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著者	Nishino Yoshiharu
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Study of Love Waves Reflected at a Corner by Finite Difference Method

YOSHIHARU NISHINO

Osaka Shin-ai Jogakuin Junior College, Joto-ku, Osaka 536, Japan

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Abstract: The problem of the transmission and reflection of Love waves in a wedge-shaped medium is studied by the use of a finite difference method. Reflection coefficients of Love waves are obtained for wedge angle range 72° to 108°. Reflection coefficient for acute-angled corner is greater than that for obtuse-angled corner. Reflection coefficient is not symmetric about wedge angle 90°. The accuracy of numerical calculation was examined in various ways.

1. Introduction

Wave propagation in wedge-shaped medium is important and interesting because of its bearing on effects of topographic irregularities and crustal discontinuities on seismic disturbances. A number of investigators have studied the problems of Rayleigh wave propagation in wedges both by theoretical and experimental means (e. g. De Bremaeker, 1958; Knopoff and Gangi, 1960; Hudson and Knopoff, 1964b; Mal and Knopoff, 1966; Lewis and Dally, 1970).

Hudson and Knopoff (1964a) discussed transmission and reflection of Love waves in a wedge using a Green's function technique, in which they neglected the contribution from multiple reflections and diffracted waves. They pointed out that reflection coefficients for acute-angled and obtuse-angled corners are identical when deviation of wedge angle from 90° is the same.

Yamazaki and Ishii (1973) investigated Love waves in a wedge-shaped medium overlying an elastic medium by using a ray theoretical method. Their discussion is concerned with phase and group velocities at observation points far from an apex and is valid for small wedge angles for which the diffracted wave amplitude is negligible.

Since model experiments are difficult for the study of Love wave propagation, numerical experiments play an important role. Boore (1970) studied Love wave propagation in a medium with a sloping boundary between superficial layer and basement by the use of finite difference method.

In the present study reflection coefficients of Love waves at a corner are obtained as a function of wedge angle by applying a finite difference method. In the finite difference method space grids having the shape of parallelograms with sides parallel to the two free surfaces of the wedge are adopted. Coefficients of Love waves reflected at an obtuse-angled corner and acute-angled corner are discussed in comparison with the results of Hudson and Knopoff (1964a). Computed seismograms and phase velocities of reflected Love waves are obtained. Wave displacements including

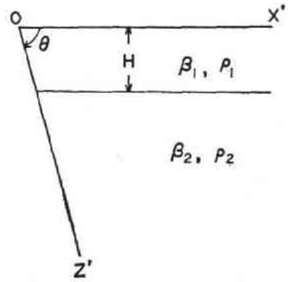

Fig. 1. The geometry of the problem.

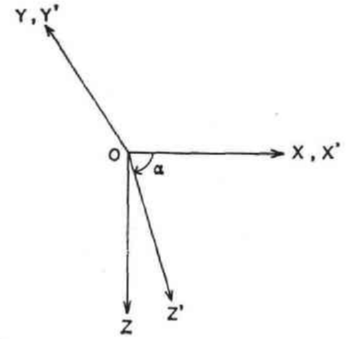

Fig. 2. The rectangular coordinate system (x, y, z) and the oblique coordinate system (x', y', z').

reflected Love waves and refracted body waves in the entire spatial region of interest are also discussed.

2. Equation of Motion and Boundary Conditions

Consider the geometry shown in Fig. 1, where a wedge-shaped elastic medium is bounded by its two stress free surfaces. A low-velocity surface layer of thickness H with shear wave velocity β_1 and density ρ_1 is in welded contact with a high-velocity substratum with elastic parameters β_2 and ρ_2. The wedge angle is denoted by θ.

We introduce an oblique coordinate system (x', y', z') related to a rectangular cartesian coordinate system (x, y, z) as shown in Fig. 2, in which $x'=x$, $y'=y$. An angle between oblique coordinate axes ox' and oz' is denoted by α. In this problem only SH motion is considered, the motion being independent of y. Equation of motion in the rectangular coordinates

$$\frac{\partial^2 v_i}{\partial t^2} = \beta_i^2\left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial z^2}\right), \qquad i = 1,2, \tag{1}$$

becomes in the oblique coordinates

$$\frac{\partial^2 v_i}{\partial t^2} = \frac{\beta_i^2}{\sin^2\alpha}\left(\frac{\partial^2 v_i}{\partial x'^2} - 2\cos\alpha\frac{\partial^2 v_i}{\partial x'\partial z'} + \frac{\partial^2 v_i}{\partial z'^2}\right), \qquad i = 1,2\,, \tag{2}$$

where v_i is the horizontal displacement in y direction in the two regions $(i=1, 2)$. The boundary conditions are that stresses on the free surfaces of the wedge vanish and stresses and displacements on the interface are continuous. Hence the boundary conditions in the oblique coordinates are given as follows,

$$-\cot\alpha\cdot\frac{\partial v_1}{\partial x'} + \operatorname{cosec}\alpha\cdot\frac{\partial v_1}{\partial z'} = 0\,, \qquad \text{on } z' = 0\,, \tag{3}$$

$$-\operatorname{cosec}\alpha\cdot\frac{\partial v_i}{\partial x'} + \cot\alpha\cdot\frac{\partial v_i}{\partial z'} = 0\,, \qquad i = 1,2\,, \text{ on } x' = 0\,, \tag{4}$$

$$v_1 = v_2\,, \qquad \text{on} \qquad z' = H\operatorname{cosec}\alpha\,, \tag{5}$$

$$\mu_1\Big(-\cot\alpha\cdot\frac{\partial v_1}{\partial x'}+\operatorname{cosec}\alpha\cdot\frac{\partial v_1}{\partial z'}\Big)=\mu_2\Big(-\cot\alpha\cdot\frac{\partial v_2}{\partial x'}+\operatorname{cosec}\alpha\cdot\frac{\partial v_2}{\partial z'}\Big),\quad \text{on}\quad z'=H\operatorname{cosec}\alpha, \tag{6}$$

where μ_1 and μ_2 are rigidities of the low-velocity layer and high-velocity medium, respectively. The angle α related to the oblique coordinates is taken equal to the wedge angle θ.

3. Finite Difference Formulation

Finite difference techniques have recently been applied to elasticity problems (Alterman and Karal, 1968; Alterman and Rotenberg, 1969; Alterman and Loewenthal, 1970; Boore, 1970; Ottaviani, 1971; Satô, 1972; Munasinghe and Farnell, 1973). Although we are familiar with rectangular space grids, when a boundary or an interface does not pass through grid points, special devices must be made at a boundary or an interface in finite difference approximation of boundary conditions and wave equation (Boore, 1970). We now proceed to deal with space grid having a shape of parallelogram with sides parallel to the free surfaces of the wedge, or, parallel to the oblique coordinate axes ox' and oz'. In this case it has the advantage that all the boundary conditions and wave equation are replaced by finite difference approximation without introduction of special devices. Further details of such grid are discussed by Morley (1963).

Applying standard centered finite difference approximation to equation (2), displacement at a point (x', y', z') at time $t+k$ can be written in the form

$$v_i(x',y',z',t+k)=2v_i(x',y',z',t)-v_i(x',y',z',t-k)+k^2L(v_i), \tag{7}$$

where k is the time increment, and $L(v_i)$ is the finite formulation of the term of the righthand side of equation (2) and is expressed by

$$\begin{aligned}L(v_i)=\Big(\frac{\beta}{h\sin\alpha}\Big)^2[&-4v_i(x',y',z',t)+v_i(x'+h,y',z',t)\\&+v_i(x'-h,y',z',t)+v_i(x',y',z'+h,t)+v_i(x',y',z'-h,t)\\&-2\cos\alpha\cdot\{v_i(x'+h,y',z'+h,t)-v_i(x'+h,y',z'-h,t)\\&-v_i(x'-h,y',z'+h,t)+v_i(x'-h,y',z'-h,t)\}],\end{aligned} \tag{8}$$

where h is the mesh size both in the x' and z' directions. When the displacements at time t and $t-k$ are known, the displacements at next time step $t+k$ can be computed by using equations (7) and (8). Once each displacement value at time $t+k$ has been calculated, the corresponding values at time t and $t-k$ may be replaced by the new values.

The recursive finite difference scheme must satisfy a certain criterion for the stability of the system, that is,

$$\frac{\beta_m k}{h}<\frac{\sin\alpha}{\sqrt{2}}, \tag{9}$$

where h is the grid spacing and β_m is the maximum shear wave velocity. If $\alpha=90°$, equation (9) becomes $\beta_m \cdot k/h < 1/\sqrt{2}$, which is the stability condition in the case of square space grid.

The corner point is a kind of singular point and a number of methods are suggested for the treatment of the condition at this point. Satô (1972) briefly summarized the methods and proposed a new device for the study of Rayleigh wave propagation in the elastic quarter space. In this study we smooth the corner by a curve passing through the point in such a way as that discussed by Alterman and Rotenberg (1969).

At time $t=0$, displacements of sinusoidal Love waves with wave length λ and phase velocity c, truncated by a rectangular window with length of 4λ, are given by

$$v(x,y,z)=\begin{cases} 0 & \text{for } x<0,\ x>4\lambda,\ 0\leqq z\leqq H, \\ A\cos(2\pi/\lambda\cdot\sqrt{c^2/\beta_1^2-1}\ z)\sin(2\pi/\lambda\cdot x) & \\ & \text{for } 0\leqq x\leqq 4\lambda,\ 0\leqq z\leqq H, \end{cases} \tag{10}$$

and

$$v(x,y,z)=\begin{cases} 0 & \text{for } x<0,\ x>4\lambda,\ z>H, \\ A\cos(2\pi/\lambda\cdot\sqrt{c^2/\beta_1^2-1}\ H)\, e^{-2\pi/\lambda\sqrt{1-c^2/\beta_2^2}\,(Z-H)}\sin(2\pi/\lambda\cdot x) & \\ & \text{for } 0\leqq x\leqq 4\lambda,\ z>H, \end{cases} \tag{11}$$

where A is a constant.

4. Accuracy of Numerical Computation

It is necessary and important to examine the accuracy and the validity of the numerical method. Checks are made for two cases for which independent solutions are

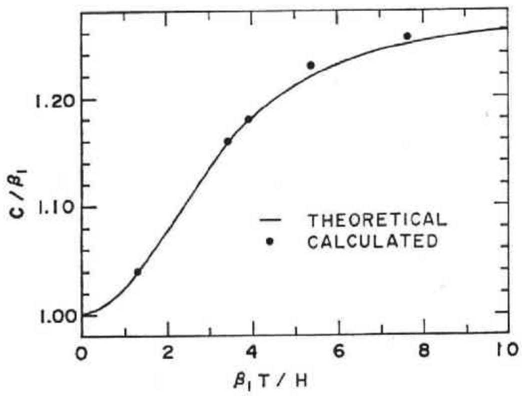

Fig. 3a

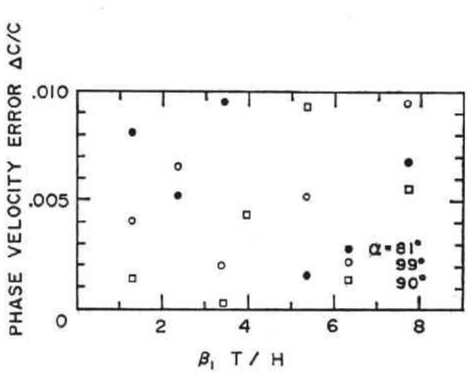

Fig. 3b

Fig. 3a. Numerically calculated and theoretical dispersion curves in a layered half-space when $\beta_1=3.51$ km/sec, $\rho_1=2.84$ g/cm^3, $\beta_2=4.50$ km/sec, $\rho_2=3.10$ g/cm^3, $H=35$ km. Calculated phase velocities are derived from Fourier transform of computed seismograms. Phase velocity is normalized by the shear wave velocity β_1 and period is normalized by H/β_1.

Fig. 3b. Relative phase velocity errors $\Delta c/c$ as a function of normalized period $\beta_1 T/H$, where T is period. Phase velocities are computed by using parallelogramic space grids with various angles 90°, 99° and 81°.

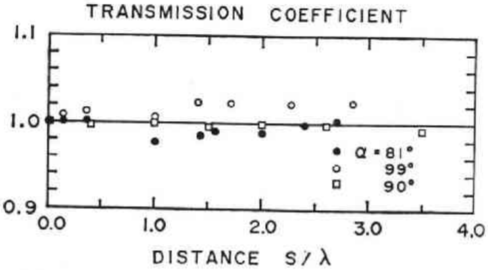

Fig. 4. Transmission coefficients plotted against the normalized distance s/λ.

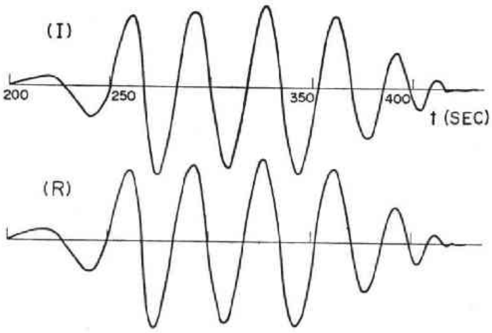

Fig. 5. Computed displacements of Love waves on a layered half-space and Love waves reflected at a 90° corner, denoted by (I) and (R), respectively.

available. The first case is a wellknown horizontally layered half-space and the second case is a quarter space with a superficial layer (wedge angle $\theta=90°$ in Fig. 1). Fig. 3a shows the comparison of numerical and the theoretical phase velocities. Numerically calculated phase velocities of Love waves are in an excellent agreement with the theory. The geometrical shape of our space grid is a parallelogram with sides parallel to two free surfaces of the wedge. By using the space grid having various shapes of parallelogram, we may obtain practically the same result. The phase velocity error $\Delta c/c$ is shown in Fig. 3b with various values of α, α being an angle related to rhombic space grid. Here, Δc is the difference between numerical and theoretical phase velocities. In Fig. 3b phase velocity error $\Delta c/c$ falls within 1% for different shapes of space grid.

The transmission factors are given in Fig. 4 with different angles of the oblique coordinate system. The abscissa in Fig. 4 is a normalized distance s/λ, where λ is the predominant wave length and s is the path length along which Love waves traversed. The transmission factor is defined as the spectral amplitude at a given station normalized by the amplitude at a reference station. Theoretically for any angle α of the oblique coordinate system it should be unity during wave propagation along the free surface of the layered half-space. It is found in Fig. 4 that the errors of the transmission factors are within 2% and inaccuracies in the calculation do not seriously increase during the numerical computation.

Let us now consider Love waves propagating for a 90° corner. When Love waves travelling on the free surface ox' reach the corner, it may be expected that Love waves are perfectly reflected back along ox', reflection coefficient being unity. Calculated seismogram for a layered half-space is shown in the upper curve of Fig. 5 and the lower curve shows calculated wave form of Love waves reflected at a 90° corner. In both cases of Fig. 5, distances along which Love waves travelled are the same. Here the predominant wave length is 141 km and the corresponding phase velocity is 4.07 km/sec. As obviously seen in Fig. 5 wave forms in the two cases are identical. Fig. 6 represents phase velocity of waves reflected at a 90° corner obtained by means of

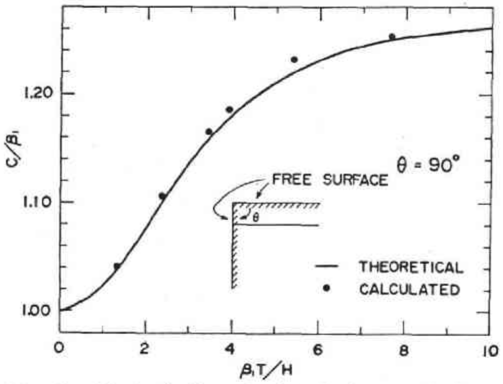

Fig. 6. Numerically calculated phase velocities of Love waves reflected at the 90° corner, compared with the theoretical dispersion curve for the layered half-space.

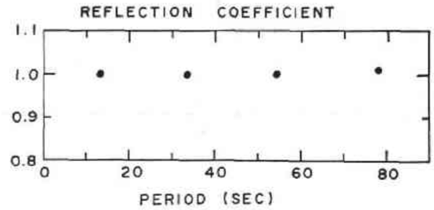

Fig. 7. Reflection coefficients of reflected Love waves on the quarter space with a superficial layer, plotted against period.

the Fourier transform of the seismograms shown in Fig. 5. Numerical phase velocity fits well to the theoretical dispersion curve, and it implies that reflected waves are certainly Love waves. Reflection coefficient defined as a spectral amplitude ratio of reflected Love wave to incident Love wave is shown in Fig. 7. It is concluded from the above results for Love wave propagation on the layered half-space and the quarter space that this numerical method can be safely applied to more complicated models. The accuracy of computation in a more complicated model will be discussed in Section 6.

5. Results

Consider Love waves propagating in a wedge-shaped medium (Fig. 1). Incident Love waves propagate along the surface of the superficial layer into a corner. Interacting at the corner, a part of Love wave energy is reflected back into the surface layer and the remainder of the energy is diffracted in the form of body waves.

Reflected Love waves measured at the free surface of the superficial layer are illustrated in Fig. 8 for wedge angles 81°, 99° and 90°. In both models of wedge angles 99° and 81°, deflection angles from 90° are identical, say, 9°. The period given to incident Love wave is 34.7 sec and the corresponding wave length is 141 km. In these three cases of wedge angles 90°, 99° and 81°, sums of a distance from a point at which incident Love waves begin to travel to the apex and a distance from the apex to an observation point are identical. Fig. 8 reveals some features of reflected Love waves for an obtuse-angled corner and for acute-angled corner, as compared with features of Love waves for the 90° corner. For the obtuse-angled wedge 99°, the reflected Love waves arrive later than those for the 90° corner, and the first one cycle of the wave trains becomes longer than that for the 90° corner. On the other hand, for the acute-angled wedge, the arrival time of reflected Love wave is earlier and period of the first motion is shorter than that for the 90° wedge. It is seen that Love wave amplitude of predominant period for 81° wedge is slightly greater than that

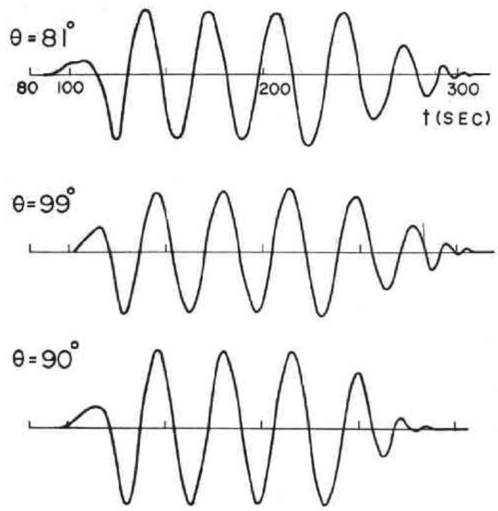

Fig. 8. Computed displacement seismograms of reflected Love waves for 81°, 99° and 90° wedges.

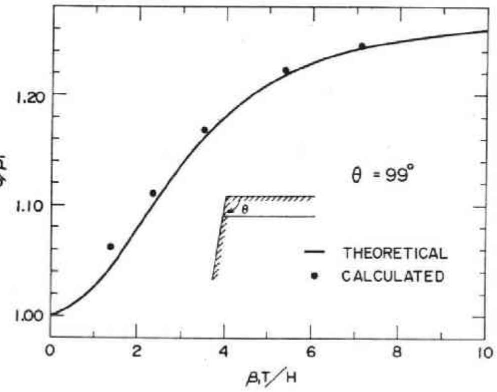

Fig. 9a

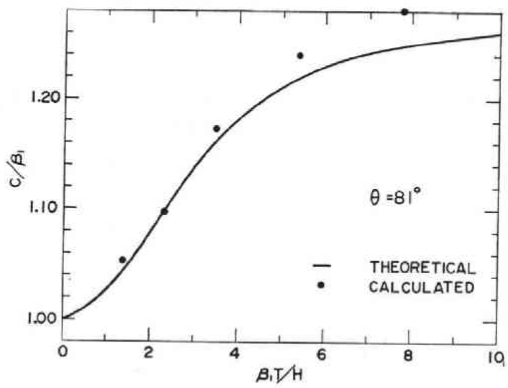

Fig. 9b

Fig. 9a. Numerically calculated and theoretical phase velocities of reflected waves for 99° wedge.

Fig. 9b. Numerically calculated and theoretical phase velocities of reflected waves for 81° wedge.

for wedge angle 99°.

Calculated phase velocities of reflected waves for the 99° wedge and the 81° wedge are illustrated in Figs. 9a and 9b, respectively. Numerical phase velocity in the case of $\beta_1 T/H=7.76$ in Fig. 9b does not fit the theoretical value. In this case of $\beta_1 T/H=7.76$ for the 81° wedge, however, measured points are not far from the corner and Love waves reflected at the corner superpose on Love waves travelling towards the corner. Accordingly, it is difficult to distinguish reflected waves from incident waves. Numerical phase velocities agree fairly well with the theoretical curve as a whole, and the reflected waves may be concluded to be Love waves.

Fig. 10 shows the comparison between the reflection coefficients obtained here

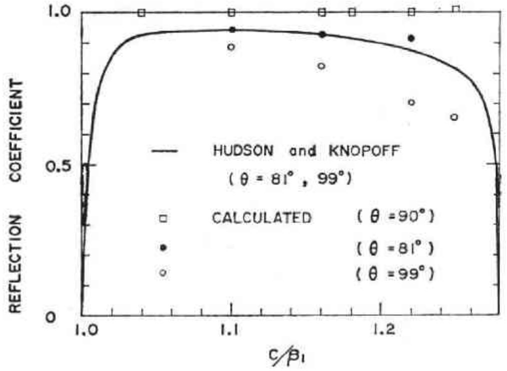

Fig. 10. Comparison of reflection coefficient with that obtained by Hudson and Knopoff (1964a).

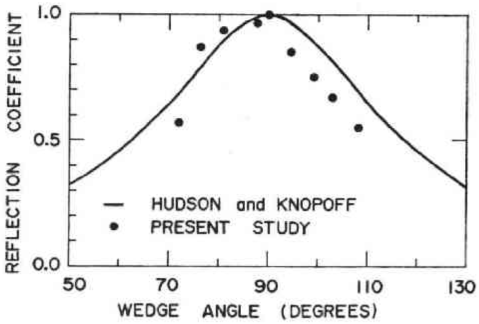

Fig. 11. Reflection coefficients as a function of wedge angle, when incident wave length is 230 km, $c/\beta_1=2.2$ and corresponding period is 54 sec.

and the result obtained by Hudson and Knopoff (1964a). The abscissa is a velocity ratio c/β_1, where c is phase velocity. As the ratio c/β_1 increases, the corresponding period becomes longer. According to Hudson and Knopoff, the reflection coefficient for 99° wedge is equal to that for 81° wedge. But it is easily seen in Fig. 10 that the reflection coefficients for acute-angled wedge are greater than those for obtuse-angled corner. The reflection coefficient for 99° wedge decreases with an increase in period, however, the coefficient for 81° wedge is nearly constant irrespective of period. In Fig. 11 the reflection coefficient is shown as a function of wedge angle θ, as compared with that by Hudson and Knopoff. Our result shows that the reflection coefficient is not symmetric with respect to 90° over wedge angle range 72° to 108°.

Fig. 12 shows displacement field of Love waves propagating toward the corner and Fig. 13 shows displacement field of Love waves reflected at the corner, which travel in the opposite direction from the corner. Differences of the features between the two displacement fields are obvious. In Fig. 12 Love wave displacements, decreasing monotonously with depth from the free surface, becomes insignificantly small at the depth of about two times of the wave length. On the contrary, amplitudes of reflected waves shown in Fig. 13 are not so small at that depth and there exists some phase lag from those on the surface. In Fig. 12 the wave fronts of incident Love waves are straight while those of reflected waves in the lower layer are curved. It is inferred that the curved wave front is due to body waves converted from Love waves.

As a final check of the computation we adopted several grid sizes. Figures 14a and 14b show reflection coefficients as a function of grid points per wave length for the cases of $c/\beta_1=1.22$, corresponding wave length $\lambda=230$ km and $c/\beta_1=1.16$, $\lambda=141$ km, respectively. In Fig. 14a there is a negligible difference between result obtained by 38 points and that by 40 grid points per wave length. Deviations of the result derived by coarse mesh (26 grid points) from that by fine mesh (40 points) are 3% and 7% against the latter result for 81° and 99° wedge, respectively.

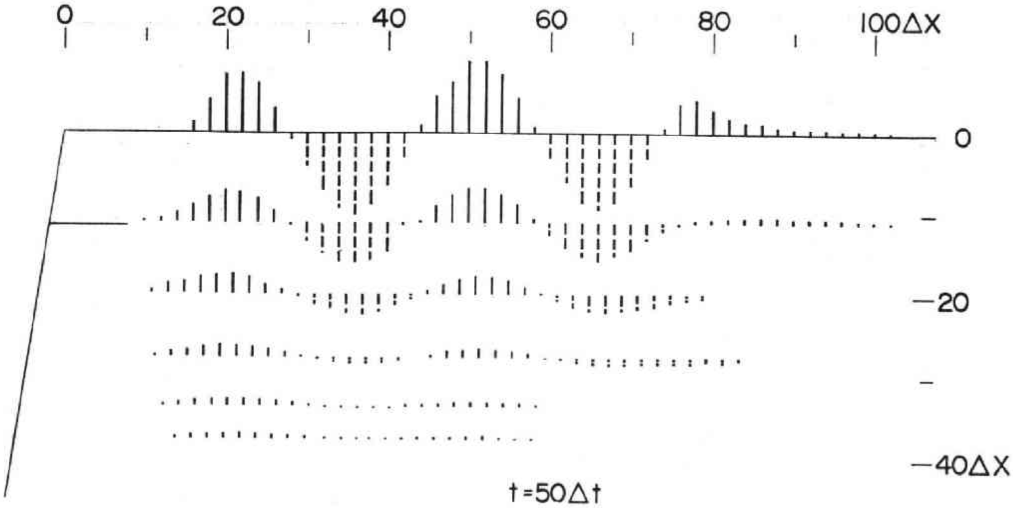

Fig. 12. Spatial distribution of displacement of Love waves before they reach the corner for the moment $t=50\Delta t$, where Δt is time increment. The wedge angle is 99°. Δx is mesh size, $c=3.86$ km/sec, $\lambda=90.8$ km. Full line shows displacements in y direction and broken line shows displacements in the opposite direction.

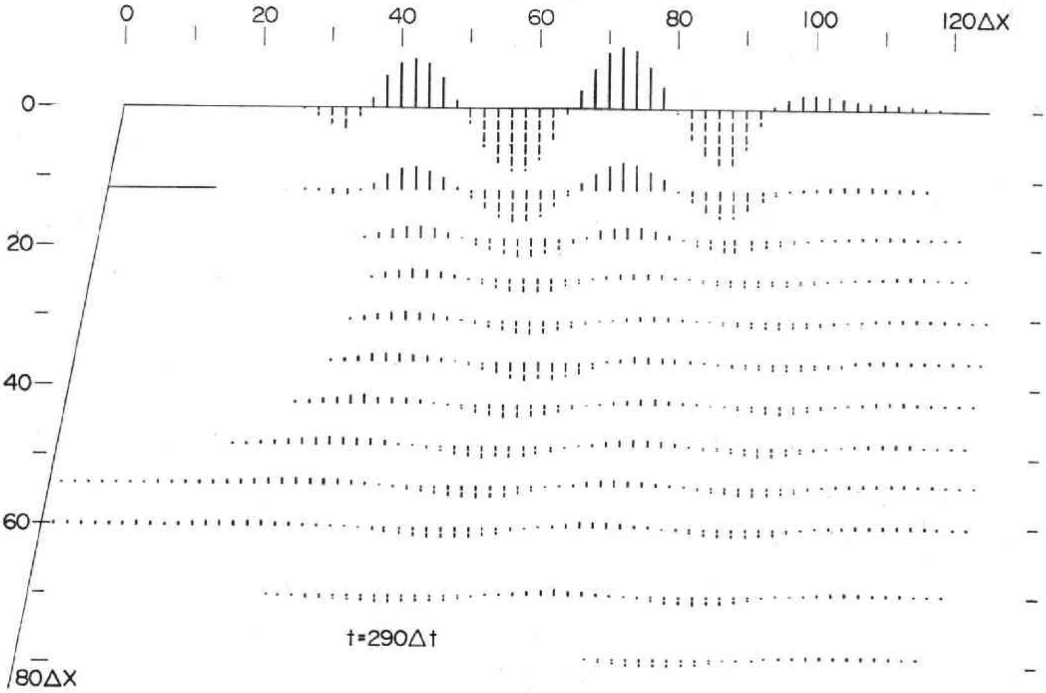

Fig. 13. Displacement field of Love waves reflected at the 99° corner for the moment $t=290\Delta t$, $c=3.86$ km/sec.

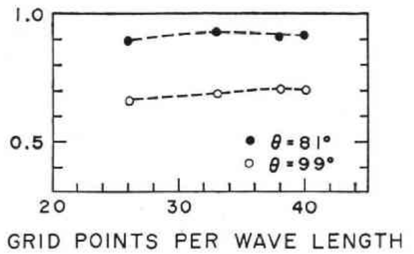

Fig. 14a

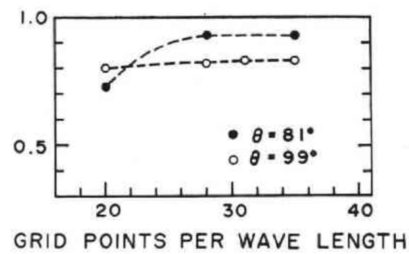

Fig. 14b

Fig. 14. Effects of mesh size on the reflection coefficient: (a) for the case of $c/\beta_1=1.22$, (b) for the case of $c/\beta_1=1.16$. The ordinate is the reflection coefficient and the abscissa is number of grid points per wave length.

6. Conclusion

In this paper study of a numerical experiment on the problem of the transmission and reflection of Love waves in a wedge-shaped medium was carried out by the use of a finite difference method. When we employ this numerical method, it is necessary to check the accuracy of computation. As stated in Section 4, the solutions with satisfactorily high accuracy are obtained for the problems in a layered half-space and a quarter-space with a superficial layer. Furthermore, in order to insure a high accuracy of the computed reflection coefficient of Love waves reflected at a corner, we take progressively finer meshes until there is no appreciable change in numerical results.

The results obtained are as follows:

1) Reflection coefficient of Love waves for acute-angled corner is greater than that for obtuse-angled corner. This is inconsistent with the result by Hudson and Knopoff (1964a) that the reflection coefficient is symmetric about 90°.

2) The reflection coefficient for 99° wedge decreases as period of Love waves becomes longer, whereas that for 81° wedge is almost constant (i.e. independent of period) in the period range treated here.

3) As deviation of wedge angle from 90° becomes larger, less of the incident Love wave energy is reflected back into the superficial layer. The remainder of the energy is diffracted into the high velocity medium.

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