Theoretical study on the Crustal Movememts Part ?. The Influence of Surface Topography (Two-Dimensional SH-Torque Source).

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Theoretical Study on the Crustal Movements
Part I. The Influence of Surface Topography
(Two-Dimentional SH-Torque Source)

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Abstract: There exists a SH-Torque source in a semi-infinite elastic half sapce where the surface form is of the sine, cosine, arctangent and exponent types. The displacement of the surface is approximately evaluated by the perturbation method when $\varepsilon/H \approx 0$ and $\varepsilon/H = 0.1$ in which ε denotes the parameter of perturbation and H the depth of the source.

1. Introduction

The deformation of a semi-infinite elastic half space with a plane surface has been studied by many investigators. For example, Honda and Miura (1935) investigated on the problems of various surface sources, while Yamakawa (1955) and Nakano (1959) dealt with the problem of the internal source. A great deal of observations have been made on the crustal movements in areas of variant topography, but a little has been known about the movements at a place with a complex plane. Therefore, the present authors made a study of the influence of the surface topography on the crustal movements, using the same tow-dimensional SH-Torque line source as the strike-slip one employed by Kasahara (1964). In view of the fact that no mathematical solution under the topographical considerations has been given in this field, we made a first attempt to elucidate the influence of surface topography on the crustal movements by means of a perturbation method.

2. Theory

We take the Y-axis vertically downward (Fig. 1). The elastic constant is denoted by μ. A torque soruce is located at the point of origin, and the movement caused by it is assumed to be unchanged in the direction of the Z-axis. Let the displacements have only the Z component and be independent of Z. When there are no external forces, the equation of equilibrium of a homogeneous and isotropic elastic medium is expressed by

$$\mu \nabla^2 w = 0, \tag{1}$$

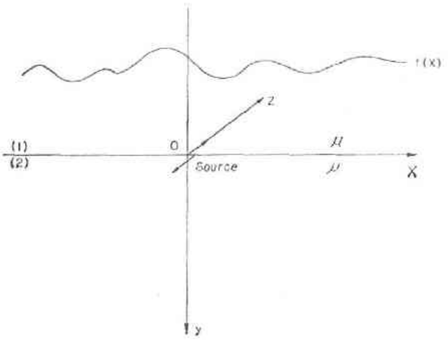

Fig. 1

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

μ and w represent the rigidity and the displacements of the Z-component, respectively. The stress component is

$$Z_y = \mu \frac{\partial w}{\partial y},$$

$$Z_x = \mu \frac{\partial w}{\partial x}. \tag{2}$$

Assuming the displacements to be

$$w^{(1)} = A\, e^{-ky} \cos k x + B\, e^{ky} \cos k x\,,$$

$$w^{(2)} = C\, e^{-ky} \cos k x\,. \tag{3}$$

The subscripts 1 and 2 represent the quantities above and below the source. The stress component on the free surface is

$$Z_\nu = Z_y - Z_x f'(x)\,. \tag{4}$$

The boundary condition at the free surface requires zero tangential stress expressed as

$$\frac{\partial w^{(1)}}{\partial y} - \frac{\partial w^{(1)}}{\partial x} f'(x) = 0 \qquad \text{at} \quad y = f(x)\,. \tag{5}$$

The boundary condition at the source plane requires continuity of the displacements and the stress component.

$$w^{(1)} = w^{(2)} \qquad \text{at} \quad y = 0,$$

$$Z_y{}^{(1)} - Z_y{}^{(2)} = Z \cos k x \qquad \text{at} \quad y = 0. \tag{6}$$

Substituting (3) into (5) and (6), we obtain

$$A=\frac{Z}{2\mu k}e^{-2kf(x)}\frac{\cos kx+f'(x)\sin kx}{\cos kx-f'(x)\sin kx},$$

$$B=\frac{Z}{2\mu k},\tag{7}$$

$$C=A+B.$$

Putting $f(x)=-H+\varepsilon F(x)(\varepsilon\ll 1)$, performing the power expansion and omitting the terms of more than second order, we have

$$A=\frac{Z}{2\mu k}\left[1-(2kF(x)-2F'(x)\tan kx)\,\varepsilon\right]e^{-2kH}.\tag{8}$$

Inserting the expressions for A, B and C into (3)

$$w^{(1)}=\frac{Z}{2\mu k}\left[1-(2kF(x)-F'(x)\tan kx)\,\varepsilon\right]e^{-k(y+2H)}\cos kx+\frac{Z}{2\mu k}e^{ky}\,\mathrm{cps}\,kx,$$

$$w^{(2)}=\frac{Z}{2\mu k}\left[1-(2kF(x)-F'(x)\tan kx)\,\varepsilon\right]e^{-k(y+2H)}\cos kx+\frac{Z}{2\mu k}e^{-ky}\cos kx.\tag{9}$$

In order to resolve the point source, we perform the operation

$$\int_0^\infty dk.$$

The displacement $W^{(1)}$ due to a point soruce is expressed as

$$W^{(1)}=\frac{Z}{2\mu}\int_0^\infty\frac{1}{k}(e^{-k(y+2H)}+e^{ky})\cos kx\,dk+\varepsilon\frac{Z}{\mu}\left[F'(x)\int_0^\infty\frac{e^{-k(y+2H)}}{k}\sin kx\,dk-F(x)\int_0^\infty e^{-k(y+2H)}\cos kx\,dx\right].\tag{10}$$

Therefore, the displacement $W_x{}^{(1)}$ due to the couple force of the X direction is given by

$$W_x{}^{(1)}=-\frac{Z}{2\mu}\int_0^\infty(e^{-k(y+2H)}+e^{ky})\sin kx\,dk+\varepsilon\frac{Z}{\mu}\left[F''(x)\int_0^\infty\frac{e^{-k(y+2H)}}{k}\sin kx\,dk+F(x)\int_0^\infty k\,e^{-k(y+2H)}\sin kx\,dk\right].\tag{11}$$

Making use of the following relations,

$$\int_0^\infty e^{-ak}\sin bk\,dk=\frac{b}{a^2+b^2}, \qquad a>0$$

$$\int_0^\infty k\,e^{-ak}\sin bk\,dk=\frac{2ab}{(a^2+b^2)^2}, \qquad a>0$$

$$\int_0^\infty \frac{e^{-ak}\sin bk}{k}\,dk=\tan^{-1}\frac{b}{a}, \qquad a>0 \tag{12}$$

we obtain,

$$W_x{}^{(1)}=-\frac{Z}{2\mu}\left\{\frac{x}{\{(y+2H)^2+x^2\}}+\frac{x}{y^2+x^2}\right\}$$

$$+\varepsilon\frac{Z}{\mu}\left[F''(x)\tan^{-1}\frac{x}{y+2H}+F(x)\frac{2x(y+2H)}{\{(y+2H)^2+x^2\}^2}\right]. \tag{13}$$

At the free surface, $y=f(x)=-H+\varepsilon F(x)$. The first term represents the form of the solution for the free surface plane, and the second term the influence of surface topography. Then,

$$\frac{2\mu H}{Z}W_x{}^{(1)}=-\bar{x}\left[\frac{1}{\{(\bar{y}+2)^2+\bar{x}^2\}}+\frac{1}{\bar{y}^2+\bar{x}^2}\right]$$

$$+\varepsilon\cdot 2\left[F''(H\cdot\bar{x})\tan^{-1}\frac{\bar{x}}{\bar{y}+2}+\frac{1}{H}F(H\cdot\bar{x})\frac{2\bar{x}(\bar{y}+2)}{\{(\bar{y}+2)^2+\bar{x}^2\}^2}\right], \tag{14}$$

where $\bar{x}=\dfrac{x}{H}$, $\quad\bar{y}=\dfrac{y}{H}\quad$ and $\quad\bar{y}=-1+\varepsilon\dfrac{1}{H}F(H\cdot\bar{x})$

at the free surface.

Putting the following expressions into each term of Eq. (14),

$$D_1=-\bar{x}\left[\frac{1}{\{(\bar{y}+2)^2+\bar{x}^2\}}+\frac{1}{\bar{y}^2+\bar{x}^2}\right],$$

$$D_2=\left[F''(H\cdot\bar{x})\tan^{-1}\frac{\bar{x}}{\bar{y}+2}+\frac{1}{H}F(H\cdot\bar{x})\frac{2\bar{x}(\bar{y}+2)}{\{(\bar{y}+2)^2+\bar{x}^2\}^2}\right],$$

$$=D_3+\frac{1}{H}D_4. \tag{15}$$

The calculation of the above terms will satisfactorily represent the influence of surface topography on the crustal movements.

3. Illustration

D_1 in the preceding section represents the form of the solution for the free surface plane. At $\varepsilon=0.1$, however, the value on the free surface are almost equal to those on the

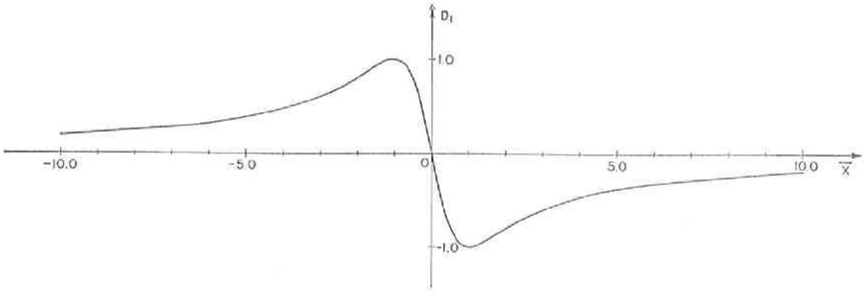

Fig. 2 Displacement for the plane boundary.

plane surface. Figure 2 shows the displacement for the plane boundary, where the term for the influence of surface topography, D_2, is calculated. For both cases, $\varepsilon \approx 0$ and $\varepsilon = 0.1$, D_2 is also calculated to examine the influence due to the variation of ε. D_3 dominates over other terms when H is large, while D_4 is predominant when H is small. In this calculation $H=1$ is chosen. For the following figures, the ordinates are devoted by

‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐	topography $10\,F\,(H \cdot x)$
————————	D_2
—— ‐ —— ‐ —— ‐ ——	$\dfrac{2\mu H}{Z} W_x{}^{(1)}$.

(1) *Sine and cosine types of topography*

The following cases were calculated:

$F(H \cdot \bar{x})$	$+\sin 2\pi \dfrac{H}{\lambda} \bar{x}$	$+\cos 2\pi \dfrac{H}{\lambda} \bar{x}$	$\cos 2\pi \dfrac{H}{\lambda} (\bar{x}-1.25)$
Figure	Fig. 3	Fig. 5	Fig. 7
$F(H \cdot \bar{x})$	$-\sin 2\pi \dfrac{H}{\lambda} \bar{x}$	$-\cos 2\pi \dfrac{H}{\lambda} \bar{x}$	
Figure	Fig. 4	Fig. 6	

where $H/\lambda = 0.1$ is chosen. D_2 for $\varepsilon \approx 0$ and $\varepsilon = 0.1$ are shown in the same figures, in which the figure of D_2 depends only on the amplitude of topography but also on the form of topography. As is evident from these figures, the influence of topography is exerted in such a way that the displacement of the plane surface is reduced in a convex part of the free surface and is reversely increased in a concave part of the free surface (refer to Fig. 2). But this tendency is reversed on the spot above the source. As can be seen from Figs. 4c, 5c, 6c, and 7c, D_4 concentrates upon the spot above the source.

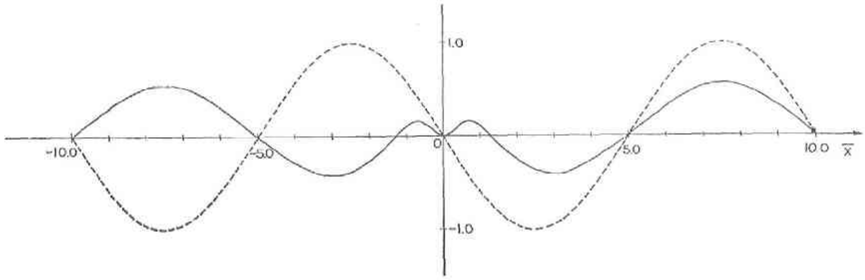

Fig. 3a $\varepsilon \approx 0$

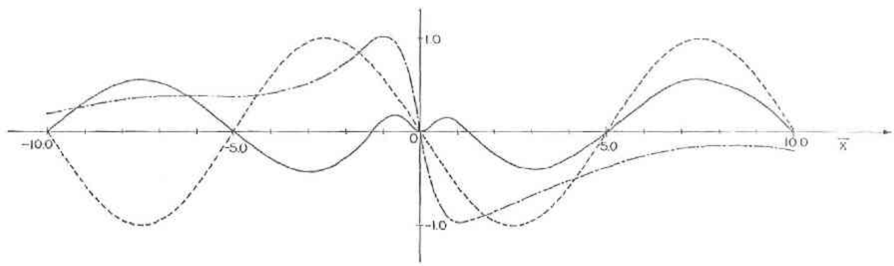

Fig. 3b $\varepsilon = 0.1$

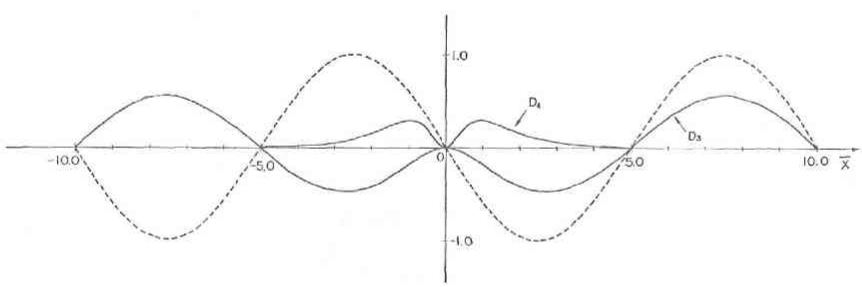

Fig. 3c

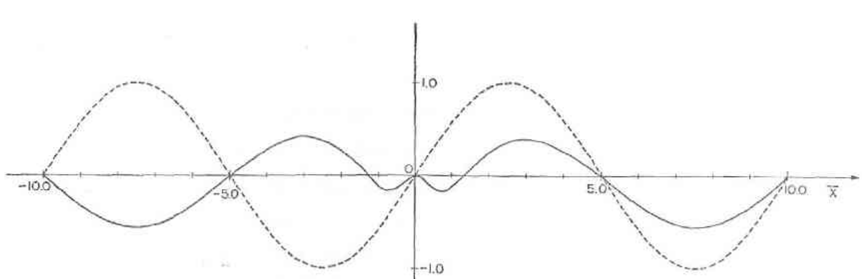

Fig. 4a $\varepsilon \approx 0$

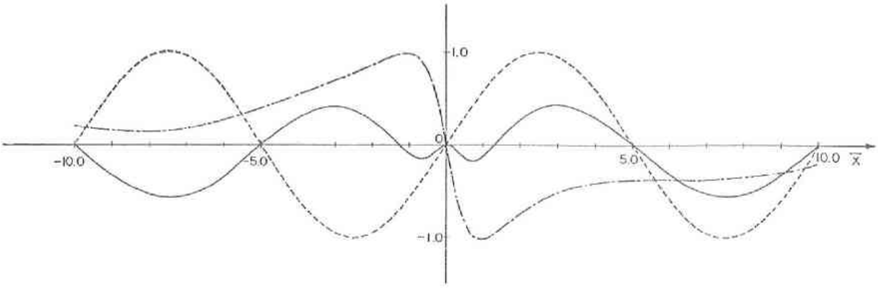

Fig. 4b $\varepsilon=0.1$

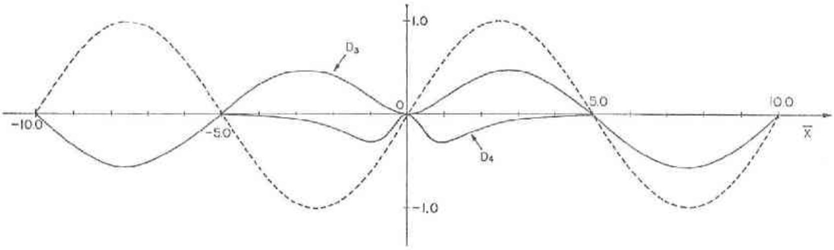

Fig. 4c

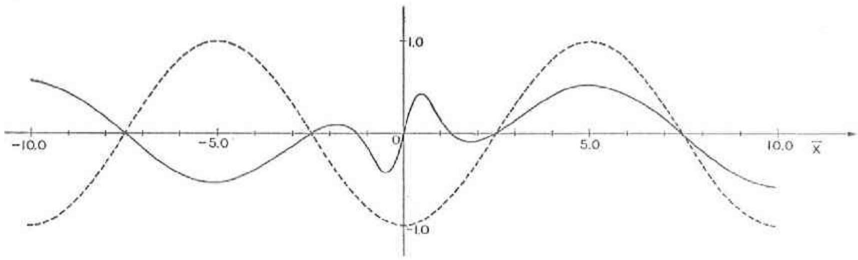

Fig. 5a $\varepsilon\approx 0$

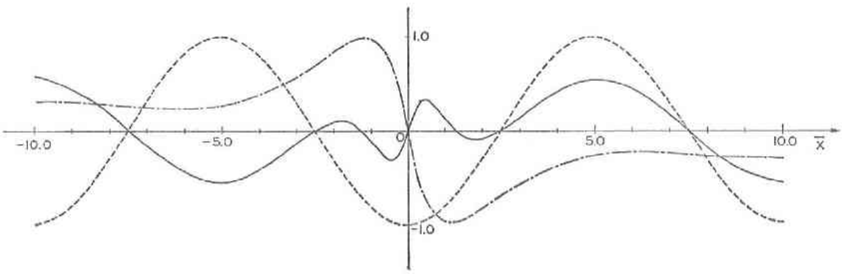

Fig. 5b $\varepsilon=0.1$

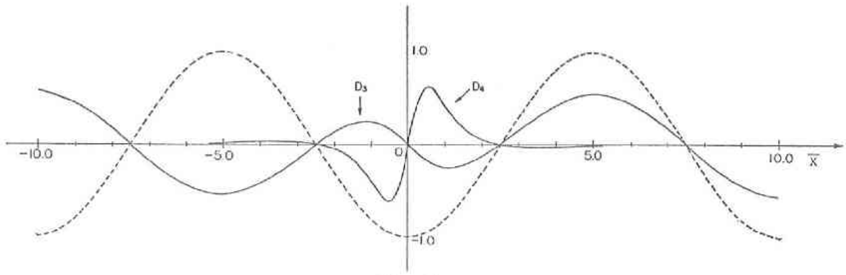

Fig. 5c

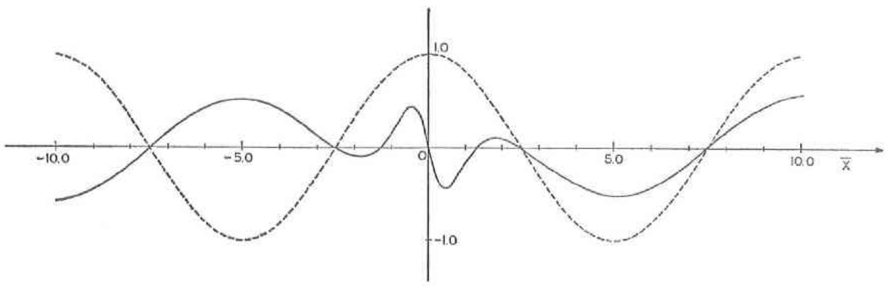

Fig. 6a $\varepsilon \approx 0$

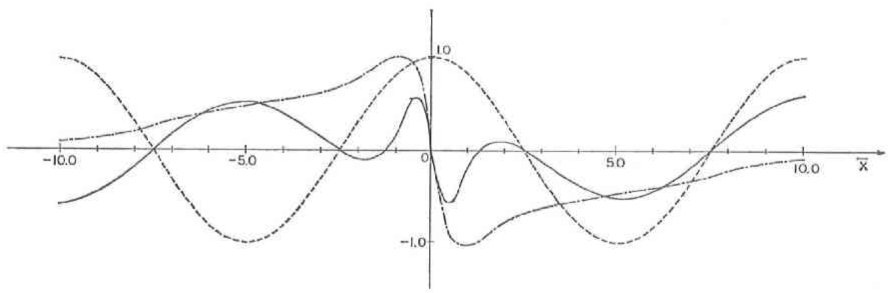

Fig. 6b $\varepsilon = 0.1$

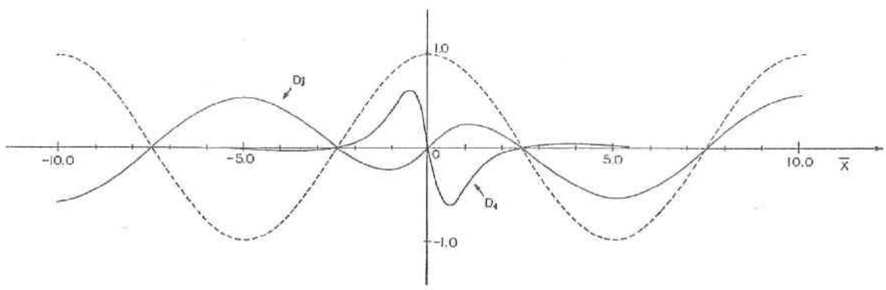

Fig. 6c

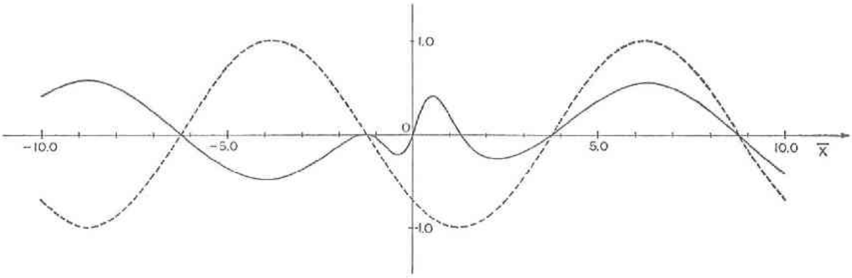

Fig. 7a $\varepsilon \approx 0$

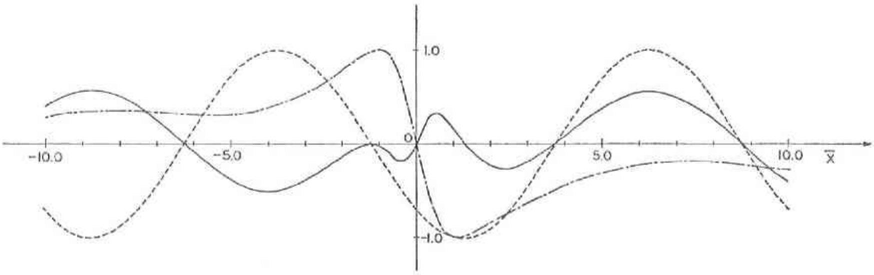

Fig. 7b $\varepsilon = 0.1$

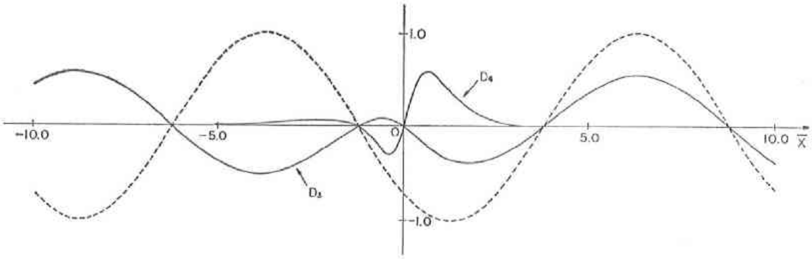

Fig. 7c

(2) *Arctangent type of topography*

The following cases were calculated:

$F(H \cdot \bar{x})$	$\tan^{-1}(\bar{x}-5)$	$-\tan^{-1}(\bar{x}-5)$
Figure	Fig. 8	Fig. 9

The values of D_2 for $\varepsilon \approx 0$ and $\varepsilon = 0.1$ are shown in the same figures, in which it is recognized that D_2 shows the only difference in respect of the amplitude at the spot above a source. A characteristic feature is noted at the place where topography croses over the X axis. There is almost no topographical influence on the plane of more than $\bar{x} = 10$,

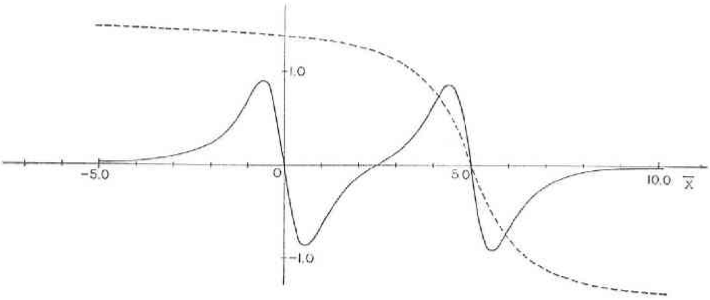

Fig. 8a $\varepsilon \approx 0$

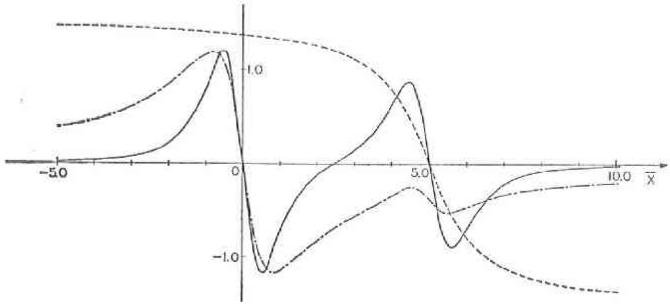

Fig. 8b $\varepsilon = 0.1$

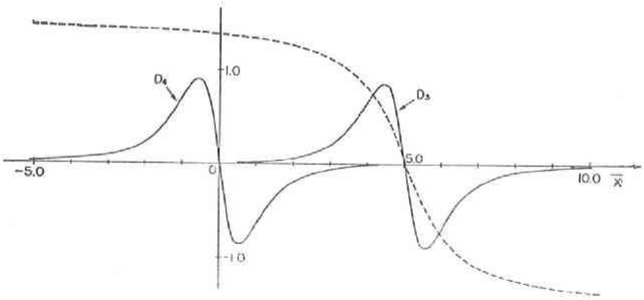

Fig. 8c

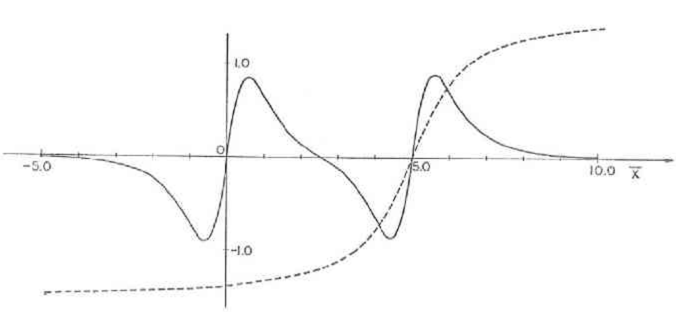

Fig. 9a $\varepsilon \approx 0$

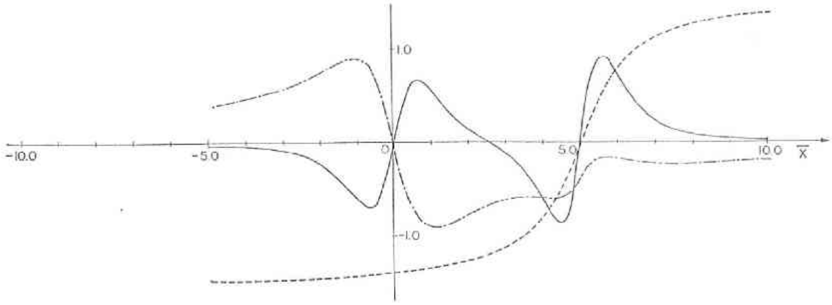

Fig. 9b $\varepsilon=0.1$

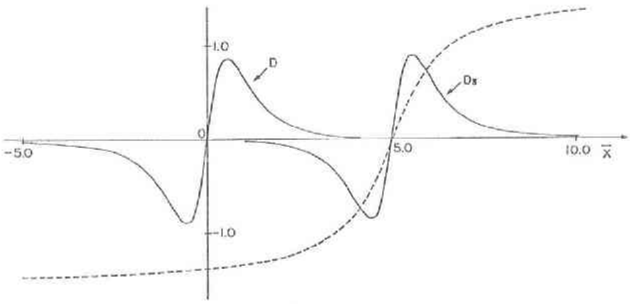

Fig. 9c

(3) *Exponent type of topography*

The following cases were calculated:

$F(H\cdot\bar{x})$	$e^{-\bar{x}^2}$	$-e^{-\bar{x}^2}$	$e^{-(\bar{x}-1)^2}$	$-e^{-(\bar{x}-1)^2}$	$e^{-(\bar{x}-4)^2}$	$-e^{-(\bar{x}-4)^2}$
Figure	Fig. 10	Fig. 11	Fig. 12	Fig. 13	Fig. 14	Fig. 15

The values of D_2 for $\varepsilon\approx0$ and $\varepsilon=0.1$ in Figs. 10, 11, 14, and 15 are shown, from which D_2 is known to depend on the form of topography. In the case of the $\pm e^{-(\bar{x}-4)^2}$ type, viz., the isolated convex and concave types, it is noteworthy that the top of the convex part diminishes remarkably the displacement of the plane surface and the top of the concave part increases remarkably the displacement of the plane surface.

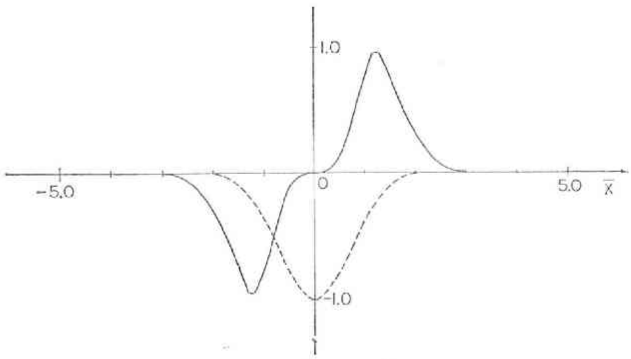

Fig. 10a $\varepsilon\approx0$

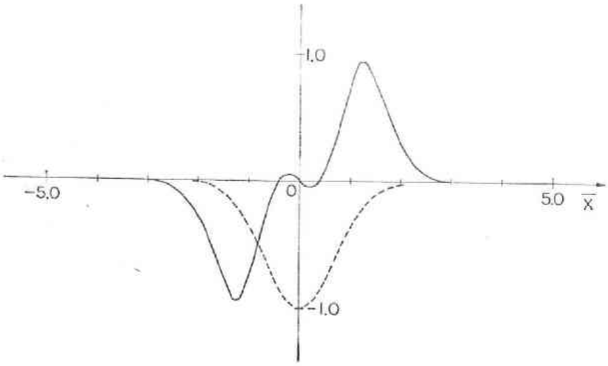

Fig. 10b $\varepsilon=0.1$

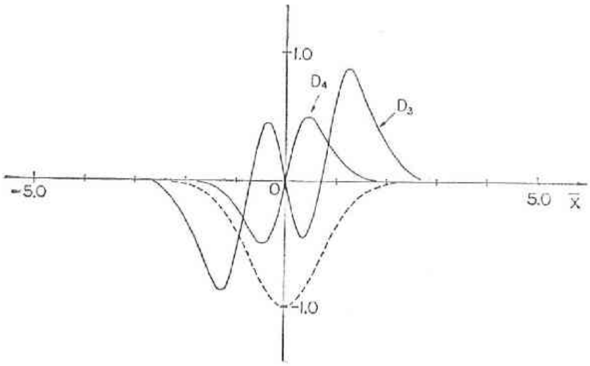

Fig. 10c

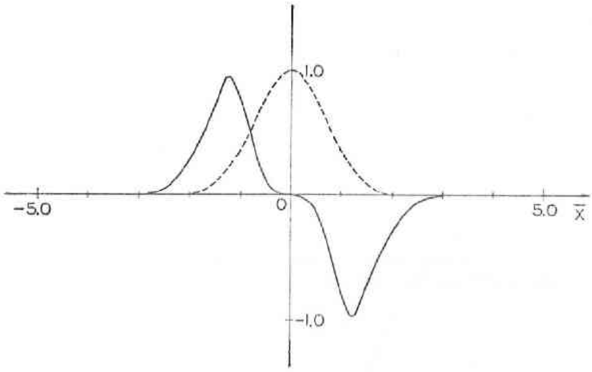

Fig. 11a $\varepsilon\approx 0$

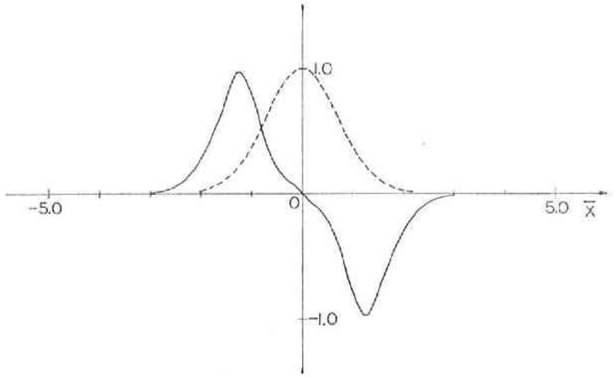

Fig. 11b $\varepsilon=0.1$

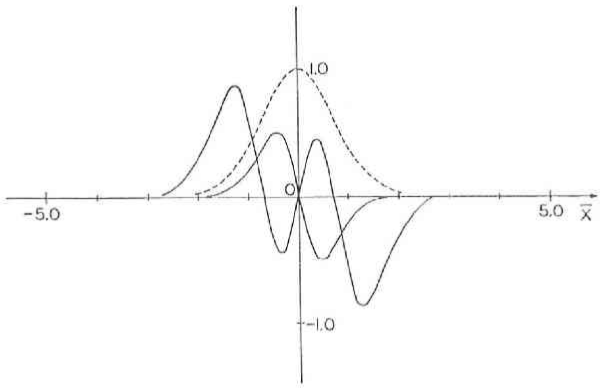

Fig. 11c

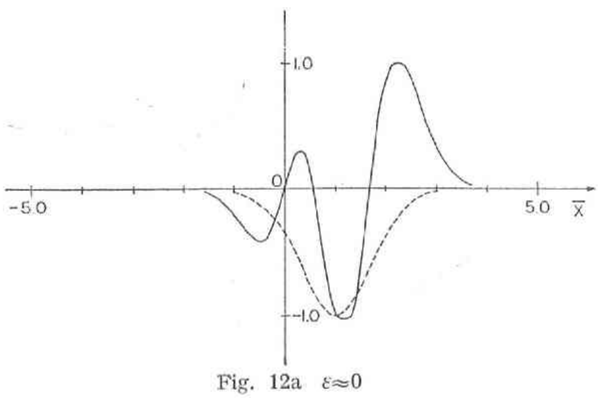

Fig. 12a $\varepsilon\approx 0$

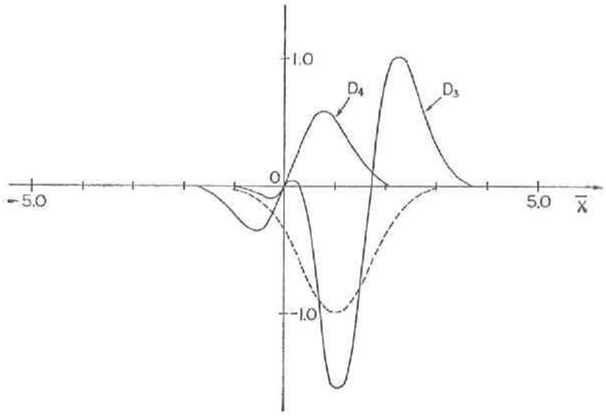

Fig. 12b

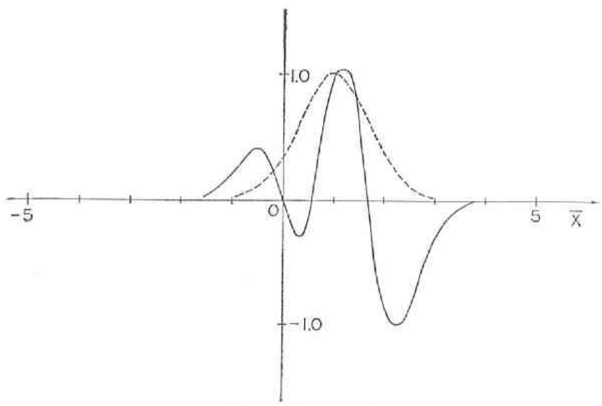

Fig. 13a $\varepsilon \approx 0$

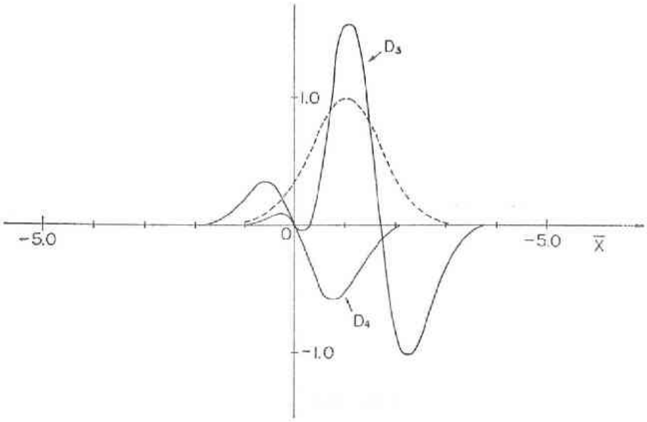

Fig. 13b

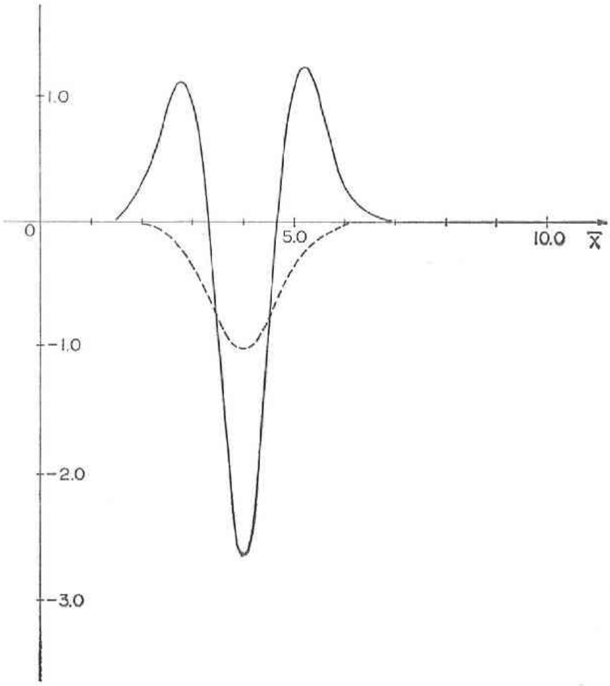

Fig. 14a $\varepsilon \approx 0$

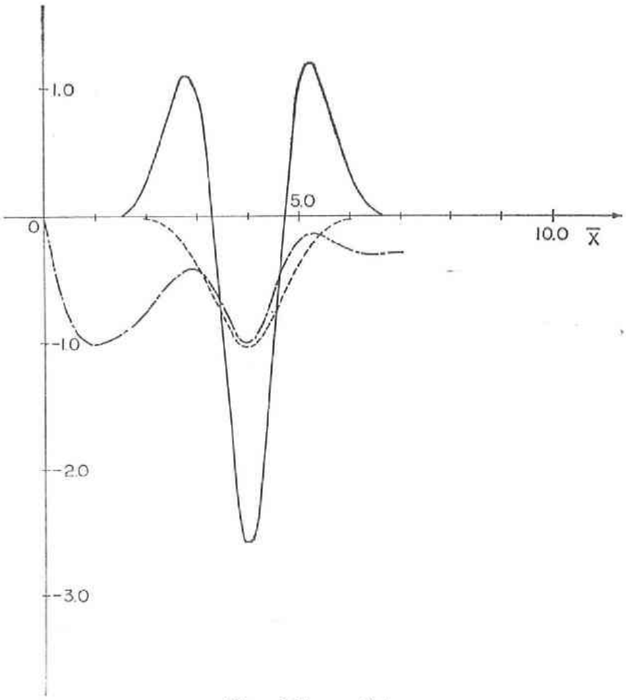

Fig. 14b $\varepsilon = 0.1$

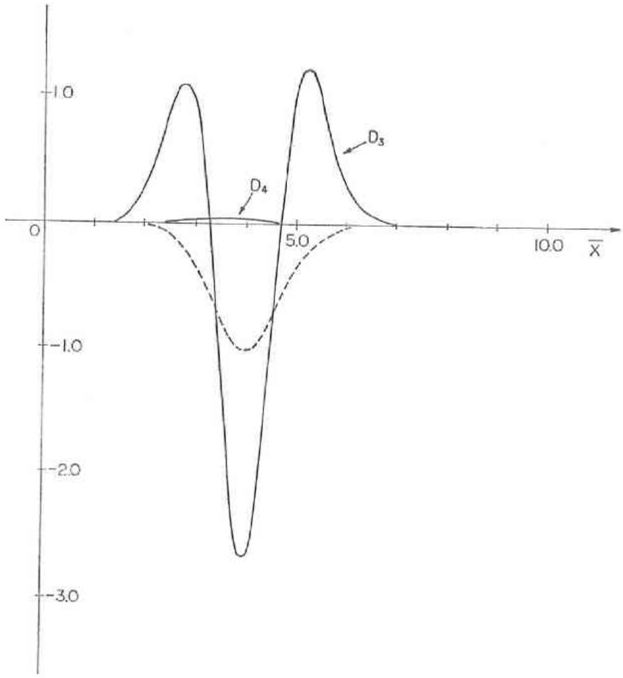

Fig. 14c

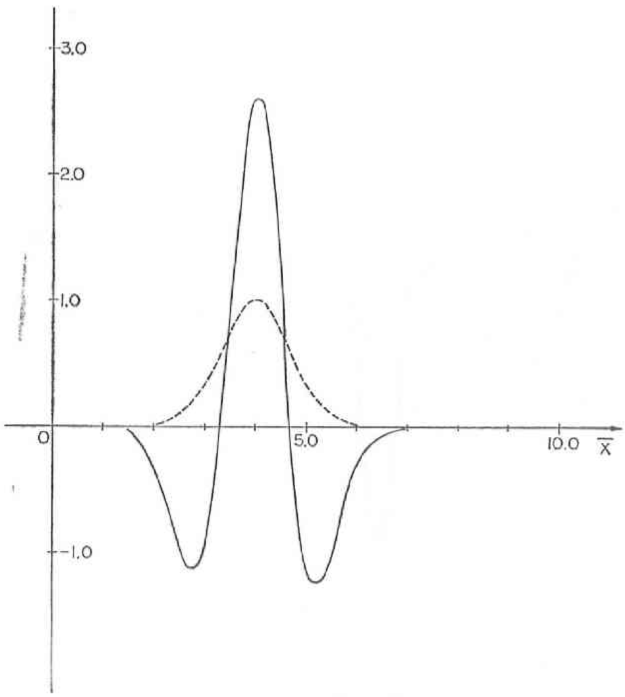

Fig. 15a $\varepsilon \approx 0$

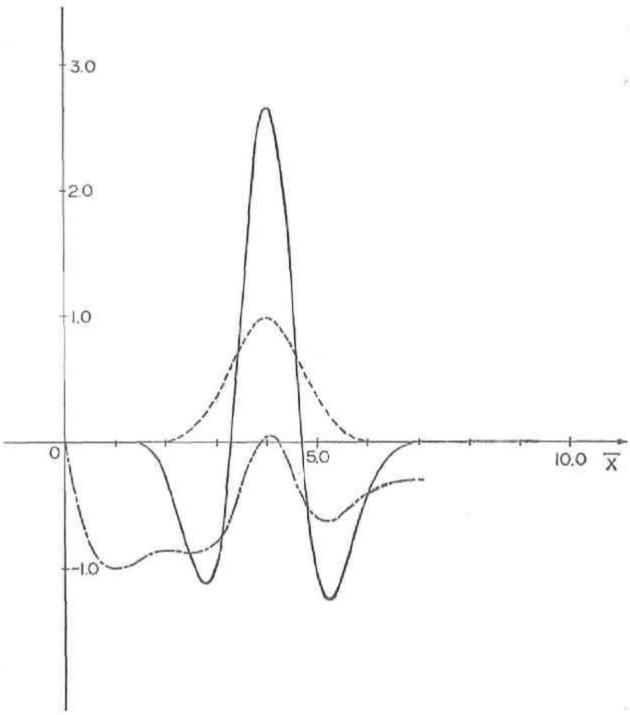

Fig. 15b $\varepsilon=1.0$

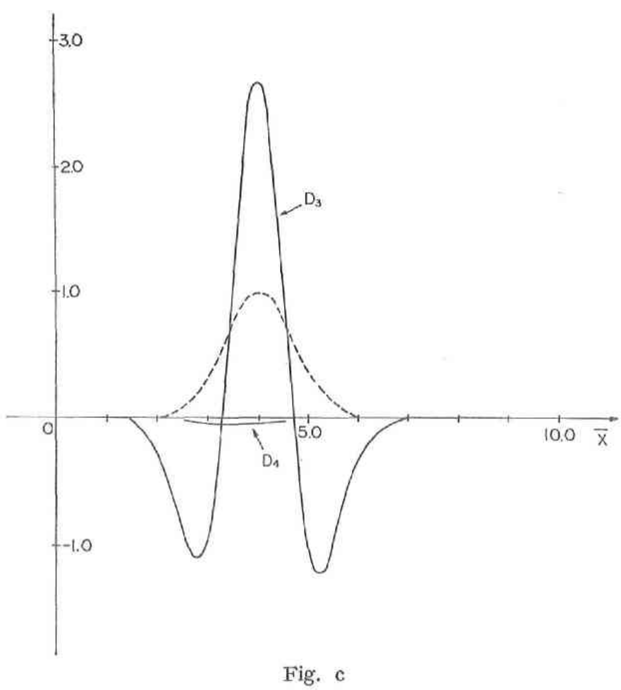

Fig. c

4. Summary

The displacements of the elastic half space on the surfaces of the sine, cosine, arc-tangent and exponent types have been investigated theoretically for the two-dimentional SH-torque source and the results obtained are illustrated graphically.

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