Research on Akita-Komaga-take (?)-Focal Depth of Explosion Earthquake

著者	Tanaka Kazuo, Kasahara Minoru, Hori Shuichiro
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	22
号	1
ページ	9-18
発行年	1974-05
URL	http://hdl.handle.net/10097/44716

The Science Reports of the Tôhoku University, Ser. 5, Geophysics, Vol. 22, No. 1, 1974.

Equilibrium Boundary Layer between a Hot Plasma and a Magnetic Field

NOBUO WATARI and HIROSHI KAMIYAMA

Geophysical Institute, Tohoku University, Sendai, Japan

(Received December 10, 1973)

Abstract: The structure of the steady state boundary layer between a magnetic field and a hot plasma is described in a one-dimensional model. An arbitrary velocity distribution is assumed for particles in the plasma. The problem is formulated by combining the equations of motion, Maxwell's equations, and the equation of continuity. It is shown that an equilibrium boundary layer is established when the initial velocity distribution of protons is equal to that of electrons. The structure of the magnetic field in the layer is evaluated for some typical velocity distributions. The characteristics of the layer are shown to be quite dependent of the velocity distribution function.

Introduction

The confinement of a magnetic field by a plasma flow is of the fundamental interest in the magnetospheric physics. This problem was first considered by Chapman and Ferraro (1931) and succeeded by a number of authors. As a fundamental approach, the interaction between a cold plasma flow and a magnetic field was studied by Ferraro (1952), Dungey (1958), Mjolsness et al. (1961), Longmire (1963), and Watari and Kamiyama (1971). However, the problem is needed to be re-examined by considering the effect of a velocity distribution of the particles in a plasma. Under this situation, attempts were made by Grad (1961), Hurley (1963), Longmire (1963), and Sestero (1964) to describe the problem, but the exact solution has not yet been obtained.

For a better understanding of the problem, the analysis is simplified by employing a one-dimensional model in which all quantities depend on the single variable x. The magnetic field $\boldsymbol{B}$ is composed of a given uniform field and the field induced by the diamagnetic current, and is assumed to be directing to the z-axis in the right-handed coordinate system. When the boundary surface lies in the y-z plane, the electric field $\boldsymbol{E}$ resulting from the charge separation is in the x-direction. A plasma flow coming from $x=-\infty$ is assumed to be composed of protons and electrons having the common velocity distribution function $f(\boldsymbol{V})$. These conditions are sufficient, in general, for finding a unique solution. This paper analyzes the magnetic structure of the transition layer for some typical velocity distributions.

Analysis

Considering that a proton and an electron have a same initial velocity $\boldsymbol{V}_0=(V_{ox}, V_{oy}, V_{oz})$ at $x=-\infty$, the velocity in the layer is given by the equations of motion for a proton (mass M) as

$$V_x^2+V_y^2+V_z^2=V_0^2-\frac{2e}{M}\phi \tag{1}$$

$$V_y=V_{0y}-\frac{e}{Mc}\eta \tag{2}$$

$$V_z=V_{0z} \tag{3}$$

and similarly for an electron (mass m) as

$$v_x^2+v_y^2+v_z^2=V_0^2+\frac{2e}{m}\phi \tag{4}$$

$$v_y=V_{0y}+\frac{e}{mc}\eta \tag{5}$$

$$v_z=V_{0z} \tag{6}$$

where, e, c, ϕ and η are the electric charge, the light velocity, the electric potential, and the y-component of the magnetic vector potential ($B=d\eta/dx$), respectively. From these equations one finds directly the conservation of momentum in the y-direction,

$$MV_y+mv_y=(M+m)V_{0y} \tag{7}$$

and the conservation of the initial kinetic energy,

$$\frac{1}{2}MV^2+\frac{1}{2}mv^2=\frac{1}{2}(M+m)V_0^2 \tag{8}$$

or from $V_z=v_z=V_{0z}$,

$$\frac{1}{2}M(V_x^2+V_y^2)+\frac{1}{2}m(v_x^2+v_y^2)=\frac{1}{2}(M+m)(V_{0x}^2+V_{0y}^2) \tag{9}$$

Substituting (7) into (9), one obtains

$$V_x^2=V_{0x}^2-\mu^2\xi^2+\mu^2(V_{0x}^2-\mu^2\xi^2-v_x^2) \tag{10}$$

where $\xi=v_y-V_{0y}$ and $\mu^2=m/M$. Since v_x does not seem to exceed V_{0x}, the approximation can be made that

$$V_x^2=V_{0x}^2-\mu^2\xi^2 \tag{11}$$

At the turning point of a proton, $V_x=0$, $\xi=V_{0x}/\mu$, and $V_y=V_{0y}-\mu V_{0x}$, then $v_x=0$. This indicates that a proton and an electron having a same initial velocity penetrate into the same depth.

At an arbitrary point x, the number of protons, dN, having initial velocities between $\boldsymbol{V}$ and $\boldsymbol{V}+d\boldsymbol{V}$ is given by the equation of continuity as

$$dN=V_x(V_x^2-\mu^2\xi^2)^{-1/2}f(\boldsymbol{V})\,d^3\boldsymbol{V}$$

If V_{cx} denotes the initial velocity of a proton which turns at x, protons having initial velocities exceeding V_{cx} penetrate to further depths. Thus, the proton number density $N(x)$ contributed by the inward flow is expressed by the integrated form as

$$N(x)=\int_{V_{cx}}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}V_x(V_x^2-\mu^2\xi^2)^{-1/2}f(\boldsymbol{V})\,d^3\boldsymbol{V}$$

$$=2N_0\int_{a\eta}^{\infty}V_x(V_x^2-a^2\eta^2)^{-1/2}f(V_x)\,dV_x \tag{12}$$

where

$$f(\boldsymbol{V})=N_0f(V_x)f(V_y)f(V_z),\qquad \iint_{-\infty}^{\infty}f(V_y)f(V_z)\,dV_y\,dV_z=1$$

and N_0 is the proton density in the plasma flow moving towards the boundary. The factor of two appeared in the expression (12) is introduced by a consideration of the return flow. Denoting the electron density by n, the electric charge density is given by

$$\rho=e(N-n) \tag{13}$$

From the condition of over-all charge neutrality, $\int_{-\infty}^{\infty}\rho\,dx=0$, the electric field at x is given by

$$E(x)=-4\pi\int_{x}^{\infty}\rho\,dx \tag{14}$$

Substituting (2), (3), (13), and (14) into (1), one obtains the expression for the electron density,

$$n(x)=N(x)+\frac{m}{8\pi e^2}\frac{d^2}{dx^2}\{(1-\mu^2)\xi^2+2V_{0y}\xi\}$$

$$=N(x)+\frac{m}{8\pi e^2}\frac{d^2}{dx^2}(\xi+V_{0y})^2 \tag{15}$$

Since the diamagnetic current in the layer flows in the y-direction, its y-component is

$$j=\frac{e}{c}(NV_y-nv_y) \tag{16}$$

and the magnetic field satisfies

$$\frac{dB}{dx}=\frac{d^2\eta}{dx^2}=-4\pi j \tag{17}$$

Then, combining (2), (12), (15), (16), and (17), we find that the magnetic vector potential satisfies

$$\frac{d^2\eta}{dx^2}=-\frac{4\pi e}{c}(NV_y-nv_y)$$

$$=\frac{8\pi e^2N_0}{mc^2}\eta\int_{a\eta}^{\infty}V_x(V_x^2-a^2\eta^2)^{-1/2}f(V_x)\,dV_x$$

$$+\frac{m}{2ec}\left(V_{0y}+\frac{e}{mc}\eta\right)\frac{d^2}{dx^2}\left(V_{0y}+\frac{e}{mc}\eta\right)^2 \tag{18}$$

When $v_y^2 \ll c_2$, the integration of (18) under the boundary conditions that $\eta = d\eta/dx = 0$ at $x = -\infty$ gives approximately (see Appendix)

$$\frac{d\eta}{dx} = \sqrt{16\pi MN_0}\left\{\int_0^\infty V_x^2 f(V_x)dV_x - \int_{a\eta}^\infty V_x(V_x^2 - a^2\eta^2)^{1/2} f(V_x)\,dV_x\right\}^{1/2} \tag{19}$$

Thus only the x-component of the velocity is participating in the interaction with the magnetic field. Then, for a given velocity distribution, the vector potential η is uniquely determined as a function of x. The solutions of (19) will be evaluated for some representative velocity distributions in the following section.

Structure of Magnetic Field within the Boundary Layer

(1) Rectangular distribution function

Let us first consider the distribution function $f(V_x)$ given by

$$f(V_x) = \begin{cases} (V_2 - V_1)^{-1} & (V_1 \le V_x \le V_2) \\ 0 & ((V_x < V_1,\ V_2 < V_x) \end{cases} \tag{20}$$

as illustrated in Fig. 1 (a). The deepest penetration point reached by particles having the largest velocity V_2 is assigned to be $x=0$, and a plasma is assumed to be emitted from $x = -\infty$. Then (20) becomes

$$\frac{d\eta}{dx} = \left(\frac{16\pi MN_0}{V_2 - V_1}\right)^{1/2}\left\{\int_{V_1}^{V_2} V_x^2\,dV_x - \int_S^{V_2} V_x(V_x^2 - a^2\eta^2)^{1/2}\,dV_x\right\}^{1/2} \tag{21}$$

where $S = a\eta$ when $V_1 < a\eta \le V_2$, and $S = V_1$ when $a\eta < V_1$. Since $\eta = V_2/a$ and $S = V_2$ at $x=0$, the magnetic field intensity at $x=0$ is given by

$$B_0 = \left(\frac{d\eta}{dx}\right)_{x=0} = \left\{\frac{16}{3}\pi MN_0(V_1^2 + V_1V_2 + V_2^2)\right\}^{1/2} \tag{22}$$

which in turn leads to the pressure balance equation

$$\frac{B_0^2}{8\pi} = \frac{2}{3}MN_0(V_1^2 + V_1V_2 + V_2^2) \tag{23}$$

Introducing the following variables and parameters,

$$Y = \left(1 - \frac{a^2\eta^2}{V^2}\right)^{1/2}, \qquad X_R = \frac{x}{\lambda_R}, \qquad \alpha = \frac{V_1}{V_2}$$

$$\lambda_R = \left\{\frac{2}{3}(1 + \alpha + \alpha^2)\right\}^{-1/2}\lambda \quad \text{and} \quad \lambda = \left(\frac{mc^2}{8\pi N_0 e^2}\right)^{1/2}$$

we find Eq. (21) is reduced to the dimensionless equations

$$Y(1-Y^2)^{-1/2}\left\{1 - \frac{Y^3}{1-\alpha^3}\right\}^{-1/2} dY = -dX_R \tag{24}$$

when $V_1 \le a\eta \le V_2$ or $0 \le Y \le (1-\alpha^2)^{1/2}$, and

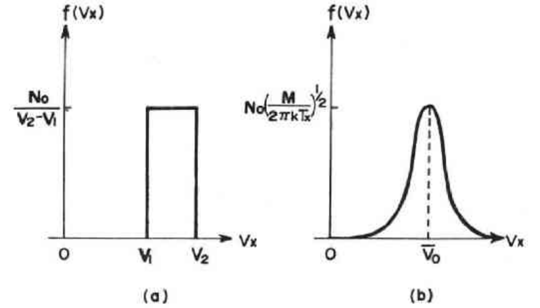

Fig. 1. Initial velocity distribution functions given at $x=-\infty$; (a) the rectangular distribution and (b) the Maxwellian distribution superposed on a bulk velocity $\bar{V}_0$.

$$Y(1-Y^2)^{-1/2}\left\{1-\frac{Y^3}{1-\alpha^3}+\frac{(\alpha^2-1+Y^2)^{3/2}}{1-\alpha^3}\right\}^{-1/2}dY=-dX_R \tag{25}$$

when $a\eta<V_1$ or $(1-\alpha^2)^{1/2}<Y\leq 1$.

Numerical integrations of Eqs. (24) and (25) have been obtained for several values of α. The magnetic fields normalized by their boundary values B_0 are shown in Fig. 2 as a function of the converted distance x/λ. The result shows that the thickness of the boundary is remarkably influenced by the number density N_0 and by the parameter α characterizing the distribution function. The thickness decreases as α tends to 1. In the extreme case when $\alpha=1$, the result is coincident with that obtained in the study of the interaction with a cold plasma flow.

(2) Maxwellian distribution

Fig. 1 (b) illustrates the Maxwellian distribution given by

$$f(V_x)=\left(\frac{M}{2\pi kT_x}\right)^{1/2}\exp\left[-\frac{M}{2kT_x}(V_x-\bar{V}_0)^2\right] \tag{26}$$

where T_x is the effective kinetic temperature of a plasma measured in the direction perpendicular to the boundary surface and $\bar{V}_0$ is the bulk velocity of a plasma moving in the x-direction. When the velocity distribution is thermally isotropic, T_x may be replaced by T. If $M\bar{V}_0^{\,2}\gg kT_x$ the approximation may be made that $\int_{-\infty}^{0}f(V_x)dV_x\simeq 0$. Then, one has

$$\int_{-\infty}^{\infty}f(V_x)dV_x=\int_{0}^{\infty}f(V_x)\,dV_x$$

and (19) becomes

$$\frac{d\eta}{dx}=2\left(\frac{8\pi M^3N_0}{kT_x}\right)^{1/2}\left\{\int_0^\infty V_x^2\exp\left[-\frac{M}{2kT_x}(V_x-\bar{V}_0)^2\right]dV_x\right.$$
$$\left.-\int_{a\eta}^{\infty}V_x(V_x^2-a^2\eta^2)^{1/2}\exp\left[-\frac{M}{2kT_x}(V_x-\bar{V}_0)^2\right]dV_x\right\}^{1/2} \tag{27}$$

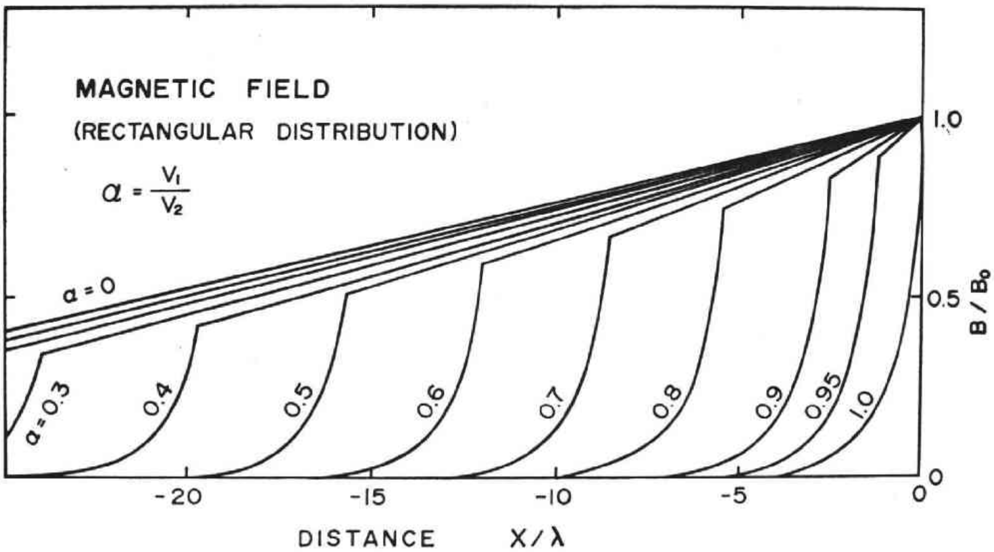

Fig. 2. Distribution of the magnetic field in the boundary layer formed by the interaction of a plasma flow in which the rectangular velocity distribution is assumed. The field intensity and the distance are given in units normalized by B_0 and λ, respectively.

If the penetration depth of particles having the mean velocity $\bar{V}_0$ is assigned to be $x=0$, the deepest penetration point of the injected flow becomes $x=+\infty$, where the magnetic pressure balances the dynamic pressure of the flow. The pressure balance equation can be btained easily from (27) as

$$\begin{aligned}\frac{B_0^2}{8\pi} &= MN_0\left(\frac{2M}{kT_x}\right)^{1/2}\int_0^\infty V_x^2\exp\left[-\frac{M}{2kT_x}(V_x-\bar{V}_0)^2\right]dV_x \\ &= 2N_0M\bar{V}_0^2+2N_0kT_x \\ &= 2N_0M(\bar{V}_0^2+V_{tx}^2) \end{aligned} \tag{28}$$

where V_{tx} denotes the x-component of the thermal velocity $\boldsymbol{V}_t$, and is given by $V_{tx}=(kT_x/M)^{1/2}$. Eq. (27) is also transformed as

$$\left\{1-\frac{1}{\left(1+\dfrac{1}{2\beta}\right)\sqrt{\dfrac{\pi}{\beta}}}\int_{z-1}^\infty(1+r)[(1+r)^2-Z^2]^{1/2}\exp(-\beta r^2)\,dr\right\}^{-1/2}dZ=dX_M \tag{29}$$

where

$$Z=a\eta/V_0,\quad X_M=x/\lambda_M,\quad r=(V_x-\bar{V}_0)/\bar{V}_0$$
$$\beta=M\bar{V}_0^2/2kT_x\quad\text{and}\quad\lambda_M=(2+1/\beta)^{-1/2}\lambda$$

Under the condition that $Z=1$ at $x=X_m=0$, the integrations have been made numerically for several values of β. The results are shown in Fig. 3 in units normalized by B_0 as a function of the converted distance x/λ. The magnetic shielding distance increases towards smaller values of β. This result is consistent with that obtained on the basis of the rectangular distribution.

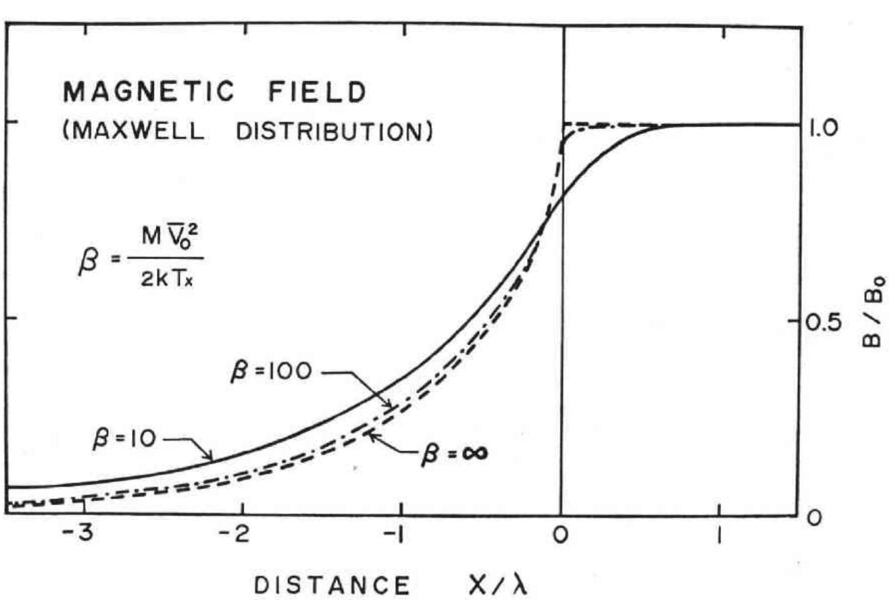

Fig. 3. For representative values of the β-factor which is given by $M\bar{V}_0^2/(2kT_x)$, distribution of the magnetic field in the boundary layer formed by the interaction of a plasma flow in which the Maxwellian distribution is assumed. The normalized units are employed as in Fig. 2.

Summary

The steady state transition layer formed by the interaction between a magnetic field and a plasma flow having an arbitrary velocity distribution is examined in a one-dimensional model. It is shown that the self-consistent solution of the nonlinear plasma-field equation can be obtained for any velocity distribution given at a point sufficiently distant from the boundary region when the initial velocity distribution is assumed to be common both for protons and electrons. The quantities participating in the interaction are essentially the density of the flowing plasma and the velocity component normal to the boundary surface. It is also noted that the equation of continuity in the entire velocity space is not satisfied in the boundary layer, since the penetration depth of particles ranges widely according to their initial velocities. The thickness of the layer is shown to be remarkably dependent of the velocity distribution function. The thickness minimizes when a cold plasma flow incidences.

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Appendix

Eq. (19) in the text is derived through the following steps.

Multiplying by $d\eta/dx$, one finds that (18) becomes

$$\frac{1}{2}\frac{d}{dx}\left(\frac{d\eta}{dx}\right)^2=\frac{8\pi e^2}{mc}N\eta\frac{d\eta}{dx}+\frac{m^2}{8e^2}\frac{d}{dx}\left\{\frac{d}{dx}\left(V_{0y}+\frac{e}{mc}\eta\right)^2\right\}^2 \qquad \text{(A1)}$$

The integration of (A1) under the conditions that $\eta=d\eta/dx=0$ at $x=-\infty$ gives exactly

$$\left(\frac{d\eta}{dx}\right)^2=16\pi MN_0\left\{\int_0^\infty V_x^2 f(V_x)dV_x-\int_{a\eta}^\infty V_x(V_x^2-a^2\eta^2)^{1/2}f(V_x)\,dV_x\right\}$$
$$+\frac{m^2}{4e^2}\left\{\frac{d}{dx}\left(V_{0y}+\frac{e}{mc}\eta\right)^2\right\}^2 \qquad \text{(A2)}$$

Since

$$v_y=V_{0y}+\frac{e}{mc}\eta \quad \text{and} \quad v_y^2\ll c^2$$

one finds that

$$\left(\frac{d\eta}{dx}\right)^2-\frac{m^2}{4e^2}\left\{\frac{d}{dx}\left(V_{0y}+\frac{e}{mc}\eta\right)^2\right\}^2=\frac{m^2c^2}{e^2}\left[\left(\frac{dv_y}{dx}\right)^2-\frac{1}{4c^2}\left(\frac{d}{dx}v_y^2\right)^2\right]$$

$$=\frac{m^2c^2}{e^2}\left(\frac{dv_y}{dx}\right)^2\left(1-\frac{v_y^2}{c^2}\right)\simeq\frac{m^2c^2}{e^2}\left(\frac{dv_y}{dx}\right)^2=\left(\frac{d\eta}{dx}\right)^2$$

Thus, (A2) becomes

$$\left(\frac{d\eta}{dx}\right)^2\simeq 16\pi MN_0\left\{\int_0^\infty V_x^2 f(V_x)\,dV_x-\int_{a\eta}^\infty V_x(V_x^2-a^2\eta^2)^{1/2}f(V_x)\,dV_x\right\} \qquad \text{(A3)}$$

which leads easily to Eq. (19).