Theoretical Study on the Crustal Movements ?. Extension and Tilt Produced by a Temperature Distribution Due to a Thermal Point Source in a Semi-infinite Elastic Medium

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Theoretical Study on the Crustal Movements III. Extension and Tilt Produced by a Temperature Distribution Due to a Thermal Point Source in a Semi-infinite Elastic Medium

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Abstract: A problem of extension and tilt due to hyperbolically decreasing temperature distribution from a thermal point source in a semi-infinite elastic medium is theoretically investigated. An example of extension and tilt on a free surface is graphycally presented. It is clarified that tilt has a maximum absolute value at a point of 1.3 times source depth and extension decreases more rapidly than tilt with increasing distance.

1. Introduction

Extension and tilt are continuously observed by extensometer and tiltmeter in observatories of crustal movements. However, observed extension and tilt include various influences like temperature, atmospheric pressure, rainfall and so on. Therefore, we must know how various factors influence on extension and tilt, in order to take out a signal that we desire from observed data. In a previous paper (Ishii and Takagi, 1967), the influence of horizontal discontinuity was studied. In this paper we will investigate a thermal influence on the crustal deformations. The effect of surface temperature on the crustal deformations was investigated by some workers (e.g. Nishimura (1930), Arakawa (1931), Matsuzawa (1942) and Nakano (1963)). Shima (1958) solved displacement by application of potential theory in the case where the spheroidal or spherical region of constant temperature is embedded in a semi-infinite elastic body. However, a temperature distribution will be produced by a thermal source for long period. Then the temperature distribution may influence on extension and tilt being observed. Therefore, in the present paper we will investigate extension and tilt produced by a temperature distribution of hyperbolic type due to a thermal point source in a semi-infinite elastic medium.

2. Formulation

We take the cylindrical coordinates (r, θ, z) as in Fig. 1, and $z=0$ as free surface. We assume symmetrical displacement about a z-axis, namely that displacement may take place in planes through the axis and be same in all such planes. In this case the conditions would be expressed by reference to cylindrical coordinates r, θ, z by the equations

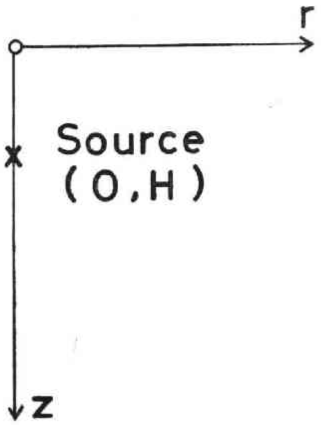

Fig. 1. Cylindrical coordinate system used for formulation. Free surface is $z=0$.

$$u_\theta = 0\,, \quad \frac{\partial u_r}{\partial \theta} = \frac{\partial u_z}{\partial \theta} = 0\,. \tag{1}$$

It will be convenient to write U for u_r and w for u_z. The strain components are then expressed by equations

$$\left.\begin{aligned} e_{rr} &= \frac{\partial U}{\partial r}\,, \quad e_{\theta\theta} = \frac{U}{r}\,, \quad e_{zz} = \frac{\partial w}{\partial z}\,, \\ e_{rz} &= \frac{\partial U}{\partial z} + \frac{\partial w}{\partial r}\,, \quad e_{r\theta} = e_{\theta z} = 0\,. \end{aligned}\right\} \tag{2}$$

Cubical dilatation and rotation are expressed by equations

$$\left.\begin{aligned} &\varDelta = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial w}{\partial r}\,, \quad 2\varpi_\theta = \frac{\partial U}{\partial z} - \frac{\partial w}{\partial r}\,, \\ &\varpi_r = \varpi_z = 0\,. \end{aligned}\right\} \tag{3}$$

We will write ϖ for ϖ_θ hereafter.

Then we have the following forms of equations

$$(\lambda + 2\mu)\,\frac{\partial \varDelta}{\partial r} + 2\mu\,\frac{\partial \varpi}{\partial z} = \alpha\,\frac{\partial T}{\partial r}\,, \tag{4}$$

$$(\lambda + 2\mu)\,\frac{\partial \varDelta}{\partial z} - \frac{2\mu}{r}\,\frac{\partial}{\partial r}\,(r\varpi) = \alpha\,\frac{\partial T}{\partial z}\,, \tag{5}$$

where T is the difference of temperature, and

$$\alpha = \left(\lambda + \frac{2}{3}\mu\right)c\,, \tag{6}$$

where c is the cubical expansion coefficient of the body. Transforming equations (4) and (5), we have

$$(\lambda + 2\mu)\left[\frac{\partial^2 \Delta}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta}{\partial r} + \frac{\partial^2 \Delta}{\partial z^2} \right] = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right], \tag{7}$$

$$\frac{\partial^2 \varpi}{\partial r^2} + \frac{1}{r}\frac{\partial \varpi}{\partial r} - \frac{\varpi}{r^2} + \frac{\partial^2 \varpi}{\partial z^2} = 0 . \tag{8}$$

We take as a solution of equation (8)

$$2\varpi = A \frac{\partial J_0(kr)}{\partial r} e^{-k|z-H|}, \tag{9}$$

where A is an arbitrary constant. To obtain an expression at a free surface, we use the case

$$2\varpi = A \frac{\partial J_0(kr)}{\partial r} e^{k(z-H)} \qquad z \leqq H . \tag{10}$$

In order to determine a form of T in equations (4) and (5), we must first solve the equation of the conduction of heat such as

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right), \tag{11}$$

where κ is the thermal conductivity. In this paper we deal with T satisfying the condition of the conduction at every moment and the conditions of the problem to be solved. Now we assume that the temperature distribution of the body is expressed by the following form:

$$T = a\, J_0(kr) e^{k(z-H)} \qquad z \leqq H, \tag{12}$$

where a is an arbitrary constant. Inserting (12) into (7), we have

$$\frac{\partial^2 \Delta}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta}{\partial r} + \frac{\partial^2 \Delta}{\partial z^2} = 0 . \tag{13}$$

We take as a solution of equation (13)

$$\Delta = B J_0(kr) e^{k(z-H)} \qquad z \leqq H . \tag{14}$$

Displacement (U_1, w_1) satisfying Δ expressed by equation (14) and $\varpi = 0$, is expressed by

$$\left.\begin{aligned} U_1 &= \frac{B}{2k} z\, e^{k(z-H)} \frac{\partial J_0(kr)}{\partial r}, \\ w_1 &= \frac{B}{2k} (1 + kz) J_0(kr) e^{k(z-H)} . \end{aligned}\right\} \tag{15}$$

Displacement (U_2, w_2) satisfying ϖ expressed by equation (10) and $\Delta = 0$, is expressed by

$$\left.\begin{aligned} U_2 &= \frac{A}{2}\, z\, \frac{\partial J_0(kr)}{\partial r}\, e^{k(z-H)}\,, \\ w_2 &= -\frac{A}{2}\,(1-kz)\, J_0(kr) e^{k(z-H)}\,. \end{aligned}\right\} \tag{16}$$

Displacement (U_3, w_3) answering to $\Delta=2\varpi=0$ is expressed by

$$\left.\begin{aligned} U_3 &= C\, \frac{\partial J_0(kr)}{\partial r}\, e^{k(z-H)}\,, \\ w_3 &= CkJ_0(kr) e^{k(z-H)}\,. \end{aligned}\right\} \tag{17}$$

Therefore, the general solution for displacement is

$$\left.\begin{aligned} U &= U_1+U_2+U_3\,, \\ w &= w_1+w_2+w_3\,. \end{aligned}\right\} \tag{18}$$

Substituting (10), (12) and (14), we have the relation

$$(\lambda+2\mu)B+\mu Ak = a\alpha\,. \tag{19}$$

Stress components are denoted as follows

$$\left.\begin{aligned} \widehat{zz} &= \lambda\Delta+2\mu\,\frac{\partial w}{\partial z}-aT = \lambda\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right)+(\lambda+2\mu)\,\frac{\partial w}{\partial z}-aT\,, \\ \widehat{\theta z} &= 0\,, \\ \widehat{rz} &= \mu\left(\frac{\partial U}{\partial z}+\frac{\partial w}{\partial r}\right). \end{aligned}\right\} \tag{20}$$

The condition that free surface be free of forces requires

$$\widehat{zz}=\widehat{\theta z}=\widehat{rz}=0 \qquad \text{at} \qquad z=0\,. \tag{21}$$

Substituting equation (18) into the boundary condition (21), we obtain

$$\left.\begin{aligned} &(\lambda+2\mu)B+2\mu k^2C = a\alpha\,, \\ &B+2k^2C = 0\,. \end{aligned}\right\} \tag{22}$$

We have from equations (19), (21) and (22)

$$\left.\begin{aligned} A &= -\frac{1}{k}\,\frac{a\alpha}{(\lambda+\mu)}\,, \\ B &= \frac{a\alpha}{\lambda+\mu}\,, \\ C &= -\frac{a\alpha}{2k^2(\lambda+\mu)}\,. \end{aligned}\right\} \tag{23}$$

From equations (18) and (23) we have an expression for extension and tilt as follows

$$\left.\begin{aligned}\frac{\partial U}{\partial r} &= -\frac{a\alpha}{2(\lambda+\mu)}\, e^{k(z-H)}\,\frac{\partial^2 J_0(kr)}{k^2\partial r^2}\,,\\ \frac{\partial w}{\partial r} &= \frac{a\alpha}{2(\lambda+\mu)}\, e^{k(z-H)}\,\frac{\partial J_0(kr)}{k\partial r}\,.\end{aligned}\right\} \tag{24}$$

Performing the operation

$$\int_0^\infty dk$$

to the temperature distribution (12) and to the extension and tilt component (24), and using the identical formulae

$$\left.\begin{aligned}\int_0^\infty e^{k(z-H)} J_0(kr)dk &= \frac{1}{\sqrt{r^2+(H-z)^2}}\,,\\ \int_0^\infty e^{k(z-H)} J_1(kr)dk &= \frac{r}{\{r^2+(H-z)^2\}^{3/2}}\,,\end{aligned}\right\} \tag{25}$$

we have

$$T = \frac{\alpha}{\sqrt{r^2+(H-z)^2}}\,, \tag{26}$$

$$\frac{\partial U}{\partial r} = \frac{a\cdot\alpha}{2(\lambda+\mu)}\Bigg[\frac{1}{\sqrt{r^2+(H-z)^2}} - \frac{\sqrt{H^2+r^2}}{r^2} + \frac{H-z}{r^2} + \frac{\sqrt{r^2+H^2}-\sqrt{r^2+(H-z)^2}}{r^2}\Bigg], \tag{27}$$

$$\frac{\partial w}{\partial r} = -\frac{a\alpha}{2(\lambda+\mu)}\Bigg[\frac{1}{r} + \frac{z-H}{r\sqrt{r^2+(H-z)^2}}\Bigg]. \tag{28}$$

3. Numerical Computation and Discussion

The temperature distribution due to a thermal point source is shown in Fig. 2 using equation (26). Fig. 2 representatively shows a form of the curve which moves depending on α. The temperature decreases hyperbolically with increasing distance from the heat source. Extension and tilt on free surface due to the temperature distribution are shown in Fig. 3. The ordinate is the value multiplied by $H\cdot\frac{2(\lambda+\mu)}{c\cdot\alpha}$ and the abscissa is $n=r/H$ the distance divided by H. H is the depth of the point source, λ and μ Lame's constant, α constant appeared in the temperature distribution, and c cubical expansion coefficient. We have known the relation $c=3a_l$ where a_l is the linear expansion coefficient. In this computation we employed the linear expansion coefficient of basalt which is equal to 8.3×10^{-6}, and $\sigma=0.25$. Computed results are graphically presented in Fig. 3. As can be seen in Fig. 3 extension decreases more rapidly than tilt with increasing distance. Tilt has a maximum absolute value at 1.3 times the source depth. Next, we computed $A=1\Big/\Big(H\cdot\frac{2(\lambda+\mu)}{c\cdot\alpha}\Big)$, for practical

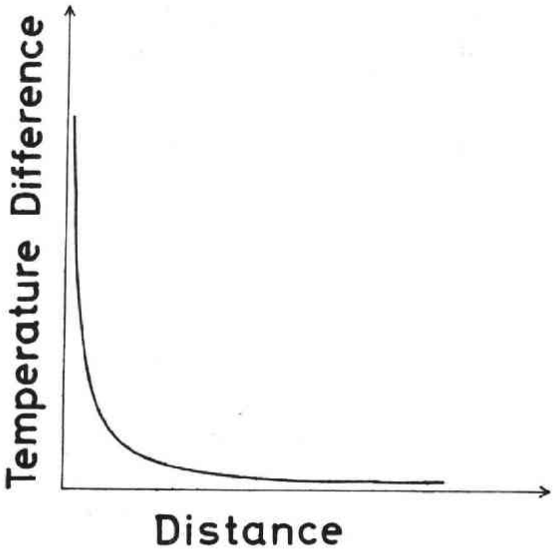

Fig. 2. Distribution of temperature difference due to a thermal point source.

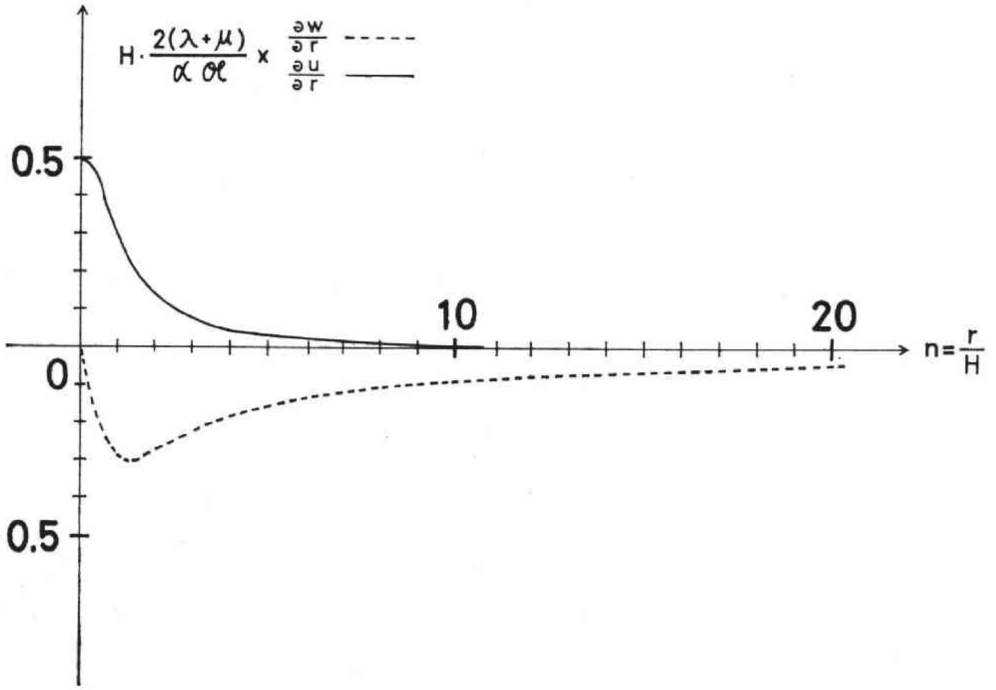

Fig. 3. Extension and tilt on free surface due to the temperature distribution shown in Fig. 2.

purpose. Namely, the value of Fig. 3 multiplied by the value A is the value of an extension and a tilt observed on the surface. Table 1 shows the value of A for various $\mathfrak{a}$ and H. We can estimate extension and tilt on the surface using Fig. 3 and Table 1. For example, in the case of $\mathfrak{a}=10^4{}^\circ\mathrm{C}\cdot\mathrm{cm}$ and $H=10$ km, the temperature difference at

Table 1. Value of $A=1/\left(H\cdot\frac{2(\lambda+\mu)}{c\cdot a}\right)$. H is the depth of the point soruce, λ and μ Lame's constant, a constant appeared in the temperature distribution, and c cubical expansion coefficient.

a (°C·cm)		10^3	10^4	10^5
Temperature Difference (°C) (at 1 km from center)		0.01	0.1	1.0
$\frac{1}{H\cdot\frac{2(\lambda+\mu)}{c\cdot a}}$	$H=1$km	1.038×10^{-7}	1.038×10^{-6}	1.038×10^{-5}
	$H=10$km	1.038×10^{-8}	1.038×10^{-7}	1.038×10^{-6}

a point of 1 km from the point source is 0.1°C and the value of A is 1.038×10^{-7} for basalt, so that extension at $n=r/H=0.0$ (just above the source) is $0.5\times1.038\times10^{-7}=5.19\times10^{-8}$.

4. Summary

Surface extension and tilt of semi-infinite elastic medium are theoretically investigated in the case where a hyperbolical temperature distribution from an internal thermal point source exists. It is found that extension diminishes more quickly than tilt with increasing distance. It is also found that the largest tilt appear at 1.3 times the source depth. For some situations values of extension and tilt are presented for practical estimation.

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REFERENCES

Arakawa, H., **1931**: The effect of temperature on the deformation of infinite or semi-infinite elastic body, *Geophys. Mag.*, **4**, 297–306.

Ishii, H and A. Takagi, 1967: Theoretical study on the crustal movements Part II. The influence of horizontal discontinuity, *Sci. Rep. Tôhoku Univ. Ser.* **5**, *Geophys.* **19**, 95–106.

Matsuzawa, T., 1942: Temperatur verlauf an der Bodenfläche und der Spannungszustand in der Erdkruste, *Bull. Earthq. Res. Inst.*, **20**, 20–29.

Nakano, S., 1963: The effect of surface temperature on the crustal deformations, *Disaster Prevension Res. Inst. Kyoto Univ. Bull.*, **60**, 2–20.

Nishimura, G., 1930: The effect of temperature distribution on the deformation of a semi-infinite elastic body, *Bull. Earthq. Res. Inst.*, **8**, 91–141.

Shima, M., **1958**: On the thermoelasticity in the semi-infinite elastic solid, *Disaster Prevension Res. Inst. Kyoto Univ. Bull.*, **25**, 2–17.