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Conversion of Explosive Sound into Seismic Waves at the Ocean Bottom

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Abstract: The refraction of explosive sound from a line source in a liquid into a solid is solved by an exact evaluation of formal solution of wave equation, especially on the variation of wave amplitude due to the change in positions of source and receiver. Wave patterns at various depths in the solid are given. Amplitude decrease of the pseudo Rayleigh and the Stoneley waves with increasing depth indicates that most energy of the waves are confined within the depth of the order of one wave length.

The theoretical results are applied to an actual case. It has been frequently reported that a big amplitude of shear wave was observed in the case of explosion at the bottom of water. Theory can check whether or not this wave may be taken as pseudo Rayleigh wave of which the travel time is close to that of shear wave. This idea, however, is not supported by theory especially in the small change in amplitude of pseudo Rayleigh wave due to the change of position of source.

1. Introduction

The propagation of transient wave in liquid-solid half spaces in contact has been studied by Roever, Vining and Strick (1959) and Emura (1960). According to their results, the disturbances on the interface before the direct pulse arrives are similar to those in Lamb's study (Lamb, 1904) when concentrated force acts vertically downwards on the surface of solid. Their discussions, however, were concerned with the motions in liquid and on the interface. In the present paper, the refracted wave in a solid from a line source in liquid will be theoretically treated by the exact evaluation of integrals in formal solution of the wave equation and the effect of change in seismic wave velocities in solid and the positions of source and receiver will be also discussed.

Recently in Japan and some other countries, many observations have been done on the seismic wave generated by explosions in water. It may be of some interest, therefore, to give a mathematical basis on the wave forms in such cases. An application of the theoretical result to a practical problem will be also discussed in this paper.

2. Exact transient solution

Consider a liquid half space of density ρ_1 and compressional wave velocity α_1, superposed upon a solid half space of density ρ_2 and compressional and shear wave velocities α_2 and β_2 (Fig. 1). The subscripts 1 and 2 refer to the quantities in liquid and solid respectively. The xy-plane is taken in the horizontal plane interface and the positive z-axis is directed toward the liquid. A line source located at $x=0$ and $z=h$ is assumed to emit an explosive sound, the motion being independent of y.

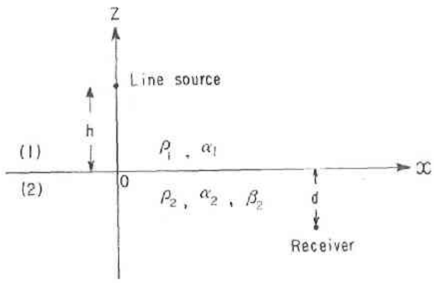

Fig. 1.

Let us define the Laplace transform by

$$\bar{f}\,(x, z, p) = \int_0^\infty f\,(x, z, t)\, e^{-pt}\, d\,t \tag{1}$$

and all the transformed quantities are denoted by superimposed bars in this paper. The potential $\bar{\phi}_0$ for the initial pulse satisfying the transformed equation of motion,

$$\frac{\partial^2 \bar{\phi}_0}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{\phi}_0}{\partial R} - \frac{p^2}{{\alpha_1}^2}\bar{\phi}_0 = 0\,, \qquad R = \sqrt{x^2 + (h - z)^2} \tag{2}$$

is given by

$$\bar{\phi}_0 = \bar{f}\,(p)\, K_0\,(p\, R\,/\,\alpha_1)\,, \tag{3}$$

where $\bar{f}(p)$ is the transform of a time function which should be selected as to represent the time variation of initial pulse, and K_0 the modified Bessel function of the second kind and zero order. Taking the explosive source, we put $\bar{f}(p)=-1/p$, where the negative sign is introduced for the algebraic convenience. Then the radial component of displacement for the initial pulse is expressed by

$$\begin{aligned} \frac{\partial \phi_0}{\partial R} &= 0\,, & t &< R\,/\,\alpha_1\,, \\ &= \frac{t}{R\sqrt{t^2 - (R\,/\,\alpha_1)^2}}\,, & t &> R\,/\,\alpha_1\,. \end{aligned} \tag{4}$$

The transforms of displacements in solid are given as follows (Emura, 1960).

$$\begin{aligned} \bar{u}_2 = &- \frac{2\,\mu^2}{\alpha_1}\, Im \int_0^\infty u\, \left[(2\, u^2 + \mu^2)\, e^{p\eta_{2p} z/\alpha_1} - 2\, \eta_{2p}\, \eta_{2s}\, e^{\,p\eta_{2s} z/\alpha_1}\right] \\ &\times \frac{1}{F\,(u)}\, e^{-p(\eta_{1p} h + iux)/\alpha_1}\, du\,, \end{aligned} \tag{5}$$

$$\begin{aligned} \bar{w}_2 = &- \frac{2\,\mu^2}{\alpha_1}\, Re \int_0^\infty \eta_{2p}\, \left[(2\, u^2 + \mu^2)\, e^{p\eta_{2p} z/\alpha_1} - 2\, u^2 e^{p\eta_{2s} z/\alpha_1}\right] \\ &\times \frac{1}{F(u)}\, e^{-p(\eta_{1p} h + iux)/\alpha_1}\, du\,, \end{aligned} \tag{6}$$

$$
\begin{gathered}
F(u) = (\rho_2/\rho_1)\,\eta_{1p}\left\{(2u^2+\mu^2)^2 - 4u^2\eta_{2p}\eta_{2s}\right\} + \mu^4\eta_{2p}\,, \\
\eta_{1p} = \sqrt{u^2+1}\,, \qquad \eta_{2p} = \sqrt{u^2+\nu^2}\,, \qquad \eta_{2s} = \sqrt{u^2+\mu^2}\,, \\
Re\,\eta_{1p} > 0\,, \qquad Re\,\eta_{2p} > 0\,, \qquad Re\,\eta_{2s} > 0\,, \\
\nu = \alpha_1/\alpha_2, \qquad \mu = \alpha_1/\beta_2\,.
\end{gathered}
\tag{7}
$$

The inverse transformation of these expressions is performed by means of Garvin's method (Garvin, 1956). The integration with respect to the variable u is reduced to that over the time t_ν or t_μ by the replacement,

$$t_\nu = \left\{\sqrt{u^2+1}\,h - \sqrt{u^2+\nu^2}\,z + iux\right\}/\alpha_1\,, \tag{8}$$

or

$$t_\mu = \left\{\sqrt{u^2+1}\,h - \sqrt{u^2+\mu^2}\,z + iux\right\}/\alpha_1\,. \tag{9}$$

Since the integrand is independent of p except for the exponent, the inverse transformation gives the delta function with respect to time, and the final expressions for the displacements are

$$u_2 = -\frac{2\mu^2}{\alpha_1}\,Im\left[(2u^2+\mu^2)\frac{du}{dt_\nu} - 2\eta_{2p}\eta_{2s}\frac{du}{dt_\mu}\right]\frac{u}{F(u)}\,, \tag{10}$$

$$w_2 = -\frac{2\mu^2}{\alpha_1}\,Re\left[(2u^2+\mu^2)\frac{du}{dt_\nu} - 2u^2\frac{du}{dt_\mu}\right]\frac{\eta_{2p}}{F(u)}\,. \tag{11}$$

These displacements are computed in terms of $u(t_\nu)$ and $u(t_\mu)$ which satisfy the relations (8) and (9) for real t_ν and t_μ.

As the singularities of integrands in (5) and (6) give the major contributions to the integrals, the travel time of each pulse is given by the values at corresponding singularities as

$$
\begin{aligned}
t_{p1} &= R/\alpha_1\,, \\
t_{p1p2} &= (h\cos\theta - z\sqrt{\nu^2-\sin^2\theta} + x\sin\theta)/\alpha_1\,, \\
t_{p1p2s2} &= (h\sqrt{1-\nu^2} - z\sqrt{\mu^2-\nu^2} + x\nu)/\alpha_1\,, \\
t_{p1s2} &= (h\cos\theta - \mu z\sqrt{1-(1/\mu)^2\sin^2\theta} + (x/\nu)\sin\theta)/\alpha_1\,,
\end{aligned}
\tag{12}
$$

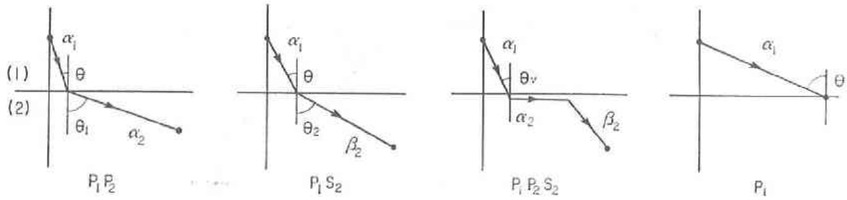

Fig. 2. Minimum time paths.

where θ is the angle of incidence. The ray paths for these pulses are illustrated in Fig. 2.

3. Numerical results

In the numerical computation we fix the values of $\alpha_1=1.5$ km/sec, $\rho_1=1.0$, $\rho_2=2.5$ and Poisson's ratio $\sigma=0.25$ and change the positions of source and receiver and seismic wave velocities in the solid medium, taking the actual case of explosions into consideration. Figs. 3 and 4 show examples of numerical results in the case where source is located at 0.2 km above the interface and the receiver is on the surface at a horizontal distance of 20 km from the source, the shear wave velocity β_2 being taken as parameter.

The wave pattern in Figs. 3 and 4 may be interpreted as follows. The initial motion is due to the head wave P_1P_2 with the travel time of t_{p1p2} and is followed by a gradual recovery. The second big event is the refracted wave P_1S_2 superposed by a big amplitude of pseudo Rayleigh wave PR. The third event is the direct pulse in liquid having an infinitely large amplitude, which is shown by a narrow gap in Figs. 3 and 4. The Stoneley wave comes after P_1 with the arrival time of t_{st} given by Strick and Ginz-

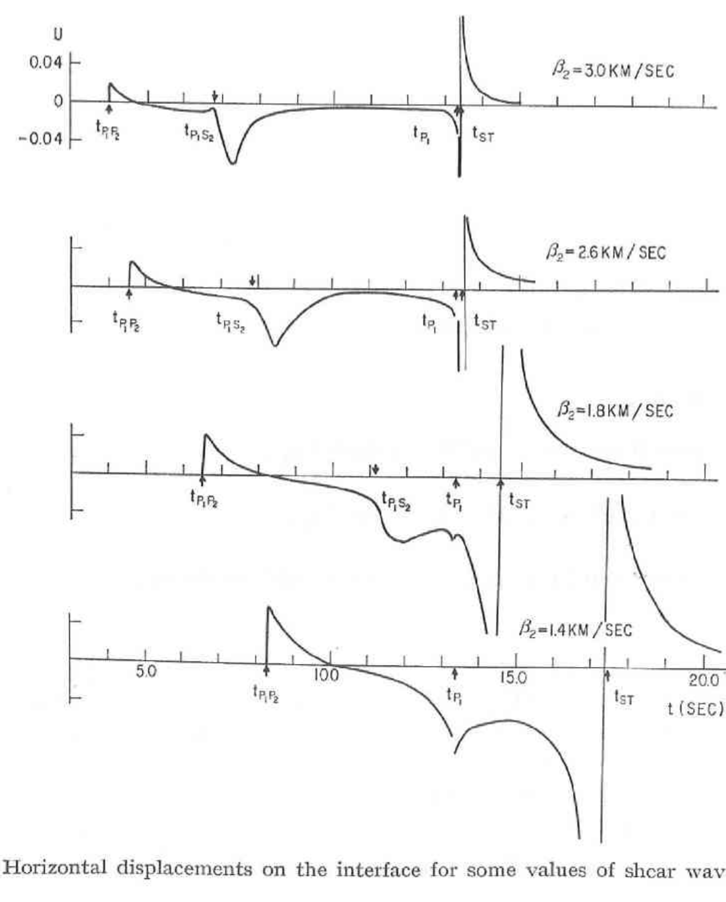

Fig. 3. Horizontal displacements on the interface for some values of shear wave velocities.

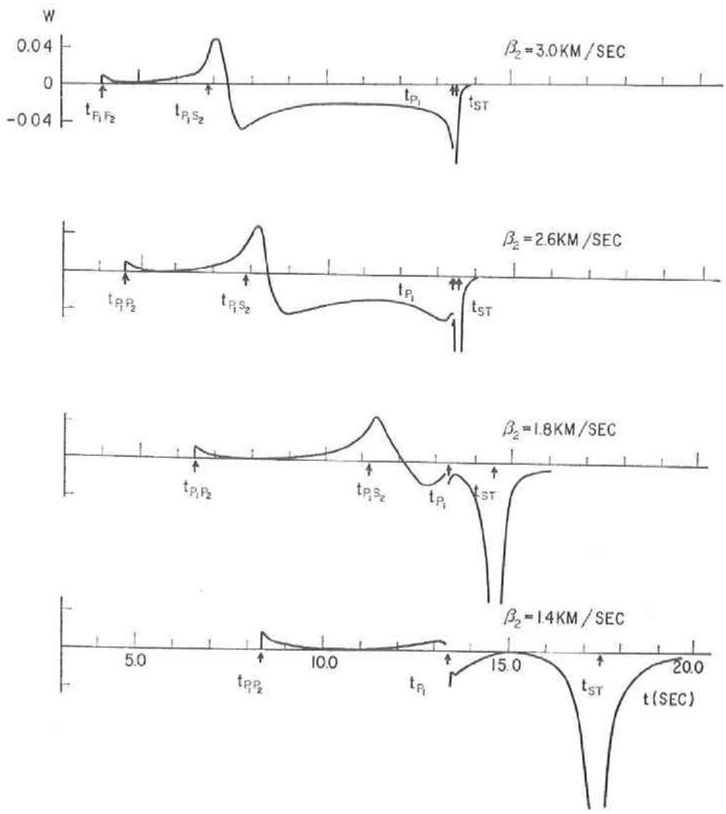

Fig. 4. Vertical displacements on the interface for some values of shear wave velocities.

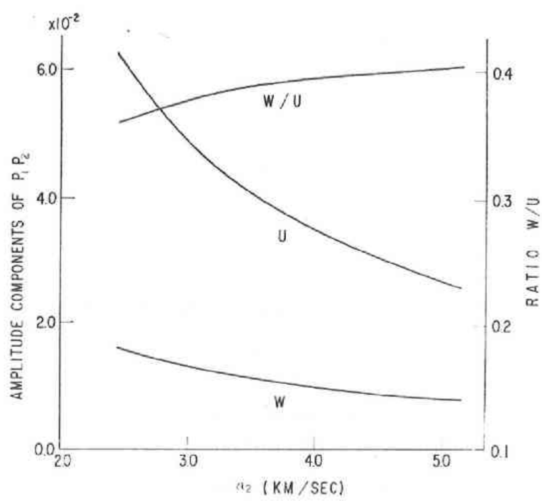

Fig. 5. Horizontal (U) and vertical (W) amplitudes and ratio (W/U) for P_1P_2 wave as a function of compressional wave velocity.

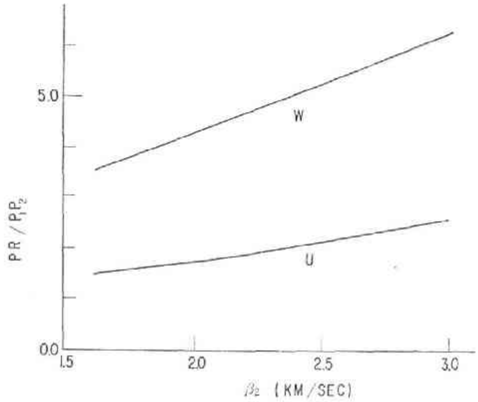

Fig. 6. Ratios of amplitudes of PR and P_1P_2 waves in horizontal and vertical components as a function of shear wave velocity.

barg (1956). In some cases, for example, when $\beta_2 = 3.0$ km/sec, t_{p1} is so close to t_{st} that these two waves are superimposed one over the other and they look like a single event. The separation of the two waves becomes evident in a lower range of β_2.

The horizontal and vertical amplitudes u and w of the head wave P_1P_2 are seen in Fig. 5 as a function of compressional wave velocity α_2, together with the ratio of w/u. Both components decrease with increasing velocity, whereas the ratio is approximately independent of the velocity. The ratios of amplitudes of PR and P_1P_2 in both horizontal and vertical components are given in Fig. 6 against the shear wave velocity β_2 as abscissa. This figure indicates that both ratios increase with velocity in this case.

The variation in wave forms in the solid are given in Figs. 7 and 8, where β_2 is fixed at 3.0 km/sec and the distance of the receiver from the interface is taken as the parameter. Big differences of these wave forms from those on the interface are

(1) The first motion P_1P_2 is followed by a small jump $P_1P_2S_2$ of which the ray is seen in Fig. 2.

(2) The wave P_1S_2 appearing before pseudo Rayleigh wave has an infinite amplitude shown by a break of line in Figs. 7 and 8.

(3) The wave P_1 has a finite amplitude and suddenly vanishes with increasing depth.

(4) The amplitudes of pseudo Rayleigh and Stoneley waves decrease rapidly with the increasing distance from interface.

The orbits of particle motion around t_{p1s2} at various depths are shown in Fig. 9, which indicates that the locus is elliptical retrograde near the interface like the free Rayleigh wave. The variation of amplitudes with depth of PR wave in both components are given in Fig. 10. The pattern is quite similar to that for the Rayleigh wave. If we take the time interval between peak and trough in the vertical component (Fig. 8) as a half period, the wave length is calculated to be 3.0km. Most of the energy of PR waves, therefore, is concluded to be confined within the depth of the order of one wave length.

Fig. 11 shows the variation of amplitude of the disturbance around t_{st} against depth

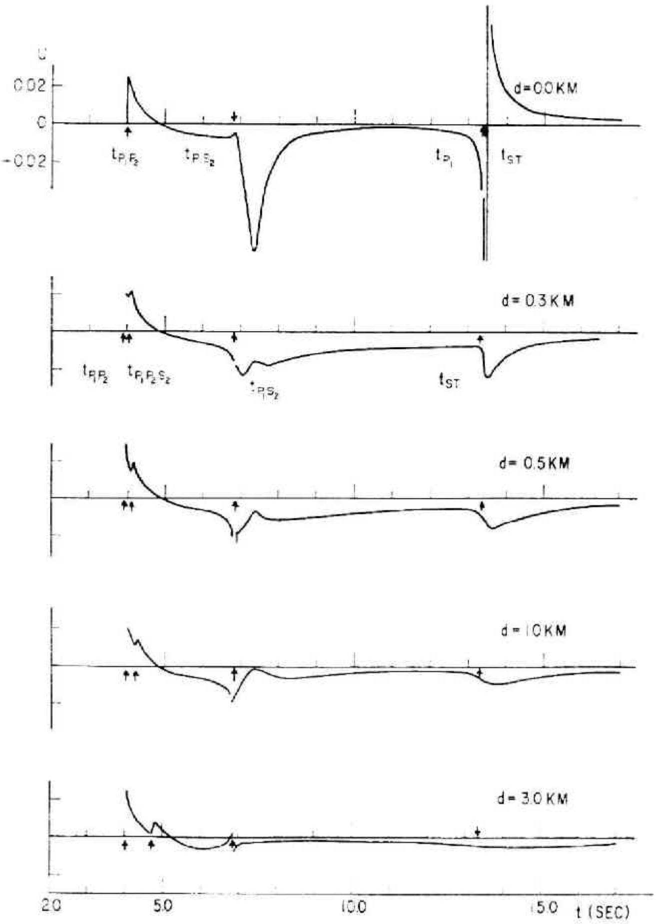

Fig. 7. Horizontal displacements at various depths d in solid.

from the interface. The amplitude decreases exponentially with increasing depth as expected from the theory of propagation of Stoneley wave.

4. Effect of position of source on the amplitude of *PR* wave

As an application of above theoretical discussion to a practical problem, we will consider the so-called shear wave generated by explosion. In recent explosion seismology, shots in water and observation on land system is frequently adopted in Japan, as well as in some other countries. One of the interesting results by this type of observation is that a big amplitude of wave is observed around the arrival time of S wave, when the explosive is detonated at the bottom of water. This wave is commonly thought to be a shear wave but the generation mechanism of a shear wave by explosion was not yet clearly explained. Steinhart and Meyer (1961) have pointed out an important feature that the amplitude of this shear wave is large in bottom explosion, while the wave

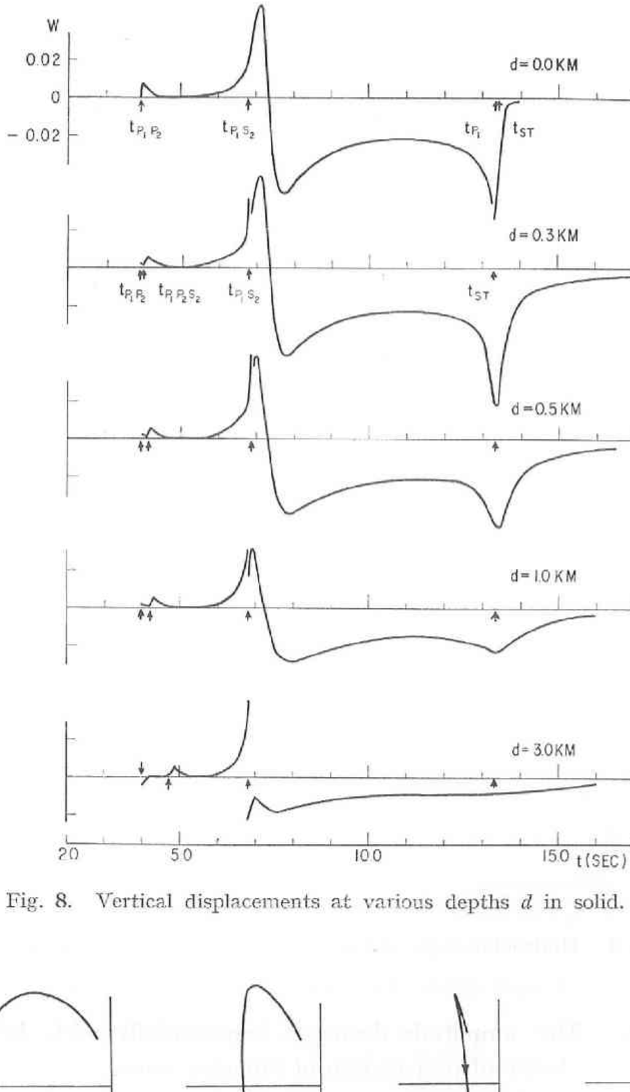

Fig. 8. Vertical displacements at various depths d in solid.

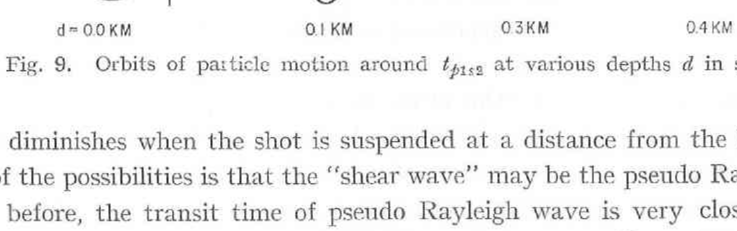

Fig. 9. Orbits of particle motion around $t_{p_1s_2}$ at various depths d in solid.

amplitude diminishes when the shot is suspended at a distance from the bottom.

One of the possibilities is that the "shear wave" may be the pseudo Rayleigh wave. As stated before, the transit time of pseudo Rayleigh wave is very close to that of converted S wave from P in water and these two waves are usually superposed one over the other. The pseudo Rayleigh wave has commonly a large amplitude in comparison

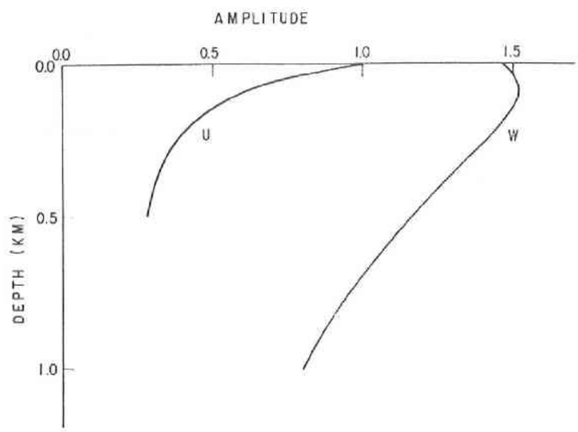

Fig. 10. Variation of amplitude of PR wave in horizontal (U) and vertical (W) components versus depth.

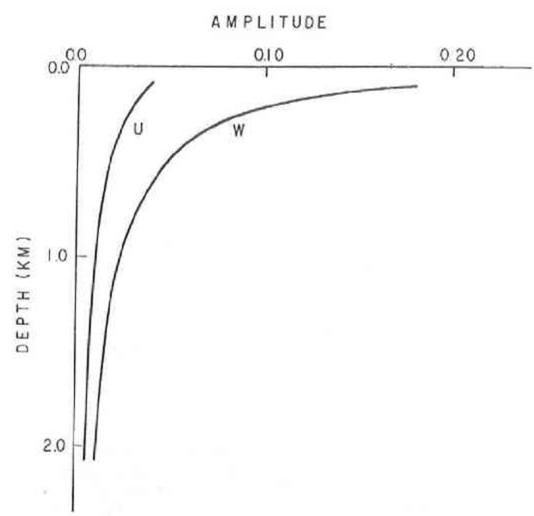

Fig. 11. Variation of amplitude of disturbance around t_{st} in horizontal (U) and vertical (W) components versus depth.

with initial motion. These results are compatible with the observation. If the theory can explain the feature of decrease in amplitude in suspended explosion, therefore, the above idea would be valid. In order to examine the idea, the relation between the amplitude of pseudo Rayleigh wave and the distance of source from the interface is studied. We compute the wave forms on the interface for several source positions ranging from $0.2 \sim 0.001$ km from the interface and measure the maximum amplitude of pseudo Rayleigh wave on the calculated seismograms. The result is given in Fig. 12 for both horizontal and vertical components, the ratio of PR/P_1P_2 being taken in ordinate for the convenience of comparison with actual observation. This figure shows obviously that the effect of source position is very small in the range of distance

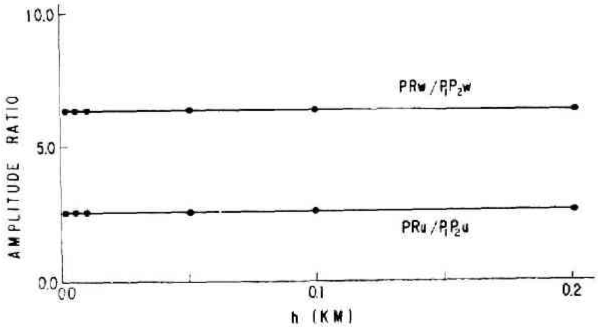

Fig. 12. Ratio of amplitudes of PR and P_1P_2 waves in each component as a function of height of source.

from the interface considered. Therefore, the possibility of a pseudo Rayleigh wave being the observed big shear wave should be abandoned.

This small change in amplitude can be deduced purely mathematically, too. Putting $z=0$ in Eqs. (8) and (9), we have

$$t_\nu = t_\mu = (h\sqrt{u^2+1} + iux) / a_1 . \tag{13}$$

If we denote $t_\nu = t_\mu = t_2$, then

$$\frac{du}{dt_2} = -i\frac{a_1\sqrt{u^2+1}}{H(x,h,u)}, \tag{14}$$

where

$$H(x,h,u) = x\sqrt{u^2+1} - ihu .$$

Solving with respect to u, we obtain

$$u = \frac{a_1}{x^2+h^2}\left\{-ixt_2 + h\sqrt{t_2^2 - \frac{x^2+h^2}{a_1^2}}\right\}. \tag{15}$$

As far as the motions before the arrival time of direct pulse P_1 are concerned, we may take the ranges

$$\begin{gathered} t_{p_1p_2} < t_2 < t_{p_1}, \\ -i\nu < u < -ix/\sqrt{x^2+h^2}, \\ x\sqrt{1-\nu^2} - h\nu > H(x,h,u) > 0 . \end{gathered} \tag{16}$$

For the disturbances expected to arrive at the time

$$t = (h\sqrt{1-l^2} + lx)/a_1, \qquad \nu < l < x/\sqrt{x^2+h^2}$$

the controling factor of amplitude is $1/H(x,h,l)$ and the amplitude decrease at large epicentral distance is given by

$$\frac{1}{x\sqrt{1-l^2}}\left(1+\frac{hl}{x\sqrt{1-l^2}}\right) \qquad \text{for } x \gg \frac{hl}{\sqrt{1-l^2}}\,. \tag{17}$$

This equation implies that the effect of the change in height of source on wave amplitude is almost negligible.

After all, the generation of shear wave whose amplitude is large in bottom explosion and small in suspended one is not expected in such a simple model of structure as treated here. The mechanism of generation should be attributed to some other factors.

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REFERENCES

Emura, K., 1960: Propagation of the d sturbances in the medium consisting of semi-infinite liquid and solid. *Sci. Rept. Tohoku Univ., Ser.* 5, **12**, 63–100.

Garvin, W.W., 1956: Exact transient solution of the buried line source problem. *Proc. Roy. Soc. London, Ser. A*, **234**, 528–541.

Lamb, H., 1904: On the propagation of tremors over the surface of an elastic solid. *Phil. Trans. Roy. Soc. London, Ser. A*, **203**, 1–42.

Roever, W.L., T.F. Vining and E. Strick, 1959: Propagation of elastic wave motion from an impulsive source along a fluid/solid interface. *ibid.*, **251**, 455–523.

Steinhart, J.S. and R.P. Meyer, 1961: *Explosion Studies of Continental Structure. Carnegie Institution of Washington Publication* 622.

Strick, E. and A.S. Ginzbarg, 1956: Stoneley-wave velocities for a fluid-solid interface. *Bull. Seism. Soc. Am.*, **46**, 281–292.