\\ \sigma_{xz}&=\mu\frac{\partial z}{\partial x}\,.\end{aligned} \tag{2-3}$$

On the fracture surface, the following boundary conditions apply:
At $y=0$ but off the crack,

$$|\sigma_{yz}| < F_s = -\nu_s \sigma_{yy} = \nu_s P_0$$
$$F_s > T_0 \tag{2-4}$$
$$\dot{w} = 0 \text{ and } w \text{ is continuous,}$$

where F_s denotes the static limiting friction, ν_s the coefficient of static friction, and dot superscript is used to indicate the time derivative.
At $y=0$ and on the crack,

$$\sigma_{yz} = F_d$$
$$F_d < T_0 \tag{2-5}$$
$$w \text{ is discontinuous across } y = 0\,,$$

where F_d is the dynamic fraction.
On the whole of $y=0$,

$$\sigma_{yz} \text{ is continuous.}$$

The dynamic friction is supposed to involve the term which increases in proportion to the slip velocity $\dot{D}=\dot{w}^+-\dot{w}^-$, and is related to the slip velocity and the normal stress as follows:

$$\begin{aligned} F_d &= -\mathrm{sgn}\,(\dot{D})\nu_d{}^0\sigma_{yy} - \dot{D}\nu_d\sigma_{yy} \\ &= \mathrm{sgn}\,(\dot{D})\,\nu_d{}^0 P_0 + \dot{D}\nu_d P_0 \end{aligned} \tag{2-6}$$

where $\nu_d{}^0$ and ν_d are the constants, *sgn* means the sign function, and D is the displacement discontinuity across $y=0$. $\dot{w}^+$ and $\dot{w}^-$ are the particle velocities referred to the positive and negative sides of $y=0$, respectively.

The direction of frictional resistance is opposite to the motion direction, so that the sign of F_d depends upon that of $\dot{D}$. We demonstrate that it is reasonably expected that $\dot{D}\geq 0$, (and never becomes negative). From the condition that initially $\sigma_{yz}{}^0=T_0>0$, slip at $x=0$ is initiated so that $\dot{D}>0$, and besides, slip at each point on the crack will occur so that $\dot{D}>0$ at the beginning, since it is hard to imagine that the total shear stress becomes negative at the tips of the crack. While $\dot{D}>0$, the shear stress due to the particle motion is given by

$$\mu\frac{\partial w}{\partial y} = -(T_0 - |F_d|) \geq -(T_0 - \nu_d{}^0 P_0) \equiv -\sigma_e{}^0 \tag{2-7}$$

On the other hand, when the particle on the crack is in the backward motion, that is $\dot{D}<0$, the following condition must hold:

$$\mu\frac{\partial w}{\partial y} = -(T_0 + |F_d|) \leq (-T_0 + \nu_d{}^0 P_0) \tag{2-8}$$

The shear stress $\mu(\partial w/\partial y)$ must necessarily decrease at least by an amount $2\nu_d{}^0 P_0$ when $\dot{D}$ changes its sign. Moreover, if the particle motion is inhibited by the static friction when $\dot{D}=0$, much more decrease in the shear stress is needed. It is considered that the medium will be at rest before the shear stress enough for the backward slip is stored up, unless we take $\nu_d{}^0 P_0$ to be very small. Thus it is reasonably expected that on the crack and for each moment,

$$\dot{D} \geq 0$$

and that

$$\mu \frac{\partial w}{\partial y} > -(T_0 + \nu_d{}^0 P_0)$$

The case where $\mu(\partial w/\partial y) < -(T_0 + \nu_d{}^0 P_0)$ is not accounted for in our calculation. The possibility of the backward slip will be discussed later in relation to the value of $\nu_d{}^0 P_0$ and the time interval needed for the stress accumulation enough to cause the backward slip.

The boundary conditions are then written as follows:
At $y=0$ but off the crack,

$$\begin{aligned} &\mu \frac{\partial w}{\partial y} < F_s - T_0 \\ &D = \dot{D} = 0\,. \end{aligned} \qquad (2\text{–}9)$$

At $y=0$ and on the crack,

$$\begin{aligned} &\mu \frac{\partial w}{\partial y} = -\sigma_e{}^0 + \dot{D}\nu_d P_0 \\ &\dot{D} > 0\,, \end{aligned} \qquad (2\text{–}10)$$

but if

$$\begin{aligned} &\mu \frac{\partial w}{\partial y} < -\sigma_e{}^0 \\ &\dot{D} = 0 \end{aligned} \qquad (2\text{–}11)$$

where we use the notation $\sigma_e{}^0$ for $(T_0 - \nu_d{}^0 P_0)$.

The problem is to solve (2–1) subject to the initial condition (2–2), the boundary conditions (2–9)–(2–11), and the condition that

$$\xi \leq L \qquad (2\text{–}12)$$

By the symmetry of the problem, we note that
w is an even function of x but an odd function of y,
σ_{yz} is an even function of x and y.
σ_{xz} is an odd function of x and y,
so that we need only consider the region $x \geq 0$, and $y \geq 0$. In the next section, the problem is reformulated in the difference form and the effect of the different mesh sizes on the accuracy of the solution is examined.

3. Reformulation in the difference form

We shall rewrite the equation of motion, the initial and the boundary conditions by replacing the various derivatives by their difference approximations. We put

$x=m\Delta L$, $y=n\Delta L$, and $t=p\Delta t$, where ΔL is incremental length along the x-, or y-axis, and Δt an increment in time. Since the region $x\geqq 0$, $y\geqq 0$ for $t\geqq 0$ is considered, m, n, and p take on positive, including zero, integral values only.

$\nabla^2 w$, sum of $\partial^2 w/\partial y^2$ and $\partial^2 w/\partial y^2$ may be replaced by its centred difference approximation as

$$\nabla^2 w = \frac{1}{(\Delta L)^2}[w_{m+1,n,p}+w_{m-1,n,p}+w_{m,n+1,p}+w_{m,n-1,p}-4w_{m,n,p}]$$

$$\equiv \frac{1}{(\Delta L)^2}\Delta_1 w \tag{3-1}$$

or in another form

$$\nabla^2 w = \frac{1}{2(\Delta L)^2}[w_{m+1,n+1,p}+w_{m-1,n-1,p}+w_{m-1,n+1,p}+w_{m-1,n-1,p}-4w_{m,n,p}]$$

$$\equiv \frac{1}{2(\Delta L)^2}\Delta_2 w \tag{3-2}$$

In the above equations the notation $w_{m,n,p}$ means the value of w at the point $(m\,\Delta L, n\Delta L)$ for time $t=p\Delta t$. Similar meanings apply to other subscripts attached to w. $\nabla^2 w$ is correct to second order in incremental length ΔL for both approximations. $\nabla^2 w$ may be also represented as a linear combination of (3–1) and (3–2),

$$\nabla^2 w = a\frac{1}{(\Delta L)^2}\Delta_1 w + b\frac{1}{2(\Delta L)^2}\Delta_2 w \tag{3-3}$$

We put $a=2/3$ and $b=1/3$, then w is correct to fourth order in ΔL. $\partial^2 w/\partial t^2$ is replaced by its centred difference form as

$$\frac{\partial^2 w}{\partial t^2} = \frac{1}{(\Delta t)^2}[w_{m,n,p+1}-2w_{m,n,p}+w_{m,n,p-1}] \tag{3-4}$$

and is correct to second order in the increment Δt.

From (3–3) and (3–4), the equation of motion (2–1) is rewritten as

$$w_{m,n,p+1} = 2w_{m,n,p}-w_{m,n,p-1}+\frac{2}{3}\left(\frac{\beta\Delta t^2}{\Delta L}\right)\Delta_1 w+\frac{1}{6}\left(\frac{\beta\Delta t^2}{\Delta L}\right)\Delta_2 w \tag{3-5}$$

It is required for ΔL and Δt that

$$\frac{\beta\Delta t}{\Delta L} = O(1)\,, \tag{3-6}$$

so that the equation (3–5) should be correct, and that

$$\frac{\beta\Delta t}{\Delta L} < 1\,, \tag{3-7}$$

so that (3–5) should be stable. As far as the condition (3–6) and (3–7) are satisfied, the equation (3–5) is an approximation of the equation of motion (2–1) in a sense that (3–5) approaches to (2–1) as ΔL, $\Delta t\to 0$, and the error in (3–5) is of the order of $(\Delta L)^4$ and of $(\Delta t)^2$.

The symmetry condition implies that for all n and p

$$w_{-1,n,p}=w_{1,n,p} \tag{3-8}$$

The initial condition (2–2) is interpreted as

$$w_{m,n,-1}=w_{m,n,0}=0 \tag{3-9}$$

for all m and n. The equation (3–5) is explicit, and we note that, by (3–5) together with the symmetry condition (3–8) and the initial condition (3–9), it is possible to compute the value of w for all $p>0$ and $n>0$. The calculation stencil for (3–5) is shown in Fig. 2. The difference equation (3–5) is applied to either medium except for on the fracture surface $y=0$. The motion at $y=0$ is determined by the boundary conditions (2–9)–(2–11).

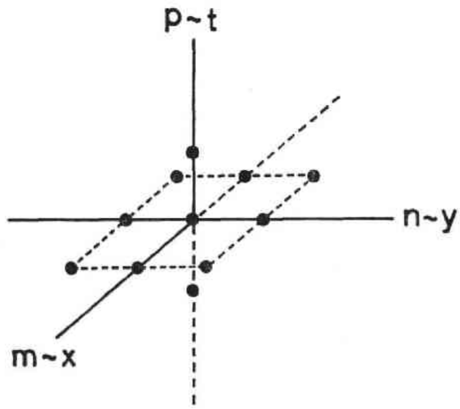

Fig. 2. The calculation stencil for the displacement w.

We may replace the derivatives $\partial w/\partial y$ and $\partial w/\partial t$ by the forward the backward difference approximation, respectively.

$$\frac{\partial w}{\partial y}=\frac{1}{\Delta L}\left[w_{m,1,p}-w^{+}{}_{m,0,p}\right],$$

$$\frac{\partial w}{\partial t}=\frac{1}{\Delta t}\left[w^{+}{}_{m,0,p}-w^{+}{}_{m,0,p-1}\right]$$

$w^{+}{}_{m,o,p}$ means the value of w referred to the positive side of the fracture surface. The boundary conditions (2–9)–(2–11) are then rewritten as:
At $y=0$ but off the crack,

$$\frac{\mu}{\Delta L}\left[w_{m,1,p}-w^{+}{}_{m,0,p}\right]<F_s-T_0$$

$$w^{+}{}_{m,0,p}=0\,, \tag{3-10}$$

At $y=0$ and on the crack

$$\frac{\mu}{\Delta L}\left[w_{m,1,p}-w^{+}{}_{m,0,p}\right]=-\sigma_e{}^0+\nu_d P_0\frac{2}{\Delta t}\left[w^{+}{}_{m,0,p}-w^{+}{}_{m,0,p-1}\right] \tag{3-11}$$

$$w^+_{m,0,p} > w^+_{m,0,p-1} ,$$

but if
$$\frac{\mu}{\Delta L}[w_{m,1,p} - w^+_{m,0,p}] < -\sigma_e^0$$

$$w^+_{m,0,p} = w^+_{m,0,p-1} . \tag{3-12}$$

In (3-11), the symmetry condition is applied, i.e.,

$$w^+_{m,0,p} = -w^-_{m,0,p}$$

By (3-11), the initial value of $w_{0,0,1}$ is computed, then it is possible to compute the value of $w_{m,n,p}$ for each time level $p \geq 2$ step by step according to (3-5) and (3-10)-(3-12). Practically the medium is bounded by rigid walls at $x = M\Delta L$ and $y = N\Delta L$ where the value of w is always zero. M and N are taken to be large enough that the reflected waves do not disturb the motion around $y=0$ and $x \leqq L$ before the final state at rest is attained.

The accuracy of a solution depends upon the choice of values of ΔL and Δt. In order to exmaine the effect of the different mesh sizes, computations are carried out for five choices of $\Delta L/L$, 0.2, 0.1, 0.05, 0.033, and 0.025, for a typical situation of friction. The value of Δt is fixed to be $\Delta L/4\beta$ for each case. Fig. 3 shows the solutions at $x=0$, $y=0$ for the five different mesh sizes. In Fig. 4(a), the values of the final displacement at several points on the crack are plotted as a function of $\Delta L/L$, and in Fig. 4(b), the effect of the mesh size on the arrival of rupture front is shown. As is seen in the figures, the deviation between the values for $\Delta L/L \leq 0.05$ is small, and it is considered that the solution for $\Delta L/L \leq 0.05$ is sufficiently accurate. $\Delta L/L$ is taken to be 0.033 in our calculation. We also exmained the effect of the value of $\beta\Delta t/\Delta L$ on the solutions to find that the deviation due to the difference in the ratio of $\beta\Delta t/\Delta L$ is much smaller

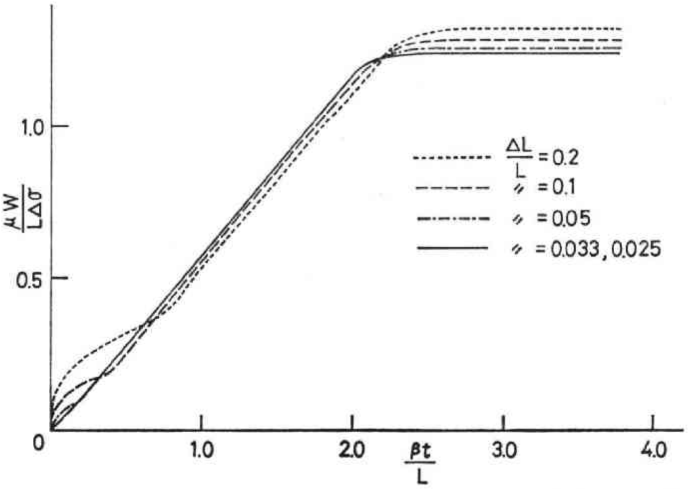

Fig. 3. The time functions of the the displacement at the centre of the crack for five different mesh sizes. It is seen that the deviation between the values for $\Delta L/L \leq 0.05$ is small.

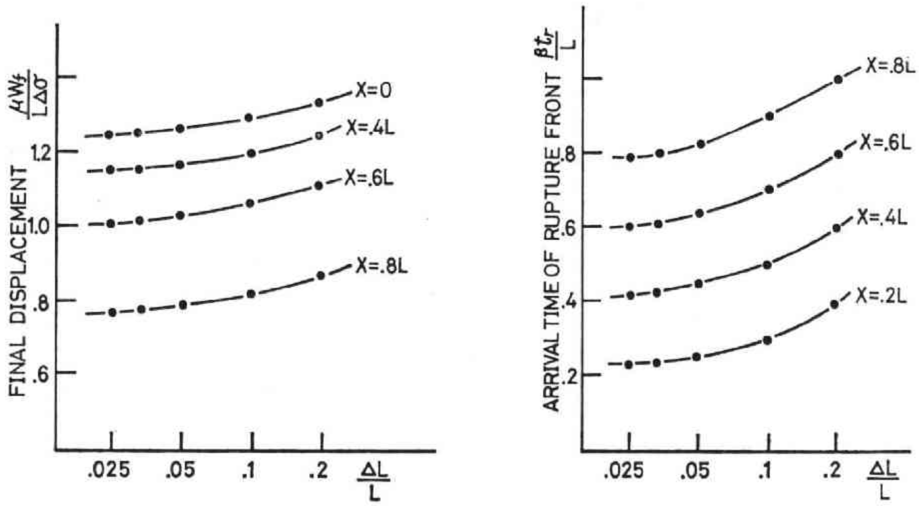

Fig. 4 (a) Fig. 4 (b)

Fig. 4. The dependency of the value of (a) final displacement and (b) arrival time of the rupture front upon the mesh size.

than that due to the difference in $\Delta L/L$ itself. $\beta\Delta t/\Delta L$ is taken to be 0.25. From Fig. 4(a) and Fig. 4(b), it is seen that errors are estimated at 0.02 in the displacement values and at 0.05 in the arrival time of rupture front for $\Delta L/L=0.033$ and $\beta\Delta t/\Delta L=0.25$.

4. Results

1) *General features of the dislocation time function*

The example for the case of no viscous friction is most typical of the dynamic features of slip motion. In Fig. 5, the particle motion at several points on the crack is shown for the case where no viscous friction acts ($\nu_d=0$). The static friction is taken as $F_s=T_0+0.25\sigma_e{}^0$.

First of all we note that the final value of displacement decreases with increasing x, approximated within the precision of our numerical solution as

$$w_f = K\frac{\sigma_e{}^0}{\mu}(L^2-x^2)^{1/2} \tag{4-1}$$

This is of the same form as the well-known solution in the static case of the problem (Bilby and Eshelby, 1968). The static solution gives $K=1$, while our solution shows that $K=1.3$, that is, the final displacement exceeds the value expected from the static solution by 30 percents. Consequently, the total shear stress on the crack at the final state is less than the value of the dynamic friction $\nu_d{}^0P_0$, and the stress drop is larger than $\sigma_e{}^0$ by a factor of 1.3. This means that the particle on the crack is stressed backward to the negative x-direction, and the backward slip may be possible. The backward slip requires that the value of the dynamic friction should be less than 0.13

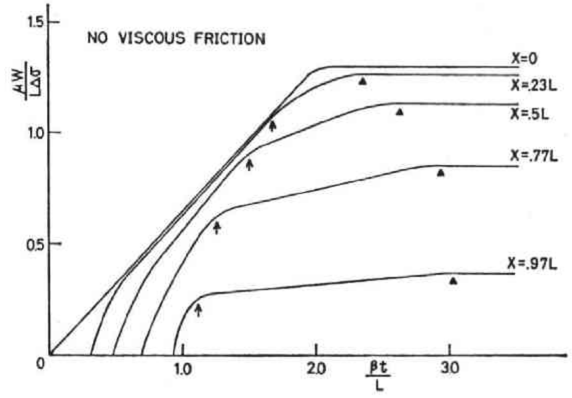

Fig. 5. The solution obtained for the case $\nu_d=0$ (no viscous friction). The propagation of the stopping phase (indicated by arrows) and the healing phase (indicated by wedges) are clearly seen. Note that the duration of slippage is two or three times longer than the time needed for the crack expansion to its maximum length.

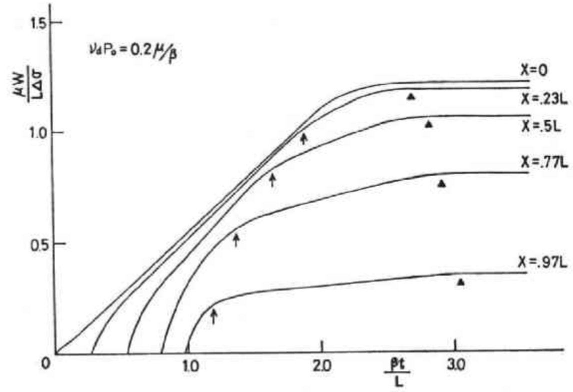

Fig. 6. The solution obtained for the case $\nu_d P_0=0.2\ \mu/\beta$

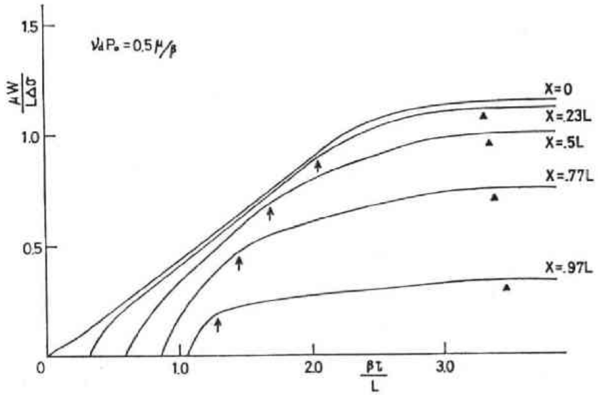

Fig. 7. The solution obtained for the case $\nu_d P_0=0.5\ \mu/\beta$

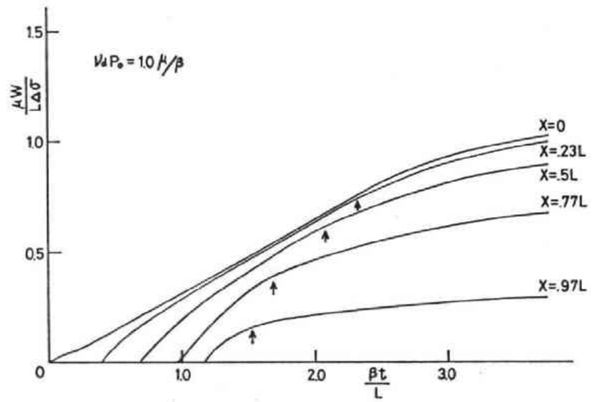

Fig. 8. The solution obtained for the case $\nu_d P_0=1.0\ \mu/\beta$

T_0. It is found, however, that it takes much time, longer than $3L/\beta$ when $\nu_d{}^0P_0=0.13T_0$, for the accumulation of the shear stress enough to cause the backward slip. Thus, it is supposed that the particle is probably blocked by the static friction again while $\dot{D}=0$. It may be concluded that the particle on the crack is frozen at its maximum displacement and that the backward slip is not likely as far as the friction resists the slip motion on the crack.

Next, it is found that the history of the particle motion at a point on the crack is characterized by the three epochs, arrival of three distinct phases; the rupture breakage, the stopping phase, and the healing phase. The rupture front arrives at $x=L$ at time $t=L/\beta$, that is, the rupture velocity $V_r=\beta$. It is found that the crack propagates at the S-wave velocity for various values of the static friction smaller than $T_0+0.4\sigma_e{}^0$, the rupture velocity being independent of the value of the static friction. This result is in harmony with those of the self-similar cracks obtained by Burridge (1973) and Burridge and Levy (1974). On the other hand, Kostrov (1964a, b) has shown that the self-similar crack propagates at a velocity lower than the Rayleigh wave velocity. The difference in the rupture velocity in these examples may be attributed to the difference in the fracture criterion. Kostrov's solution is based upon the consideration of the inelastic energy dissipation at the crack tip. Burrdige and Burrdige and Levy consider that only the friction resists the slip motion on the crack but no other inelastic effect is counted in. From our result together with the results on the self-similar cracks, it is considered that the crack propagates at either the P or the S wave velocity as far as the rupture occurs in the form of the frictional sliding and unless any other inelastic effect acts.

On the arrival of the rupture front, relative slipping takes place at the point. The slip velocity, twice the particle velocity, at the breakage increases with increasing x. It is about 1.3 $\beta\sigma_e{}^0/\mu$ at $x=0$ but it amounts to 4.0 $\beta\sigma_e{}^0/\mu$ near the edge of the crack. The slip velocity is rather high until the arrival of the second phase (indicated by arrows in the figures), when the particle is suddenly decelerated to a lower velocity. The second phase is interpreted as the stopping phase. The information that the rupture propagation is stopped at $x=L$ is transmitted backward to the centre of the crack as the reflected stress wave. The propagation velocity of the stopping phase is found to be β. The particle motion is decelerated due to no more expansion of the crack. The particle near the edge of the crack is most affected by the reflection of the shear stress, the slip velocity dropping off from 4.0 to 0.11 $\beta\sigma_e{}^0/\mu$.

Finally, at time $t=2L/\beta$, the particle at $x=0$ is set at rest and the healing phase is propagated to the edge of the crack (indicated by wedges). At $x=0$, the arrival of the stopping phase and the healing phase is almost simultaneous. The particle around the centre of the crack has a constant velocity almost all during the motion to its final state of displacement. The particle motion around the centre of the crack may be well approximated as a ramp time function. Its rise time is about $2L/\text{ß}$, twice as long as the time needed for the crack to propagate to its end $x=L$. On the contrary, the particle motion near the edge of the crack is step-like, having a high particle velocity at the breakage so that the final state of displacement is nearly attained in a short time. The duration of effective slipping, when the slip velocity is high, is very short, about 0.1 L/β.

The dislcoation function is of a complex form both temporally and spatially. It is not possible to represent the slip motion as a simple function as was used in the conventional source models. We note that at time $t=L/\beta$, just when the crack reaches its maximum length, displacement at any point on the crack is less than its final value. The duration of the particle motion on the crack is longer than that needed for the crack expansion to the final length. The slip motion consists of two processes. One is the rupturing process and the other the relaxation to the final state at rest. This is a remarkable feature of the dynamic process of expanding cracks.

2) *Effect of the viscous friction*

In Fig. 6 through 8, the dependence of the particle motion on the crack upon the viscous term of the dynamic friction is shown. The value of ν_d takes on 0.2, 0.5, and 1.0 $\mu/(P_0\beta)$, the static friction fixed as $F_s=T+0.25\sigma_e{}^0$ as in the case for $\nu_d=0$. As is clear in the figures, the viscous friction has much influence on the slip motion. The dependence of various quantities upon the value of ν_d is summarized in Table 1.

Table 1.

$\nu_d(\mu/P_0\beta)$	0	0.2	0.5	1.0
K	1.30	1.21	1.15	1.04
$V_r(\beta)$	1.00	1.00	0.92	0.83
$\dot{D}(\beta\sigma_e{}^0/\mu)$	1.3	1.1	0.9	0.6
$T_r(L/\beta)$	2.0	2.3	2.7	3.5

The dependence of the various quantities upon the value of ν_d. K, the ratio of the final displacement in our solutions to the static value; V_r, the rupture velocity; $\dot{D}$, the slip velocity; T_r, the rise time of the particle at the centre of the crack. The unit for each quantity is given in the parenthesis.

The general feature of the particle motion is nearly the same as in the case for $\nu_d=0$. The arrival of the stopping phase is also identified, but the motion is somewhat smoothed by the damping effect of the viscous friction. The slip velocity at a point on the crack is as a matter of course decreases as ν_d increases. The slip velocity at $x=0$ takes on 1.1, 0.9, and 0.6 $\beta\sigma_e{}^0/\mu$ for $\nu_d=0.2$, 0.5, and 1.0 $\mu/(P_0\beta)$, respectively. Since the slip velocity is reduced to a lower value due to the damping effect of the viscous friction, the motion of the whole medium is also damped. Consequently, it results that more time is needed for the stress consentration at the tip of the crack, causing the rupture velocity to be lower. When ν_d is small, the crack propagates at the S-wave velocity. As ν_d increases, however, the rupture velocity decreases.

The viscous friction also affects the values of final displacement on the crack and the duration of slip. The final displacement is found to be approximated as (4–1), the same form as in the case for $\nu_d=0$. The value of K is at its maximum 1.3 for $\nu_d=0$, but it takes on smaller values 1.21, 1.15, and 1.04 as ν_d increases as 0.2, 0.5 and 1.0 $\mu/(P_0 \beta)$. The duration of slip is elongated by the effect of the viscous friction. The rise time of the particle at $x=0$ is 2.3, 2.7, and 3.5 L/β for $\nu_d=0,2$, 0.5 and $1.0\,\mu/(P_0\beta)$, respectively. For a value of ν_d larger than $1.0\,\mu/(P_0\beta)$, the value of K may be no more than 1.0, the final displacement corresponding to the static displacement, and the duration of slip will be much longer. In that case, the stress drop is as same as $\sigma_e{}^0$, lower than in the case where no viscous friction acts. It may be expected that when the material on the fracture surface is much viscous, the period of observed seismic waves is longer and the amplitude smaller compared with those generated from no viscous sources for the same $\sigma_e{}^0$.

A high viscous friction results in the low effective stress during the slip motion. This causes the particle velocity to be lower, or the momentum of the particle to be decreased. The consequence of this is the decrease in the inertia of the partiale, and the energy in unit time available for the crack expansion is decreased. Therefore, a low rupture velocity is obtained and the amount of the overshoot in displacement at the final state from the static solution is decreased. This interpretation implies that these quantities, the rupture velocity, the slip velocity, the final displacement, and the duration of slip are mutually related through the effect of the viscous friction.

5. Conclusions and discussions

We have studied the dynamic fracture process on a preexisting fracture surface under prescribed stress conditions in the presence of the static and dynamic frictions as modelling the source process of rather small earthquakes. The dynamic friction is supposed to involve the viscous term which increases in proporition to the slip velocity, and the effect of this term on the slip motion is invesitaged. The fracture criterion that slip occurs when the total shear stress overcomes the static limiting friction is employed, which is considered physically reasonable as far as the fracture process during an earthquake is concerned. In order to make the physical meanings clear, no assumptions have been made but the stress boundary conditions and the fracture criterion. The problem is solved numerically by the difference method within the desired accuracy for the antiplane strain shear cracks. Some remarkable results are obtained on the dynamic features of the fracture process, and it is shown that the physical quantities which mathematically describe the fracture process, such as the rupture velocity, the slip velocity, the final displacement, and the duration of slip are mutually related through the conditions of friction.

In the exmaple for the case of no viscous friction, the dynamic nature of slip is clearly seen. It is found that the final displacement on the crack is of the same form as the static solution of the problem within the precision of our numerical solution, but that it exceeds the value expected from the static solution. The overshoot of displacement than the static position may be a consequent result of the dynamic motion of a particle on the crack. Since no viscous friction acts on the fracture surface, the effective stress during the slip motion is as much as the stress difference $\sigma_e^0=T_0-\nu_d^0P_0$. A particle has a velocity as high as is driven by the stress difference σ_e^0. The motion yields the inertia of the particle, so that a particle could not be stopped abruptly at the static position of the stress difference σ_e^0. As a result from the excess of displacement over the static position, the stress drop, the difference in the stress before and after fracturing, is larger than the effective stress during the fracture. On the other hand, when the viscous friction is strong, the final displacement on the crack is no more than the static value. A high viscous friction decreases the effective stress to a smaller value than that in the case of no viscous friction for the same stress difference σ_e^0. In this case, the stress drop is σ_e^0. Our result suggests that the viscous friction is an important factor in the study of the problem of the effective stress during the slip motion and of its relation to the stress difference σ_e^0 and the final stress drop.

The excess of displacement over the static value at the final state causes the particle to be stressed backward. It is demonstrated, however, that the backward slip is hardly possible when the particle is resisted by frictions across the fracture surface. The backward slip is only possible when the fracture surface is much lubricated. Then the particle on the crack will oscillate to its static position (Burridge, 1969). In this case, the far-field spectra of the body waves may have a peak at the corresponding frequency to the oscillation of slip at the source (Molnar et al., 1973; Archambeau, 1968). It is hard, however, to imagine that no friction acts on the fracture surface while the slip motion is in progress. It may be consluded that the backward slip is unlikely during a fracture process and consequently that the far-field spectra of body waves due to shear faulting will not have a peak.

It is also found that the rupture velocity is as fast as the S-wave velocity when the viscous friction is weak. This result is in harmony with the results of self-similar cracks obtained by Burridge (1973) and by Burridge and Levy (1974). On the contrary, as the viscous friction increases, the rupture velocity takes on a smaller value. Kostrov (1964 a, b) have shown that the crack is possible to propagate only at a lower velocity than the Rayleigh wave velocity. Kostrov's solution is based upon the consideration of inelastic energy dissipation at the crack tip, while Burridge and Burridge and Levy have considered no inelastic effects acting at the crack tip. A low rupture velocity is often obtained in the experimental studies (Archuleta and Brune, 1975; Kitagawa and Yamamoto, 1975) and in the analyses of the seismic waves (Trifunac and Udwadia, 1974; Izutani, 1974). Recently some experimental studies show that the rupture propagates as fast as the S-wave velocity (Wu et al., 1972; Johnson et al., 1973). High rupture velocities are observed in the experiments of frictional sliding on clean surfaces. The experiments which give low rupture velocities have been carried out on samples which have no pre-cut or which are considered to have much of inelastic properties. In these cases, some amounts of energy may be lost as the work done against the cohesive force or as the inelastic dissipation. It is considered that the crack will propagate at the S-wave velocity, or possibly at the P-wave velocity, so long as the material at the fracture region has little of inelastic properties other than frictions.

We have shown that the fracture process consists of two distinct processes, the rupturing process until the crack expansion is stopped and the following relaxation process to the final state at rest. Slip continues after the rupture propagation is stopped, not decelerated until the arrival of the stopping phase, which propagates at the S-wave velocity from the edge of the centre of the crack. This is an important feature of the dynamic process of fracturing in consequence of the dynamic causality condition that any information is transmitted at a finite velocity. The source time function is required to satisfy this condition. Any seismic source model has not accounted for this consequent requirement from the dynamic condition. The ratio of the displacement on the expanding crack to that on the static crack of a corresponding length is less than 1.0. This means that the stress is not in static equilibrium during rupture. This is considered due to the high rupture velocity. Some portion of the released energy must be used for the kinetic energy of the crack expansion. It is supposed that the ratio will approach to 1.0 as the rupture velocity decreases. The assumption made by Sato and Hirasawa would be valid for the cracks propagating at a very low velocity. Referring to the result by Kostrov, the ratio is nearly 1.0 for the cracks which ropagate at lower velocities than 0.2β, but it takes on smaller values 0.91, 0.88, and 0.84 for $V_r=0.5$, 0.7, and 0.98. For the cracks propagating as fast as the S-wave velocity, the displacement is always less than the static value at the moment when the crack expansion is stopped, and slip continues after the stoppage of the crack expansion. The characteristics of the theoretical predictions of seismic waves will be much affected by this dynamic feature of the slip motion. The relation between the spectral parameters of seismic waves and the physical parameters of the source must be carefully examined under the dynamic conditions of the slip motion. In particular, the relation between the corner frequency and the source dimension is considered to be most affected.

The viscous friction, as a mechanism of the inelastic energy loss on the fracture surface, is found to have much effect on the dynamic behaviour of slip. Furthermore, the rupture velocity, the slip velocity, the final value of displacement, and the duration of slip are mutually related through the effects of the viscous friction. These quantities could not be given independently, but they should be specified so that their possible interrelation is satisfied under the physical conditions considered. This condition is important easpecially in the analysis of the seismic waves to the estimation of the physical process of the source.

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