東北大学機関リポジトリ
TOUR
Tohoku University Repository

Maximum Entropy Estimates of the Spheroidal Eigenperiods of the Earth

著者	Koyama Junji
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	23
号	3-4
ページ	149-157
発行年	1976-08
URL	http://hdl.handle.net/10097/44737

Sci. Rep. Tôhoku Univ., Ser. 5, Geophysics, Vol. 23, Nos. 3–4, pp. 149–156, 1976.

Maximum Entropy Estimates of the Spheroidal Eigenperiods of the Earth

JUNJI KOYAMA

Geophysical Institute, Faculty of Science, Tôhoku University
Sendai 980, Japan

(Received April 28, 1976)

Abstract: The maximum entropy method with the Akaike criterion is adopted to measure the spheroidal eigenperiods of the earth. The data analyzed are the digital data of the Askania gravitymeter at Kyoto University, Japan, after the 1960 Chilean earthquake and the record of the TRG–1, revised North American gravitymeter, at the International Latitude Observatory of Mizusawa, Japan, following the Vladivostok deep-focus earthquake of September 29, 1973. In comparison with the spectra by conventional Fourier transforms, the method shows remarkable resolution property and has significantly enhanced the precision of the estimate of eigenperiods for higher-order overtones. The spheroidal eigenperiods of the earth for fundamental and higher modes are estimated in the period range from 200 to 1200 sec. The deviation of the estimated eigenperiods from those of the average earth model falls within 1 per cent, especially within 0.2 per cent for the period range from 300 to 800 sec.

1. Introduction

The maximum entropy method (MEM) of spectral analysis originally proposed by Burg (1972) is based on the information content of the time series under consideration. The effectiveness of the method has been demonstrated relatively to conventional techniques for the estimation of power spectrum (Smylie *et al.*, 1973). It has been shown that the method shows a remarkably high resolution in detecting sinusoidal vibrations, even when the time series is truncated (Lacoss, 1971; Ulrych, 1972).

Although the superiority of the MEM to other conventional methods, in particular for short data lengths, is well recognized, there would be two shortcomings in applying the method to real data: One is the arbitrariness for choosing the length of prediction error filter (PEF). The other is the lack of a variance estimate of the estimated spectral density. It can be expected from the representation in the following section that too short a length of PEF gives a highly smoothed spectrum with poor resolution. On the other hand, it is known from numerical experiments that an excessive length gives a spectrum with spurious detail.

It is discussed recently that the MEM is in fact equivalent to the least squares fitting of an autoregressive (AR) model to the time series (Ulrych and Bishop, 1975). This duality has allowed us to apply the statistical characteristics of the AR process to overcome the shortcomings mentioned above. Akaike (1969a, b, 1970) presented a criterion for estimating the optimum length of PEF, starting with the AR process.

We will apply the Akaike final prediction error (FPE) criterion to the time series with comparatively short data lengths obtained from the gravitymeters after the earthquakes, and estimate the spheroidal eigenperiods of the earth from spectra by FPE-criterion.

2. Maximum Entropy Spectral Estimator

According to the discussions by Akaike (1969b) and Smylie *et al.* (1973), the estimate of power spectrum, $P(f)$, for a time series is expressed in terms of the estimate of variance, $\sigma^2(M)$, of the time series after whitening by the PEF of length M and the Fourier transform of the PEF-coefficients, $a(k)$, $k=1,2,\cdots,M$, as;

$$P(f)=\frac{2\Delta t\sigma^2(M)}{\left|1+\sum_{k=1}^{M}a(k)\exp\left(-2\pi ifk\Delta t\right)\right|^2}, \tag{1}$$

$$0\leqq f\leqq f_n=1/2\Delta t\,,$$

where f_n is the Nyquist frequency and Δt the sample interval. The optimum filter length M_0, which gives the optimum power spectrum, is given by the number M which minimizes the estimate of a final prediction error;

$$(N+M+1)/(N-M-1)\,\sigma^2(M)\,,$$

where N is the data length.

Although the FPE-criterion is based on the statistical properties of the AR process, it is notable that there would be an implicit constraint that the optimum filter length M_0 is much smaller than the data length N.

It is also worth noting that the advantage of the MEM is that there is no restriction of the bandwidth in calculating spectral densities from the finite digital-data of a time series, because the denominator of eq. (1) can be calculated for an arbitrary value of frequency without assumption such as a vanishing of energy or a periodicity of the time series outside the parameter space.

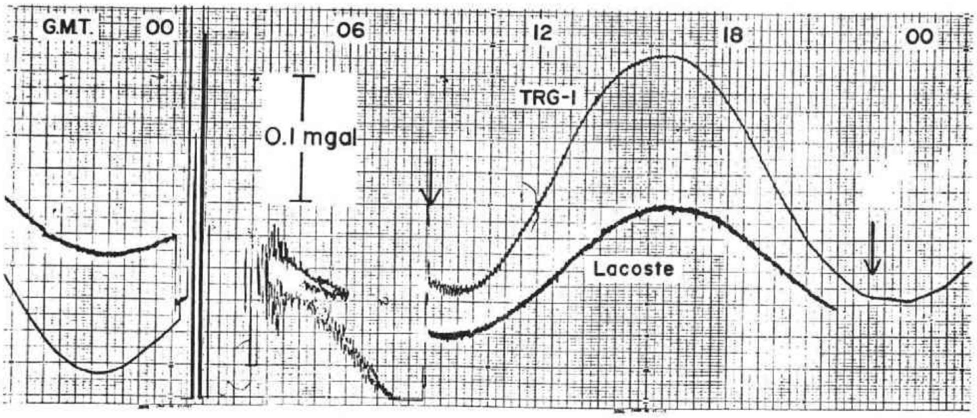

Fig. 1: The original record of the TRG–1, revised North American gravitymeter, of the International Latitude Observatory of Mizusawa following the Vladivostok deep-focus earthquake of September 29, 1973. The record between arrows is digitized and used for the analysis.

3. Data Analyzed

There is no doubt that gravitymeter records provide the estimates of spheroidal eigenperiods of the earth. Two data sets are used for analysis: One is the digitized data of the Askania gravitymeter at Kyoto University (Takeuchi *et al.*, 1962), Japan, after the 1960 Chilean earthquake, from which the main semi-diurnal and diurnal earth tides have been removed by multiplying 2-hour low-cut filter. A high-cut filter is processed on the data to cut off the component of which frequency is higher than the Nyquist frequency. The sample interval is 2 min and the resultant length of the data is 1360 for about 45 hours long. The other is the original record of the TRG–1, revised North American gravitymeter, at the International Latitude Observatory of Mizusawa, Japan, following the Vladivostok deep-focus earthquake of September 29, 1973, from which the earth tides have been removed by multiplying 1-hour low-cut filter twice. A high-cut filter is also processed. The sample interval of the data is 1.5 min and the resultant length is 405 for about 10 hours long. The original record of the TRG–1 is shown in Fig. 1.

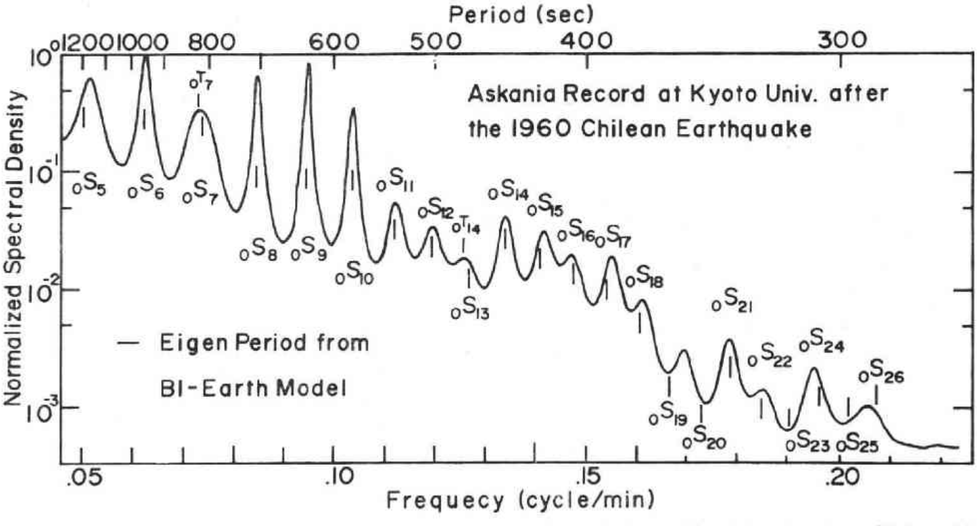

Fig. 2: The maximum entropy power spectrum estimated from the Askania data (Takeuchi *et al.*, 1962) after the 1960 Chilean earthquake. The dimension of the spectrum is $(\text{gal})^2$ sec.

4. Result and Discussion

The power spectrum estimated from the Kyoto data is illustrated in Fig. 2, which is obtained using the optimum PEF of length 71. A remarkable feature of Fig. 2 is the virtual absence of peaks which do not correspond to free oscillation modes, and the smooth and stable appearance of the spectrum. The apparent peaks of the spectrum for the higher-order overtones indicate systematically shorter eigenperiod in comparison with the theoretical one calculated from the B1-earth model (Jordan and Anderson, 1974).

Generally speaking, the Akaike criterion gives a small order of the AR process than the real one of the time series in numerical experiments (Akaike, 1970; Ulrych and Bishop, 1975). This may be the result of the dependency of the optimum filter-length M_0 by FPE-criterion on the data length N, though the dependency vanishes when M_0 is much smaller than N. An attempt is made to check the stability of the spectrum in Fig. 2: Using the FPE-coefficients obtained from the Kyoto data, future and past values of the time series are predicted. The extension of the data is made up to 2048 points. The length of the data is determined only for the sake of computer programing. The maximum entropy spectrum of the predicted Kyoto data is illustrated in Fig. 3, using the optimum PEF of length 142. The spectrum detects substantially more eigen-periods, which correspond principally to higher mode spheroidal oscillations. Fine structure of the spectrum is obtained with the enhaced precisions of resolution. We conclude that the spectrum in Fig. 3 is stable in the signal detection and is considered to be optimum. Eigenperiods are obtained from the spectrum by parabolic interpolation and are given in Table 1.

In the case of the Mizusawa data, the power spectrum estimated by using the FPE-criterion is highly smoothed and few eigenperiods are estimated. This result is considered to be the effect of the finiteness of the data on the choice of PEF length. If we could know the true length of the filter *a priori*, the spectrum would yield the peaks which correspond to the eigenfrequency. Bolt and Currie (1975) estimated the torsional eigenperiods assuming that the length of PEF is 10 per cent of the data length. In the present analysis we have used the FPE-coefficients obtained from the Mizusawa data to predict future and past values of the time series. Although the FPE filter used is not optimum, it is derived so as to maximize the entropy content of the finite time series. The final length of the Mizusawa data is taken 2048. The power spectrum estimated from the predicted data is illustrated in Fig. 4, using the optimum PEF of length 134. The stability of the spectrum in signal detections is checked in comparison with the maximum entropy spectrum obtained from 1024-point realization of the Mizusawa data. Higher-order overtones which cannot be found on the spectrum in Fig. 3 are clearly seen on the spectrum. This is the result of effective excitation of the earth's free oscillations by the deep-focus earthquake. Individual eigenperiods obtained by parabolic interpolation are given in Table 1.

Taking 400–point data from the Kyoto data, and neglecting the first and the last 480–point data, a numerical experiment has been performed to check the stability of the signal detections estimated from a short-length data by using FPE-coefficients. Two spectra in Fig. 3 are compared, which are estimated from the sets of 2048–point realization from the Kyoto data and from the 400–point Kyoto data by using FPE-criterion. The spectral peaks in Fig. 3 strictly coincide with each other. The eigenperiods from the Mizusawa data could be justified from the above experiment.

The absolute values of the spectral peaks are somewhat different in Fig. 3, for each mode. Some of the difference are due to the effect of the attenuation by the anelasticity of the earth, and some are the effect of the finiteness of the truncated time

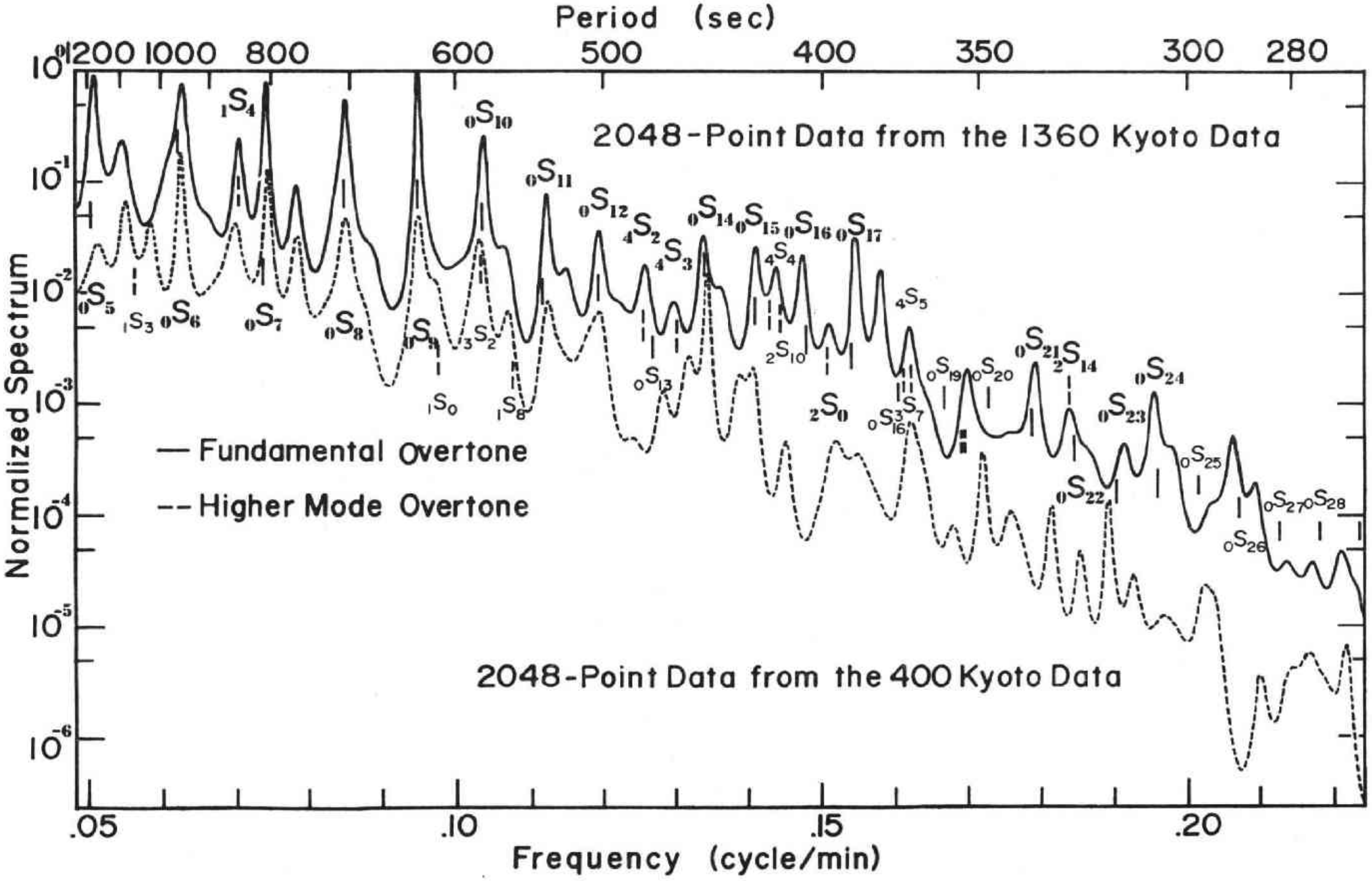

Fig. 3: The maximum entropy power spectra estimated from the 2048-point realizations of the whole Kyoto data and of the 400-point Kyoto data (broken line).

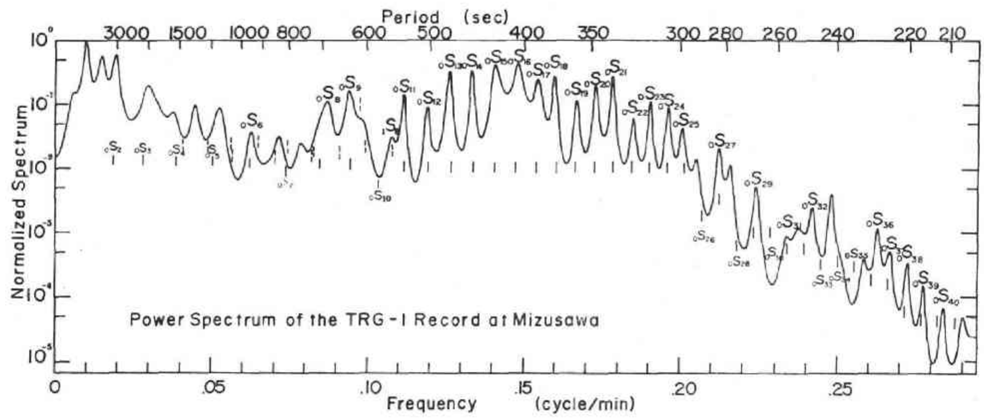

Fig. 4: The maximum entropy spectrum estimated from the 2048-point realization of the Mizusawa data after the Vladivostok deep-focus earthquake of September 29, 1973.

series on the estimates of autocorrelation functions.

The eigenperiods estimated from both the spectra are in good agreement with those theoretically-calculated. The deviation of the estimated eigenperiods from those of the average earth model falls within 1.0 per cent in the period range from 200 to 1200 sec. In the period range from 300 to 800 sec, the deviation falls within 0.2 per cent.

5. Conclusion

The maximum entropy method with the Akaike criterion has been successfully used to estimate the spheroidal eigenperiods of the earth from the gravitymeter records. In comparison with the conventional methods of Fourier analysis, the MEM has yielded highly smoothed spectrum with peaks strictly corresponding to free oscillation modes.

There is no procedural difficulties for extending the data to any length from a short length data, by using the prediction error coefficients. The predicted data synthesized from a small portion of the data gives the spectrum which is in good agreement with the one from the whole data. The attention must be paid on the absolute values of the spectrum which reflect the effect of the finiteness of the truncated time series on the estimates of autocorrelation functions, when only a small portion of the data is used to estimate the spectrum, because the time series of the damped oscillations is not stationary.

Spheroidal eigenperiods are estimated for the higher-order overtones of ${}_0S_5 \sim {}_0S_{41}$ and for some higher modes.

Ackowledgements: The author is indebted to Professor Z. Suzuki for his continuous encouragement and kind discussions. The author is extremely grateful to Dr. K. Hosoyama and his colleagues of the International Latitude Observatory of Mizusawa

Table 1. The Estimated Eigenperiods of Spheroidal Oscillations in second.

	Kyoto	Mizusawa	B1-Earth		Kyoto	Mizusawa	B1-Earth
${}_0S_5$	1186.7		1190.4	${}_0S_{19}$		359.1	360.1
${}_1S_3$	1102.6 ?		1062.8	${}_0S_{20}$		346.4	347.5
${}_0S_6$	959.3	956.2	963.7	${}_0S_{21}$	335.4	335.8	335.9
${}_1S_4$	853.6		851.8	${}_2S_{14}$	326.9		326.4
${}_0S_7$	811.4		812.2	${}_0S_{22}$		323.9	325.2
${}_0S_8$	708.3		707.7	${}_0S_{23}$	314.1	314.7	315.4
${}_0S_9$	634.3	635.8	633.7	${}_0S_{24}$	307.3	305.5	306.2
${}_0S_{10}$	579.4		579.2	${}_0S_{25}$		298.4	297.7
${}_1S_8$		555.8	556.2	${}_0S_{26}$	291.2	292.0	289.7
${}_0S_{11}$	535.3	536.4	536.9	${}_0S_{27}$		282.2	282.3
${}_0S_{12}$	503.4	502.9	502.3	${}_0S_{28}$		277.5	275.2
${}_4S_2$	478.2		478.0	${}_0S_{29}$		267.2	268.5
${}_0S_{13}$		474.7	473.2	${}_0S_{31}$		256.0	256.1
${}_4S_3$	463.4		460.9	${}_0S_{32}$		251.4	250.4
${}_0S_{14}$	449.4	449.2	448.1	${}_0S_{33}$		247.4	244.9
${}_0S_{15}$	426.8	425.7	426.2	${}_0S_{34}$		241.4	239.7
${}_4S_4$	418.4?		420.2	${}_0S_{35}$		231.6	234.7
${}_0S_{16}$	408.2	404.6	406.8	${}_0S_{36}$		227.6	229.9
${}_2S_0$	398.2		398.5	${}_0S_{37}$		224.5	225.2
${}_0S_{17}$	388.8	387.8	389.6	${}_0S_{38}$		219.8	220.8
${}_2S_{11}$	388.8	387.8	388.5	${}_0S_{39}$		215.8	216.5
${}_0S_{18}$		374.8	374.1	${}_0S_{40}$		210.9	212.4
${}_4S_5$	371.2		369.9	${}_0S_{41}$		206.3	208.5

for providing him with the original record of the TRG-1. The author is also oblidged to Professor T. Hirasawa and Dr. K. Yamamoto for enlightening discussions throughout the present study. Discussions with Drs. T. Sato and M. Izumisawa are greatly benefitted for the application of the MEM to computer programings. The author expresses his hearty thanks to Drs. H. Hamaguchi, T. Sato, and H. Oda for critically reading the manuscript. Miss Kahoru Ishizuka kindly assisted the author in digitizing the TRG-1 record. Numerical calculations have been carried out using NEAC 2200–700 at the Computer Center, Tôhoku University.

References

Akaike, H., **1969a**: Fitting autoregressive models for prediction, Ann. Inst. Statist. Math., **21**, 243–247.

Akaike, H., 1969b: Power spectrum estimation through auto-regressive model fitting, Ann. Inst. Statist. Math., **21**, 407–419.

Akaike, H., **1970**: Statistical predictor identification, Ann. Inst. Statist. Math., **22**, 203–217.

Bolt, B.A. and R.G. Currie, **1975**: Maximum entropy estimates of earth torsional eigenperiods from 1960 Trieste data, Geophys. J., **40**, 107–114.

Burg, J.P., **1972**: The relationship between maximum entropy spectra and maximum likelihood spectra, Geophyscis, **37**, 375–376.

Jordan, T.H., and D.L. Anderson, **1974**: Earth structure from free oscillations and travel times, Geophys. J., **36**, 411–459.

Lacoss, R.T., **1971**: Data adaptive spectral analysis method, Geophysics, **36**, 661–675.

Smylie, D.E., G.K.C. Clarke, and T.J. Ulrych, **1973**: Analysis of irregularities in the earth's rotation, in *Methods in Computational Physics*, Academic, New York, **13**, 391–430.

Takeuchi, H., M. Saito, N. Kobayashi, and I. Nakagawa, **1962**: Free oscillations of the earth observed on gravitymeters, Zisin, Ser. 2, **15**, 122–137 (in Japanese with English abstract).

Ulrych, T.J., **1972**: Maximum entropy power spectrum of truncated sinusoids, J. Geophys. Res., **77**, 1396–1400.

Ulrych, T.J. and T.N. Bishop, **1975**: Maximum entropy spectral analysis and autoregressive decomposition, Rev. Geophys. Space Phys., **13**, 183–200.