Plasma Wave Turbulences for the Origin of the Planets

著者	Oya Hiroshi
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	22
号	3-4
ページ	145-152
発行年	1975-04
URL	http://hdl.handle.net/10097/44725

Sci. Rep. Tôhoku Univ., Ser. 5, Geophysics, Vol. 22, Nos. 3–4, pp. 145–152, 1975.

Plasma Wave Turbulences for the Origin of the Planets

HIROSHI OYA

Upper Atmosphere and Space Research Laboratory
Tôhoku University, Katahira, Sendai 980, Japan

(Received January 11, 1975)

Abstract: After the formation of the proto-sun, the solar nebula was subjected to large solar wind that blew the material out of the solar system. The nebula became transparent for the UV radiation emitted from the proto-sun; and the cloud turned to the plasma. The plasma motion was largely controlled by the electromagnetic force being concentrated to the thin disk form and accelerated due to the centrifugal force since the plasma particles moves with the rotating magnetic field. The trapping of the plasma was started in this disk in the form of the turbulent waves. The stational wave packet was produced in the form of the electrostatic ion cyclotron waves whose positions of nodes, that were produced at given phase of the waves, coincide with the Titius-Bode's law.

1. Introduction

In formation of the proto-solar system, there is a period when the formation process of the solar system was largely dependent on the plasma phenomena. One of the remarkable evidence that requires an interpretation considering the plasma effects is the distribution of the angular momentum in the whole solar system; i.e., 92 % of the angular momentum is possessed by the planets, especially, by the giant planets. Alfvén (1954) was the first who proposed the effect of the rotating proto-solar magnetic field on the plasma for the interpretation of this angular monentum distribution. In that case, Alfvén (1954) assumed that the plasma was produced from the gas that was collected after the formation of the proto-sun while the solar system was propagating through the galactic media. Hoyle (1963) has extended Alfvén's theory to include the effects on the plasma that was originated from the proto-solar system.

The proto-sun started to blow up nebulous material out of the proto-solar system in the form of the solar wind. In this process this nebula became transparent gradually for the UV and EUV emissions emitted from the proto-sun. Thus, the nebula turned to the plasma due to the ionization effects of the UV and EUV emissions. The dynamics of the plasma was then largely controlled by the electromagnetic forces in this proto-solar system.

2. Fundamental Equation

Let's assume that the plasma in the nebula consisted of electrons and a kind of heavy ions. The governing equations for these species are given as follows;

$$N_i m_i\left(\frac{\partial \vec{V}_i}{\partial t} + \vec{V}_i \cdot \nabla \vec{V}_i\right) = N_i e(\vec{E} + \vec{V}_i \times \vec{B}) - N_i G m_i M \nabla\left(\frac{1}{r}\right) - \nabla P_i\,, \qquad (1)$$

for the ions, and

$$N_e m_e\left(\frac{\partial \vec{V}_e}{\partial t}+\vec{V}_e\cdot\nabla\vec{V}_e\right)=-N_e e(\vec{E}+\vec{V}_e\times\vec{B})-N_e G m_e M\nabla\left(\frac{1}{r}\right)-\nabla P_e, \tag{2}$$

for the electrons. The continuity of flowing material gives, for the ion

$$\frac{\partial N_i}{\partial t}+\nabla(N_i\vec{V}_i)=0, \tag{3}$$

and it follows, for the electron, that

$$\frac{\partial N_e}{\partial t}+\nabla(N_e\vec{V}_e)=0. \tag{4}$$

The equations of the state are, then, given as

$$P_i=N_i kT_i, \tag{5}$$

and

$$P_e=N_e kT_e. \tag{6}$$

The electromagnetic fields $\vec{E}$ and $\vec{B}$ in the co-ordinate system that is co-rotating with the proto-sun are related to the field in the inertia system $\vec{E}'$ and $\vec{B}'$, as

$$\begin{aligned} &E_{\parallel}'=E_{\parallel}, \quad B_{\parallel}'=B_{\parallel}, \\ &\vec{E}_{\perp}'=\gamma(\vec{E}_{\perp}+\vec{V}_{\perp}\times\vec{B}), \quad \vec{B}_{\perp}'=\gamma(\vec{B}_{\perp}-\vec{V}_{\perp}\times\vec{E}/c^2) \\ &\text{with} \quad \gamma=1/\sqrt{1-(v/c)^2}, \end{aligned} \tag{7}$$

where the suffixes $\parallel$ and $\perp$ indicate the parallel and the perpendicular components of the field; c is the light velocity. The velocity $\vec{V}$ is rotation speed of the proto-sun that is expressed by

$$\vec{V}=\vec{\Omega}\times\vec{R}, \tag{8}$$

where $\vec{\Omega}$ is the rotation vector of the planet, and $\vec{R}$ is the position measured from the center of the proto-sun.

3. Equilibrium States — Disk-like Plasma Concentration

The equations to express the equilibrium state are then summarized as

$$\begin{aligned} &N_i m_i\vec{V}_i\cdot\nabla\vec{V}_i=N_i e(\vec{E}+\vec{V}_i\times\vec{B})-N_i G m_i M\nabla\left(\frac{1}{r}\right)-\nabla P_i, \quad \cdots\cdots\cdots \text{(a)} \\ &N_e m_e\vec{V}_e\cdot\nabla\vec{V}_e=N_e e(\vec{E}+\vec{V}_e\times\vec{B})-N_e G m_e M\nabla\left(\frac{1}{r}\right)-\nabla P_e, \quad \cdots\cdots \text{(b)} \\ &\nabla(N_i\vec{V}_i)=0, \quad \cdots \text{(c)} \qquad \nabla(N_e\vec{V}_e)=0, \quad \cdots \text{(d)} \\ &\text{rot } \vec{B}=\mu_0 e(N_i\vec{V}_i-N_e\vec{V}_e), \quad \cdots \text{(e) and div } \vec{E}=e(N_i-N_e)/\varepsilon_0, \quad \cdots \text{(f)} \end{aligned} \tag{9}$$

where ε_0 and μ_0 are the dielectric constant and the magnetic permittivity of vacuum in the MKS rational unit. The ion and electron velocities are, here, given by

$$\vec{V}_i = \vec{\Omega} \times \vec{R} + \vec{v}_i , \tag{10}$$

and

$$\vec{V}_e = \vec{\Omega} \times \vec{R} + \vec{v}_e . \tag{11}$$

The velocities $\vec{v}_i$ and $\vec{v}_e$ are indicating outward flows perpendicular to the rotation axis. In Fig. 1, the coordinate system and the magnetic field configuration, that satisfies the basic equations, are given. In the co-rotating system, then, the equation is expressed by,

$$N_i e(\vec{E} + \vec{V}_i \times \vec{B}) = N_i G m_i M \nabla \left(\frac{-1}{\sqrt{r^2+z^2}} \right) + \nabla P_i + N_i m_i \vec{V}_i \cdot \nabla \vec{V}_i ,$$

and (12)

$$-N_e e(\vec{E} + \vec{V}_e \times \vec{B}) = N_e G m_e M \nabla \left(\frac{-1}{\sqrt{r^2+z^2}} \right) + \nabla P_e + N_e m_e \vec{V}_e \cdot \nabla \vec{V}_e .$$

Eq (12) is, then, separated into the component equations as,

$$(kT_i - m_i V_{ir}^2) \partial N_i / \partial r = - G N_i m_i M r / (r^2+z^2)^{3/2} + N_i m_i V_{ir}^2 / r , \quad \text{(a)}$$

$$E_\theta = -v_r B_z , \quad \text{(b)}$$

and (13)

$$\partial N_i / \partial z = - \{ z / (r^2+z^2)^{3/2} \} (G N_i m_i M / k T_i) . \quad \text{(c)}$$

The same set of the equations are obtained for electrons. From equation (13c), the

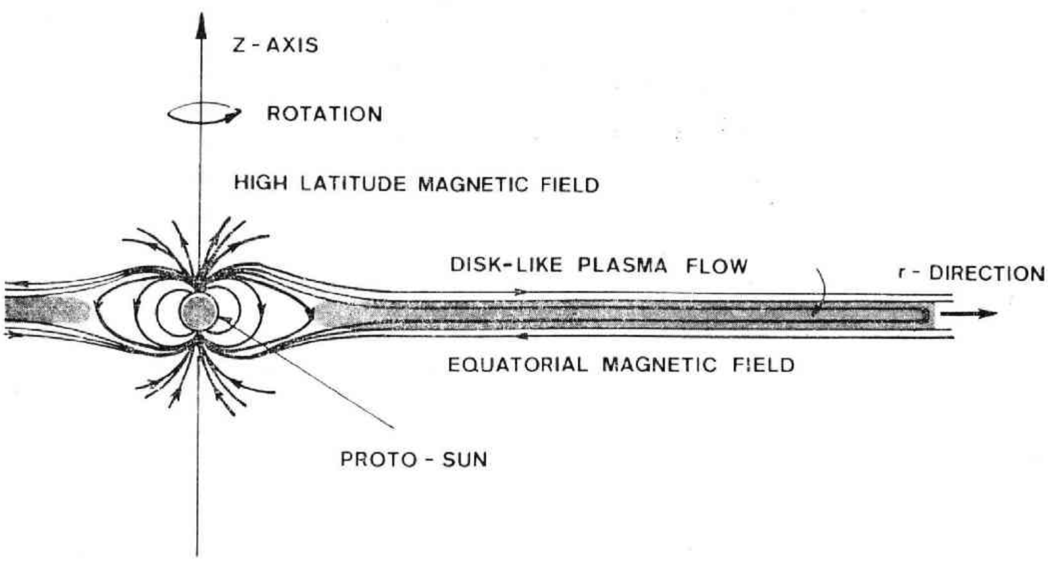

Fig. 1. The configuration of the proto-solar system and cylindrical coordinate system whose z-axis coincides with the rotation axis of the proto-sun; the plasma is flowing out in the r-direction due to the rotation of the proto-solar system.

equilibrium distribution of the plasma as the function of z is finally obtained for a given r as

$$N_i = N_0 \left[\exp \{(1/\sqrt{z^2+r^2})(Gm_e M/kT_1)\} - 1\right]. \tag{14}$$

This solution gives that the plasma is largely concentrated at the equator of the planet, i.e., $z=0$. The gravity due to the proto-sun increases this concentration tendency while the thermalization decreases the concentration. In the r-direction, it is required to obtain $N_0(r)$ by solving eq. (13 (a)). Though the accurate solutions for this r-dependence remains for future work, we can expect a widely distributed plasma density in the ecliptic plane due to the plasma flow in the r-direction.

4. Turbulent Flow–Interpretation of Titius-Bode's Law in Terms of ESCH Waves

In the equatorial region of this magnetic field, a dense plasma were concentrated in the disk form. While the plasma flowing in this disk, turbulent states are easily produced, since the region indicated high β value, as a form of perturbation. In this treatment, let's assume a weak turbulence that can only be expressed as association of linear plasma waves. The linear plasma waves might be grown up to a large turbulent waves including the nonlinear wave particle interactions; hence linear regime of the discussion is important as the start of this growing turbulences.

In a frequency range much lower than the electron cyclotron frequency the electrostatic wave with the phase, $\exp\{j(\vec{k}\cdot\vec{r}-\omega t)\}$ is characterized by the dispersion relation (Stix, 1962), as

$$k_x^2 + k_z^2 + \{\Pi_i^2 m_i e^{-\lambda} j I_n(\lambda_i)/\kappa T_i\} A_{ni} = 0\,,$$

where

$$\begin{aligned} A_{ni} &= 1 + j\left[\{(\omega - k_z V_z)/k_z\}\,(m_i/2\kappa T_i)^{1/2}\right] F_0(\alpha_{ni})\,, \\ \alpha_{ni} &= \{(\omega - k_z V_z + n\Omega_i)/k_z\}\,(m_i/2\kappa T_i)^{1/2} \\ F_0 &= \sqrt{\pi}\,(k_z/|k_z|)\exp(-\alpha_n^2) + 2jS(\alpha_n)\,, \\ S(z) &= e^{-z^2}\int_0^z e^{t^2}dt \quad \text{and} \quad \lambda_i = k_x^2 \kappa T_i/m_i\Omega_i^2\,. \end{aligned} \tag{15}$$

j indicates the unit of complex number. In this case, we can replace as $k_x = k_r$ and $k_z = 0$ considering a local perturbation in a form of the plane waves that are directed in the radial direction. The equation (15) has already been solved for the case of the electron (Crawford 1965, Oya 1971); and this is easily applied to the case of the ion. The result is reproduced in Figs. 2 (a) and 2 (b). It is well known that the temperature anisotropy produces the electrostatic electron cyclotron instabilities near 1.5 f_H, where f_H is the electron cyclotron frequency; this can also be extended to the case of the electrostatic proton cyclotron instability. The computed result (Oya, 1972) gives that we can select the turbulent condition as

$$kR \simeq 1\,, \tag{16}$$

where

$$R = \sqrt{(\kappa T_i/m_i)}/\Omega_i\,.$$

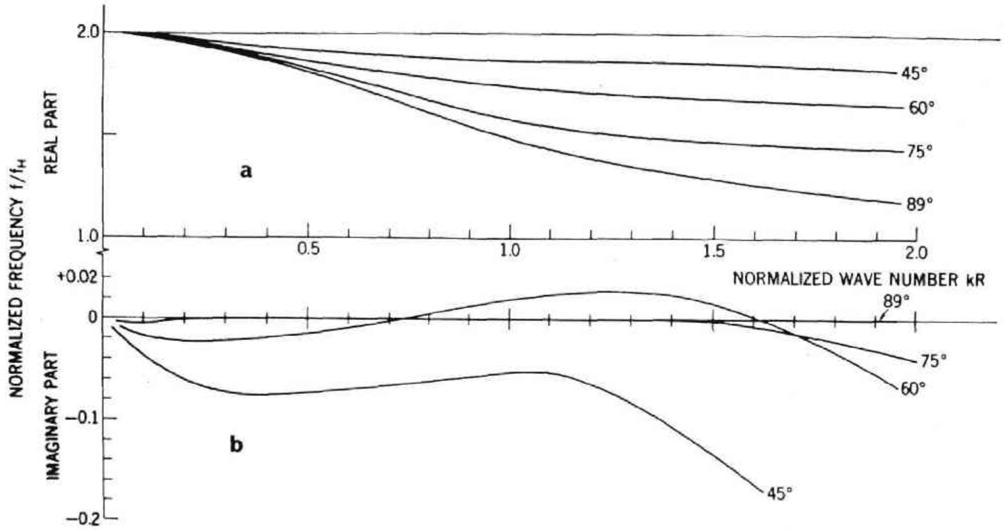

Fig. 2. The dispersion curves of the electrostatic electron cyclotron harmonic waves plotted for the normalized frequency f/f_H ,where f_H is the electron cyclotron frequency, and kR (see text); the top panel is for the real part of the f/f_H-kR relation and the bottom panel is for the imaginary part of the relation. The positive value of this imaginary part indicates growth of the turbulent waves; that is, the tubulence can grow in a range around $kR \simeq 1$ and for the frequency range $1.5\, f_H < f < 1.7 f_H$. (after Oya, 1971). This relationship is extended for the case of the ion cyclotron harmonic waves in the main text.

A solution of the magnetic field, for the radially spreading field (see Fig. 1), is obtained from div $\vec{B}=0$, as

$$B_r = (r_0/r)B_0 . \tag{17}$$

The magnetic field B_z is the component of this B_r field leaking into the equatorial plasma sheet (see Fig. 1). In this paper this leaking component is assumed to be proportional to B_r value. Thus, it follows that

$$B_z = \xi(r_0/r)B_0 . \tag{18}$$

When we assume a constant temperature model, these conditions give finally the relations for k that is expressed by

$$k = (r_0/r)k_0 , \tag{19}$$

where k_0 is the wave number at a given position r_0.

Density fluctuations ΔN in this weak turbulence is assumed to have the form, using WKB approximation, as

$$\Delta N(r',t') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(k_0\omega)\, dk_0 d\omega \exp\left[i\left\{\int_{r_0'}^{r'} kdr' - \omega t'\right\}\right]. \tag{20}$$

This relationship is described on a frame (r', t') that is fixed to the flowing plasma. Since the flow velocity is given by v_r, this coordinate system is related to the coordinate system (r, t), that is fixed to the proto-sun, as

$$r = r' + v_r t' ,$$

and (21)

$$t = t' .$$

The frequency ω is here defined by the dispersion relation of the plasma waves,

$$\omega = \omega(k) , \tag{22}$$

and $f(k_0, \omega)$ is expressed by

$$f(k_0 , \omega) = F(k_0)\delta(\omega - \omega(k_0)) . \tag{23}$$

Using eq. (23), eq. (20) is rewritten by

$$\Delta N(r',t') = \int_{-\infty}^{\infty} F(k_0)dk_0 \exp\left[i\left\{\int_{r_0'}^{r'} k_0(r_0/r)dr' - \omega(k)t'\right\}\right]. \tag{24}$$

When we select fix points r_m', where m indicates integer, in the proto-sun coordinate system, eq. (24) can be expressed as an expansion form around r_m positions; i.e., the phase value Φ in eq. (24) is given by

$$\Phi \equiv \left\{\int_{r_0'}^{r'} kdr' - \omega(k)t'\right\} = \left\{\int_{r_0}^{r_m} kdr + kr'' - \omega(k)t'\right\} , \tag{25}$$

where $r' = r_m' + r''$, $r_m = r_m' + v_r t'$ and $r'' \ll r_m'$.

This integration is obtained, using eqs. (19) and (21), as

$$\Phi = k_0 r_0 \, ln\,(r_m/r_0) + kr'' - \omega(k)t' . \tag{26}$$

For simplicity, let's here select a multi-wave model corresponding to eq. (24), in a wave number range Δk; i.e.,

$$\Delta N = Re\left[\sum_{l=0}^{n} f(k_l) \exp i\,\{k_l r_0 \, ln(r_m/r_0) + kr'' - \omega(k)t'\}\right], \tag{27}$$

where $k_l = k_0 + l\Delta k/n$ and n is integer. An extreme case of this model is the two-wave case where ΔN is given by taking $n=2$, as

$$\Delta N = Re\,[f(k_1) \exp i\,\{k_1 r_0 \, ln(r_m/r_0) + kr'' - \omega(k)t'\}$$
$$+ f(k_0) \exp i\,\{k_0 r_0 \, ln(r_m/r_0) + kr'' - \omega(k)t'\} , \tag{28}$$

and $k_1 = k_0 + \Delta k$.

This gives the result that

$$\Delta N(r',t') = f(k_0)\sqrt{(1+\beta^2) + 2\beta\cos\left[\Delta k r_0 \, ln(r_m/r_0) + \Delta k\left\{r'' - \left(\frac{\partial\omega}{\partial k}\right)t'\right\}\right]}$$
$$\cdot \cos\left[k_0 r_0 \, ln(r_m/r_0) + k_0\left(r'' - \frac{\partial\omega}{\partial k}t'\right) + \phi(t')\right], \tag{29}$$

where $\beta = f(k_1)/f(k_0)$ and $\phi(t) = \tan^{-1}\{\beta \sin\Delta\theta/(1+\beta\cos\Delta\theta)\}$,

and

$$\Delta\theta = \Delta k\left\{r_0 \, ln(r_m/r_0) + k_0\left(r'' - \frac{\partial\omega}{\partial k}t'\right)\right\} .$$

This equation indicates that the turbulence is produced with an amplitude T that is given by,

$$T = f(k_0)\sqrt{(1+\beta^2)+2\beta\cos\Delta k\left\{r_0\, ln(r_m/r_0)+\left(r''-\frac{\partial\omega}{\partial k}t'\right)\right\}}\,, \tag{30}$$

and underlying waves with the phase U; i.e.,

$$U = \cos\left[k_0 r_0\, ln(r_m/r_0)+k_0\left(r''-\frac{\partial\omega}{\partial k}t'\right)+\phi(t')\right]. \tag{31}$$

A fixed point r_f that is given on the coordinate fixed to the proto-sun is related to the flowing plasma coordinate system (r'', t') as

$$r_f = r''+v_r t (= r''+v_r t')\,. \tag{32}$$

When the wave packet moves with the group velocity $-v_r$ in the r-direction, the packet keeps a constant circular orbit with respect to the proto-san; in this case it follows that

$$\frac{\partial\omega}{\partial k} = -v_r\,, \tag{33}$$

substituting eq. (33) into eq. (30), the result is obtained as

$$T = f(k_0)\sqrt{(1+\beta^2)+2\beta\cos\Delta k\,\{r_0\, ln(r_m/r_0)+r_f\}}\,. \tag{34}$$

In this expression r_f is again a constant value. Considering $r_f/r_0 \ll 1$, the position r_m where the T value becomes maximum is obtained from

$$ln\,(r_m/r_0) = (2\pi/\Delta k r_0)m\,. \tag{35}$$

When $2\pi/\Delta k r_0 = 1.86$, the result coincides with the Titus-Bodes law; i.e., the formation of the planet was carried out at positions of the maximum plasma density.

The weak turbulence might grow up to strong and large amplitude density fluctuations. The produced dense regions were then filled with neutralized particles since the produced dense cloud shutted out the UV radiation from the proto-sun. This situation resulted in the dust collection at the position where the planets were to be produced.

5. Discussions

The plasma condition can be approximately estimated from the present data; that is, from eq. (35) it follows that

$$f_{ion} = (1/1.86)(v_{th}/r_0\alpha)\,, \tag{36}$$

where f_{ion}, v_{th}, and r_0 are the ion cyclotron frequency, the plasma thermal velocity and the distance of Mercury from the proto-sun, respectively; α is defined by $\alpha = \Delta k/k$. For the iron ion, the magnetic field intensity can be estimated to be 15 γ at the Mercury position and 6γ at the earth's position, assuming the plasma temperature of 10 eV, and $\alpha = 10^{-3}$. Though, we can assume wide variety of values for the leakage factor ξ, it seems not to be unreasonable to assume a range of ξ from 10^{-1} to 10^{-2} for the turbulent state of the plasma; the B_r value, for this case, is several tens of gamma at the earth's position when we select $\xi=10^{-1}$ for $\alpha=10^{-2}$ and $\xi=10^{-2}$ for $\alpha=10^{-1}$.

Acknowledgement: The author thanks to Drs. T. Tamao and M. Shimizu for their discussions and interests on this work.

References

Alfvén, H., 1954: Origin of the Solar System, Oxford Univ. Press, London.
Crawford, F.W., 1965: Cyclotron harmonic waves in warm plasma, Radio Sci., 69D (6), 789.
Hoyle, F., 1963: in R. Jastrow and A.G.W. Cameron (eds.) Origin of the solar system, Academic Press, New York.
Oya, H., 1971: Verification of theory on weak turbulences in space, Phys. Fluids, **14**, 2487.
Stix, T.H., 1962: The theory of plasma waves, McGraw-Hill, New York.