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Expression of a Free Rayleigh Wave Using Complex Angles

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Abstract: Complex angles were successfully used for the calculation of amplitude characteristics for incident plane P and SV waves in the case of a dipping layer by Ishii and Ellis (1970b). In this paper, it is shown that a free Rayleigh wave is also expressed using complex angles.

1. Introduction

Complex angles were used in order that the cases of total reflection and incident angles greater than the critical angles were involved in the results in the calculation of the displacements in the case where a dipping layer exists (Ishii and Ellis, 1970b).

The expression of Rayleigh waves in terms of complex angles is also of interest for further studies of surface waves. Therefore, the present study will give an interpretation of free Rayleigh waves in terms of complex angles.

2. Formulation and Discussion

Let us consider an elastic half-space with free surface $\theta=0$ (Figure 1). For plane waves propagating in the x-y plane, the motion is independent of z and the displacement has only r and θ components. The dilatation Θ and rotation ϖ satisfy

$$\begin{aligned} \nabla^2\Theta + k_a^2\Theta = 0\,, \\ \nabla^2\varpi + k_b^2\varpi = 0\,, \end{aligned} \tag{1}$$

where $k_a=\omega/C_a$, $k_b=\omega/C_b$ and C_a and C_b are the P and S wave velocities respectively (Ishii and Ellis, 1970b). The stress components are expressed by

$$\begin{aligned} \widehat{\theta\theta} &= \rho C_a^2\Theta + 2\rho C_b^2\left\{\frac{1}{k_a^2}\frac{\partial^2\Theta}{\partial r^2} - \frac{2}{k_b^2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varpi}{\partial\theta}\right)\right\}, \\ \widehat{r\theta} &= -2\rho C_b^2\left\{\frac{1}{k_a^2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Theta}{\partial\theta}\right) + \frac{1}{k_b^2}\frac{\partial^2\varpi}{\partial r^2} + \varpi\right\}, \end{aligned} \tag{2}$$

where ρ is density. The solution in the medium can be written as

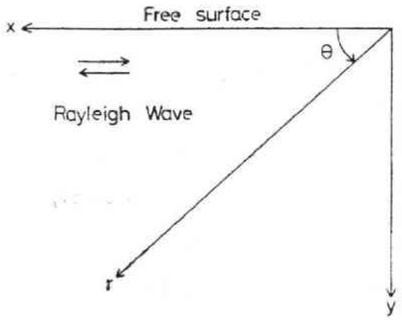

Fig. 1. Coordinate system used to calculate the complex angle of a free Rayleigh wave.

$$\Theta_R = A_R \cdot e^{ik_a r\cos(\theta-\alpha_R)} ,$$
$$\varpi_R = B_R \cdot e^{ik_b r\cos(\theta-\beta_R)} , \tag{3}$$

where A_R and B_R are constants, and α_R and β_R are propagation directions measured from $\theta=0$ as defined in the previous papers (Ishii and Ellis, 1970a and 1970b).

The boundary conditions at $\theta=0$ are

$$\widehat{\theta\theta} = 0 ,$$
$$\widehat{r\theta} = 0 . \tag{4}$$

Substituting (3) into the boundary conditions (4) using (2), we have

$$(1-2\nu_b^2 \cos^2 \alpha_R)A_R + 4\nu_b^2 B_R \sin \beta_R \cos \beta_R = 0 ,$$
$$- A_R \sin \alpha_R \cos \alpha_R + (1-2\cos^2 \beta_R) B_R = 0 , \tag{5}$$

and

$$\nu_b \cos \alpha_R = \cos \beta_R , \tag{6}$$

where $\nu_b = C_b/C_a$.
From (5), we have

$$(1-2\nu_b^2 \cos^2 \alpha_R)(1-2\cos^2 \beta_R) - 4\nu_b^2 \sin \alpha_R \cos \alpha_R \sin \beta_R \cos \beta_R = 0 . \tag{7}$$

Substituting (6), and writing $x=\cos^2 \alpha_R$ and $\nu = \nu_b^2$ gives

$$16(1-\nu)x^3 + \left(16- \frac{24}{\nu}\right)x^2 + \frac{8}{\nu^2} x - \frac{1}{\nu^3} = 0 . \tag{8}$$

Assuming Poisson's relation, $\lambda=\mu$, yields

$$\frac{32}{3} x^3 - 56x^2 + 72x - 27 = 0 . \tag{9}$$

The real root of this equation is $x=3.549$ which corresponds to $\cos \alpha_R=\pm 1.884$ and using (6), $\cos \beta_R=\pm 1.088$. Recalling the relations

$$\arccos(-z) = \pi - \arccos z\,,$$
$$\arccos p = i \operatorname{arccosh} p \qquad (p = real > 1)\,, \tag{10}$$

we obtain

$$\begin{aligned} \alpha_R &= 1.247i \quad \text{or} \quad \pi - 1.247i\,, \\ \beta_R &= 0.4068i \quad \text{or} \quad \pi - 0.4068i\,. \end{aligned} \tag{11}$$

If in equations (3), we use

$$\begin{aligned} \cos(p \pm iq) &= \cos p \cosh q \mp i \sin p \sinh q\,, \\ \sin(p \pm iq) &= \sin p \cosh q \pm i \cos p \sinh q\,, \end{aligned} \tag{12}$$

we have

$$\begin{aligned} \Theta_R &= A_R \cdot e^{\pm i k_a r (1.884\cos\theta \pm i1.597\sin\theta)} \\ &= A_R \cdot e^{\pm i \frac{\omega}{0.9194 C_b} x - \frac{0.9218}{C_b} \omega y}\,, \\ \varpi_R &= B_R \cdot e^{\pm i k_b r (1.088\cos\theta \pm i0.4278\sin\theta)} \\ &= B_R \cdot e^{\pm i \frac{\omega}{0.9194 C_b} x - \frac{0.4278}{C_b} \omega y}\,. \end{aligned} \tag{13}$$

We see that the dilatation and rotation propagate with the velocity 0.9194 C_b which coincides with the velocity of the free Rayleigh wave. As a result we see that for a Rayleigh wave written in terms of complex angles, the real part of the angle indicates the propagation direction and the imaginary part gives the decrease of amplitude with the two solutions of (11) representing waves propagating in opposite directions (0 and π).

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