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Absorption Coefficients of Stoneley Waves for Low-Loss Solid and Perfect Fluid Half-Spaces in Contact

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Abstract: The dissipation of Stoneley waves is theoretically studied for the case where a perfect fluid half-space is superposed upon an imperfect elastic solid. A relation between the absorption coefficient for Stoneley waves and those for compressional and shear waves in low-loss solid is derived.

1. Introduction

Normal mode solution for the propagation of elastic waves in a medium consisting of a fluid layer over a granitic substratum gives a high-frequency train of Stoneley waves travelling with a velocity slightly less than that of sound in water (Press and Ewing, 1950). The amplitudes of these waves due to a distant impulsive point source having a flat spectrum are zero at the arrival time of sound but increase to large amplitudes shortly thereafter. Considerable amount of energies in these waves is probably converted into surface and body waves at the shore and may be recorded with a sufficient amplitude when a seismological station is located near the shore.

Then it is of interest, even from the practical point of view, to relate the absorption coefficients of Stoneley waves to those of compressional and shear waves for a interface of fluid and solid with internal dissipation of energy. Press and Healy (1957) and Macdonald (1959) derived the expressions for low-loss solid half-space relating the absorption coefficients for Rayleigh waves to those for compressional and shear waves, the elastic velocities being taken as parameters. The conclusion of Press and Healy is that the theoretically derived expression fits reasonably well the absorption coefficients determined from ultrasonic experiment for Plexiglas sheets.

In the present paper, the absorption coefficients of Stoneley waves will be theoretically studied for a perfect fluid half-space overlying a low-loss solid half-space characterized by the specific dissipation coefficient $1/Q \ll 1$. The dependence of absorption on wave velocities, densities and Poisson's ratio, which is a parameter for the case of Rayleigh waves, will be discussed for the practically interesting ranges of these parameters.

2. Specific dissipation coefficients

Ignoring the internal dissipation, the velocity of Stoneley waves γ in fluid and solid half-spaces in contact is given by the equation

$$\delta\sqrt{1-a_1^2}\left[(2-b_2^2)^2-4\sqrt{1-a_2^2}\sqrt{1-b_2^2}\right]+b_2^4\sqrt{1-a_2^2}=0\,, \tag{1}$$

where $\delta=\rho_2/\rho_1\,,\quad a_1=\gamma/\alpha_1\,,\quad a_2=\gamma/\alpha_2,\quad b_2=\gamma/\beta_2\,,$

ρ is the density, and α and β the velocities of compressional and shear waves. The subscripts 1 and 2 refer to the quantities for fluid and solid respectively.

The absorption of progressive waves with time dependence of exp $(i2\pi ft)$ in a dissipative solid is written in the expression of displacement as

$$\exp(-kr),\quad k=\pi f/Qv\,, \tag{2}$$

where k is the absorption coefficient, f the frequency, r the distance along the ray path and v the wave velocity. It is known that Q is almost independent of frequency over the range of frequencies in seismic waves. As Press and Healy (1957) pointed out, the effect of internal friction in solid is introduced by transformation from real to complex velocities as

$$\begin{aligned}\alpha_2\rightarrow\tilde{\alpha}_2&=\alpha_2\sqrt{1+i/Q_p}\,,\\ \beta_2\rightarrow\tilde{\beta}_2&=\beta_2\sqrt{1+i/Q_s}\,,\end{aligned} \tag{3}$$

where the subscripts p and s refer to the quantities for compressional and shear waves in solid respectively.

The absorption in a viscous fluid, for which the hydrostatic viscous coefficient can be neglected, is expressed by the factor

$$\exp(-k_f r),\quad k_f=8\pi^2\nu f^2/3\rho_1\alpha_1^3\,, \tag{4}$$

where ν is the distortional viscosity coefficient. The complex velocity of sound in such a viscous fluid, which is formally analogous to the velocities of elastic waves in (3), is given by the replacement

$$\alpha_1\rightarrow\tilde{\alpha}_1=\alpha_1\sqrt{1+i/Q_f}\,, \tag{5}$$

where

$$1/Q_f=8\pi\nu f/3\rho_1\alpha_1^2\,. \tag{6}$$

It must be noted that $1/Q_f$ is proportional to f in contrast with $1/Q_p$ and $1/Q_s$, which are almost independent of f. With the assumption of low-loss media, the complex velocity of Stoneley waves may be represented as

$$\gamma\rightarrow\tilde{\gamma}=\gamma\sqrt{1+i/Q_{st}}\,. \tag{7}$$

Then $1/Q_{st}$ is determined from the corresponding Stoneley equation for dissipative media,

$$\delta\sqrt{1-\frac{\gamma^2}{\alpha_1^2}\frac{1+i/Q_{st}}{1+i/Q_f}}\left[\left(2-\frac{\gamma^2}{\beta_2^2}\frac{1+i/Q_{st}}{1+i/Q_s}\right)^2-4\sqrt{1-\frac{\gamma^2}{\alpha_2^2}\frac{1+i/Q_{st}}{1+i/Q_p}}\right.$$
$$\left.\times\sqrt{1-\frac{\gamma^2}{\beta_2^2}\frac{1+i/Q_{st}}{1+i/Q_s}}\right]+\frac{\gamma^4}{\beta_2^4}\left(\frac{1+i/Q_{st}}{1+i/Q_s}\right)^2=0\,. \tag{8}$$

Since the values of $1/Q_s$ in the Earth's interior are estimated to be of the order of 10^{-2}

or less (e.g., Anderson and Kovach, 1964), we may expand the factors in (7) neglecting terms of higher order than $1/Q$. The real part of the resulting Stoneley equation is identical with (1). To the same order of approximation, therefore, it can be shown that the Stoneley waves in low-loss media are not dispersive in the frequency range considered. The imaginary part of the equation after simplification by using (1) relates $1/Q$ of Stoneley waves to those of body waves as follows:

$$\Gamma \frac{1}{Q_{st}} = m_1 \frac{1}{Q_f} + m_2 \frac{1}{Q_p} + n_2 \frac{1}{Q_s}, \tag{9}$$

where

$$m_1 = -\frac{a_1^2 b_2^4 \sqrt{1-a_2^2}}{2(1-a_1^2)}$$

$$m_2 = -\frac{\delta a_2^2 (2-b_2^2)^2 \sqrt{1-a_1^2}}{1-a_2^2}$$

$$n_2 = 2\left[\delta \sqrt{1-a_1^2} \left\{b_2^2 (2-b_2^2) - \frac{b_2^2 \sqrt{1-a_2^2}}{\sqrt{1-b_2^2}}\right\} - b_2^4 \sqrt{1-a_2^2}\right]$$

$$\Gamma = m_1 + m_2 + n_2 .$$

Now the order of magnitude of the absorption coefficient k for body waves is considered. Substituting the typical values for the Earth's crust as

$$Q_p = 250$$

and

$$\alpha_2 = 6.0 \,\text{km} \cdot \text{sec}^{-1},$$

(2) reduces to

$$k_p \sim f \times 10^{-2} \,\text{km}^{-1} .$$

The values for fluid at the normal temperature as

$$\nu = 0.010 \text{ dyne} \cdot \text{sec} \cdot \text{cm}^{-2}$$

$$\alpha_1 = 1.5 \,\text{km} \cdot \text{sec}^{-1}$$

and

$$\rho_1 = 1.0 \,\text{gm} \cdot \text{cm}^{-2}$$

give from (4)

$$k_f \sim f^2 \times 10^{-11} \,\text{km}^{-1} .$$

Refering to (6), it is seen that the value of $1/Q_f$ is negligibly small compared with those of $1/Q_p$ and $1/Q_s$ in the frequency range considered.

Neglecting the term with $1/Q_f$, (9) becomes

$$\frac{1}{Q_{st}} = A \frac{1}{Q_p} + B \frac{1}{Q_s}, \tag{10}$$

where

$$A = \frac{m_2}{\Gamma}, \quad B = \frac{n_2}{\Gamma} .$$

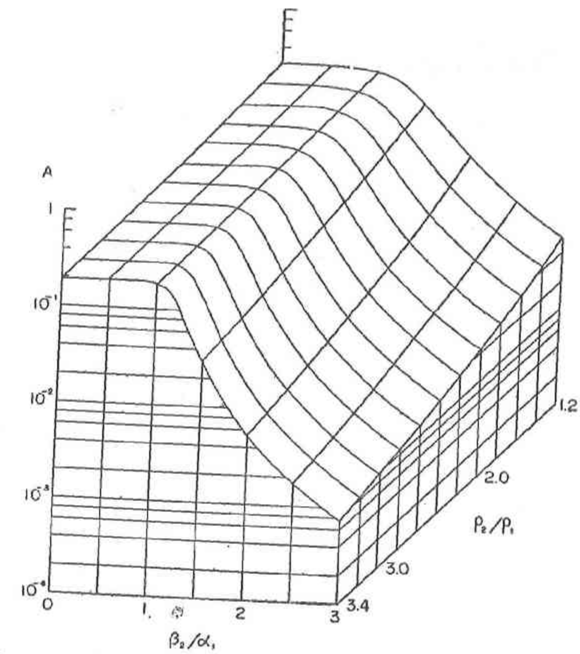

Fig. 1. Coefficient A of the two variables β_2/α_1, ρ_2/ρ_1, for $\sigma=0.25$.

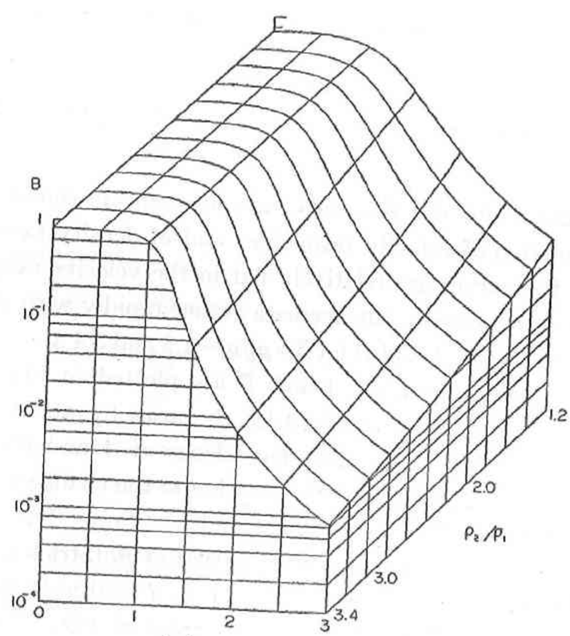

Fig. 2. Coefficient B of the two variables β_2/α_1, ρ_2/ρ_1, for $\sigma=0.25$.

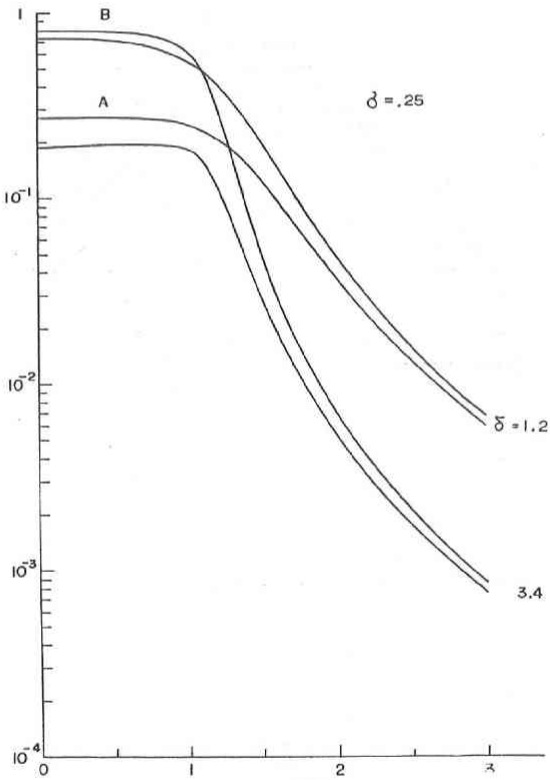

Fig. 3. Coefficients A and B as a function of β_2/a_1 for $\sigma=0.25$.

The coefficients A and B for the Poisson's ratio $\sigma=0.25$ are shown in Figs. 1 and 2 respectively as a function of velocity ratio β_2/a_1 and of density ratio ρ_2/ρ_1. It is seen in these figures that the curves are relatively flat in the velocity range of $\beta_2/a_1<1$ and in the cross section $\beta_2/a_1=$const., but decrease rather rapidly with increasing β_2 in the range of $\beta_2/a_1>1$. In Fig. 3, A and B for $\delta=\rho_2/\rho_1=1.2$ and 3.4 are seen as a function of β_2/a_1. For the same values of ρ_2/ρ_1, A and B are plotted in Fig. 4, Poisson's ratio being 0.45. Over the range of $\sigma=0.25\sim0.45$, A varies by about a factor of tenth, while B is almost independent of σ. The dependence of A and B on density ratio is comparable to each other and relatively insensitive to the change of Poisson's ratio.

The phase velocity γ of Stoneley waves should be lower than the lowest velocity of body waves. Since γ is of the same order as that of β_2 (Strick and Ginzbarg, 1956), or b_2 in (1) is nearly unity in the range of $\beta_2/a_1<1$, m_1/Γ and m_2/Γ in (9) are nearly zero and n_2/Γ is almost unity. Then $1/Q_{st}$ is of the order of $1/Q_s$. As γ in the velocity range of $\beta_2/a_1>1$ tends to a_1 with increasing β_2, a_1 tends to unity, m_2 and n_2 become vanishingly small and m_1 tends to Γ. With increasing β_2, $1/Q_{st}$ reduces to the order of $1/Q_f$ which is negligibly small. Briefly speaking, the specific dissipation coefficients

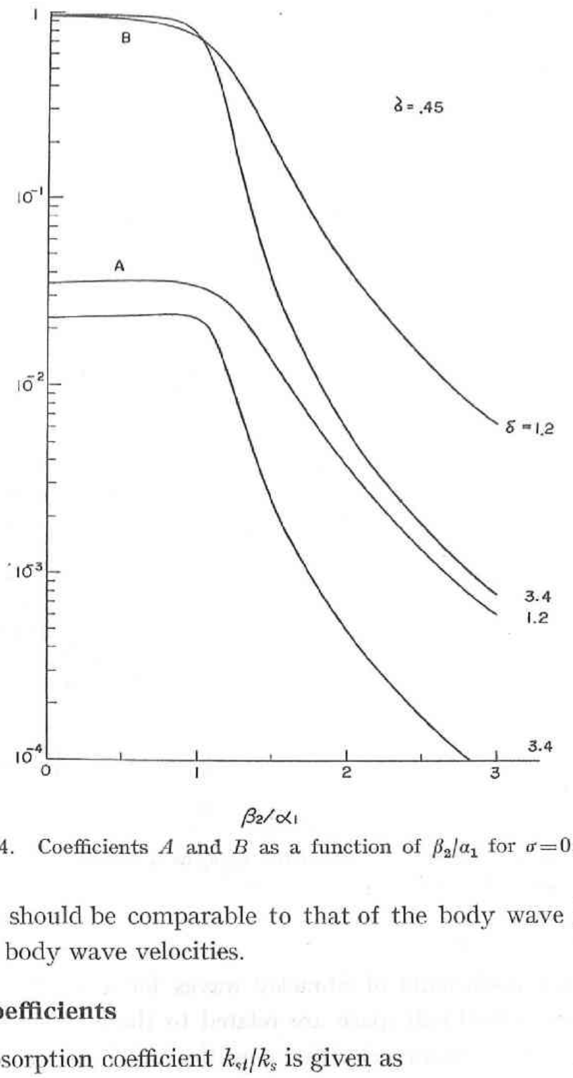

Fig. 4. Coefficients A and B as a function of β_2/α_1 for $\sigma=0.45$.

for Stoneley waves should be comparable to that of the body wave having the lowest velocity among the body wave velocities.

3. Absorption coefficients

The ratio of absorption coefficient k_{st}/k_s is given as

$$\frac{k_{st}}{k_s}=\left(A+B\frac{Q_p}{Q_s}\right)\frac{\beta_2}{\gamma} \tag{11}$$

from (2) and (10) and is seen in Fig. 5 as a function of β_2/α_1 for $Q_p/Q_s=2$ and some values of σ and ρ_2/ρ_1. The general trend of the ratio is similar to that of B in Figs. 3 and 4.

The coefficient k_{st} in the range of $\beta_2/\alpha_1<1$ is of the same order as that of shear waves and almost independent of β_2/α_1, ρ_2/ρ_1 and σ. The sharp dependence of k_{st} on β_2/α_1 in the range of $\beta_2/\alpha_1>1$ is seen for large density ratio. For a granitic substratum ($\beta_2/\alpha_1\simeq2$, $\rho_2/\rho_1\simeq2.5$ and $\sigma\simeq0.25$), k_{st} is smaller than k_s by more than a factor of tenth so that the absorption of Stoneley waves should be negligibly small, even if the energies of body waves dissipate comparatively in granitic substratum.

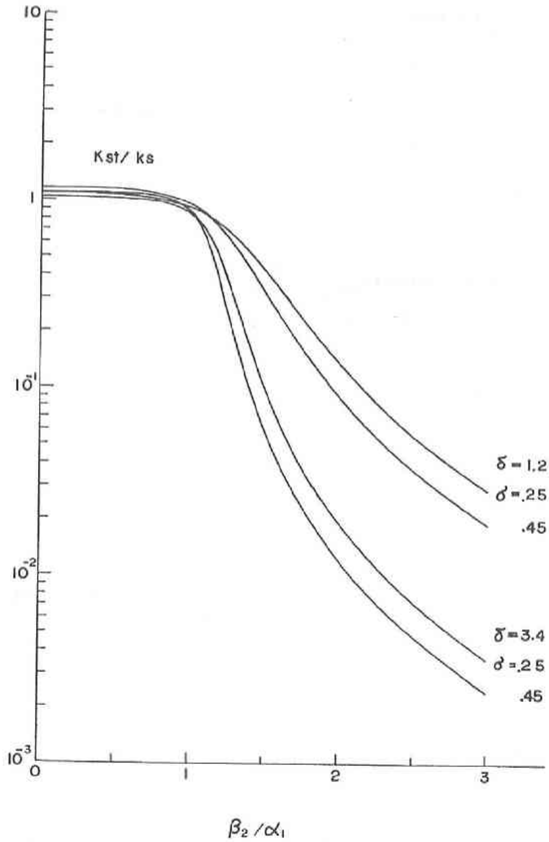

Fig. 5. Ratio of absorption coefficients k_{st}/k_s as a function of β_2/α_1 for some values of σ and ρ_2/ρ_1.

4. Conclusions

The absorption coefficients of Stoneley waves for a low-loss solid half-space in contact with a perfect fluid half-space are related to those of compressional and shear waves in solid, taking the elastic velocities, densities and Poisson's ratio as parameters. The magnitudes of absorption coefficients for Stoneley waves in practically interesting ranges of these parameters are of the same order, or less, of the shear wave absorption coefficients.

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