On Space Distribution Function of Earthquakes

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On Space Distribution Function of Earthquakes

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Abstract: The function representing space distribution of earthquakes is searched to obtain a convenient formulation of the phenomenon for quantitative comparison of natures in different regions or different ranges of magnitudes. Big earthquakes in the world, medium ones in and near Japan and small shocks in the Kanto District, Japan, are taken as data and the distribution in each case is compared. The space concerned is divided into meshes with equal area and the number of earthquakes in a mesh is counted. The frequency of mesh $P(N)$ with the number of earthquakes, N, thus constructed is well represented by

$$P(N) = \gamma N^{-\delta},$$

where γ and δ are constants. It is also found that δ has the same numerical value in every case regardless of the difference in magnitude range. It is concluded, therefore, the space distribution function, as well as the numerical value of its parameter, is independent of magnitude.

Another expression of space distribution, i.e., the frequency $f(S)$ of distance, S, between two neighbouring shocks, is written as

$$f(S) = k S^{-q},$$

as was discussed by Tomoda for small earthquakes in the Kanto District. Transforming the variable, it is proved that the relation between two constants of q and δ should be

$$q = 3 - \delta.$$

The present result is shown to be compatible with Tomoda's conclusion.

A relation between the numerical value of δ and the total number of meshes is discussed. It is noted that this relation should be considered in the comparison of results for various sizes of data.

1. Introduction

In the field of statistical seismology, the time and magnitude distributions of earthquakes have been frequently investigated and the form of distribution function, as well as the numerical values of parameters in it, has been discussed by many authors. For example, the distribution of time interval between two successive earthquakes is known to be represented by

$$f(\tau)\, d\tau = A\, e^{-\alpha\tau}\, d\tau \qquad \text{(for large shocks)},$$

and

$$f(\tau)\, d\tau = B\, \tau^{-p}\, d\tau \qquad \text{(for small shocks)},$$

where $f(\tau)$ is the frequency of interval between τ and $\tau + d\tau$, and A, α, B and p are numerical constants (e.g. Watanabe 1936, Tomoda 1954). These equations imply that rather big earthquakes occur randomly, while smaller ones have the tendency of of group occurrence. The random occurrence of big shocks can be also concluded from the fact that the distribution of earthquake number occurring in a unit time interval is expressed by a Poisson distribution (e.g. Inouye 1942, Kishinouye and Kotata 1950).

It is well-known, on the other hand, that the magnitude distribution, $f(M)$, is written in the form

$$f(M)\,dM = a\,10^{-bM}\,dM\,,$$

regardless of magnitude of earthquake and locality of seismic area (e.g. Gutenberg and Richter 1954, Asada, Tomoda and Suzuki 1953, Suzuki 1959).

Only a few number of papers, however, have been published so far on the space distribution, especially on the form of distribution function. This may be due to the situation that the seismic areas are so concentrated in geologically active zones that such an expression as a distribution function might be thought unnecessary to understand the physical or geological meaning of the activity. According to the present authors' opinion, however, it is still useful to describe the seismicity in a form of distribution function and to study the numerical value of parameter, in order to elucidate the physical properties of seismic activity. A similar expression of space distribution to time and magnitude distributions is a powerful mean for the construction of model of seismic field, and moreover, the description by a functional form is very convenient for the physical understanding of a phenomenon. From this point of view, the space distribution of big earthquakes in the world is studied and compared with those for medium and small earthquakes in this paper.

2. Data and method

Tomoda (1954) discussed the distribution function of small shocks occurring in the Kanto District, Japan and showed that the distribution of distance between two neighbouring shocks follows the formula

$$f(S)\,dS = k\,S^{-q}\,dS\,.$$

This functional form means that the earthquakes have a tendency of group occurrence, because the distribution should have an exponential form if they are randomly distributed.

Tamaki (1961) studied the earthquakes in and near Japan in another way. He divided the area under consideration into many meshes with equal area and counted the number of earthquakes in a mesh. He concluded that the distribution of mesh is well represented by the geometrical distribution, that is,

$$G(N) = q\,p^{N-1}, \quad (p + q = 1)\,,$$

where $G(N)$ denotes the number of meshes in which N earthquakes are located and q is the probability with which no shock occurs in one mesh.

In the present paper, we will follow Tamaki's way at first for great earthquakes in the world and then the relation between the results by Tamaki's and Tomoda's ways is mathematically discussed. Thus the present results can be compared with that for medium shocks by Tamaki as well as for small ones by Tomoda.

Data are taken from the table of shallow shocks (0–60km deep) in "Seismicity of the Earth" by Gutenberg and Richter (1954). The total number of earthquakes used is 2779; 628 for $M \geqq 7$, 1359 for $7 > M \geqq 6$ and 792 for $M < 6$. Although the number for $M < 6$ is too small and many shocks are probably missed, we take them as the data. This does not result an erroneous conclusion, because we classify the data according to magnitude and study the distribution for each class.

Now the whole world is divided into meshes with equal area, which refers to $10° \times 10°$ in latitude and longitude in equatorial zone. Then the total number of meshes is 414. Counting the number of earthquakes in each mesh, the frequency distribution of mesh having a specified number of shocks is constructed. When the number of earthquakes N is large, the corresponding number of mesh sometimes happens to be zero. In such a case, we take the average number of meshes corresponding to several successive numbers of shocks and then the number of meshes is not always integer.

3. Determination of distribution function

We will examine what kind of distribution function fits well to the data. First the Poisson distribution having the same mean value as that of data is tested. As seen clearly in Fig. 1, this distribution cannot explain the data. This is naturally accepted considering that the Poisson distribution implies random occurrence, while the actual earthquakes are apparently concentrated in some narrow belts.

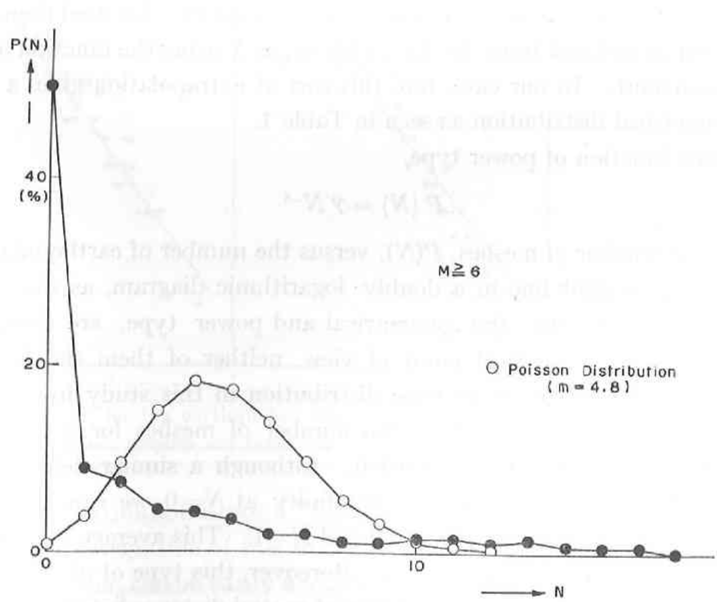

Fig. 1 Comparison of $P(N)$ for big earthquakes in the world with Poisson Distribution.

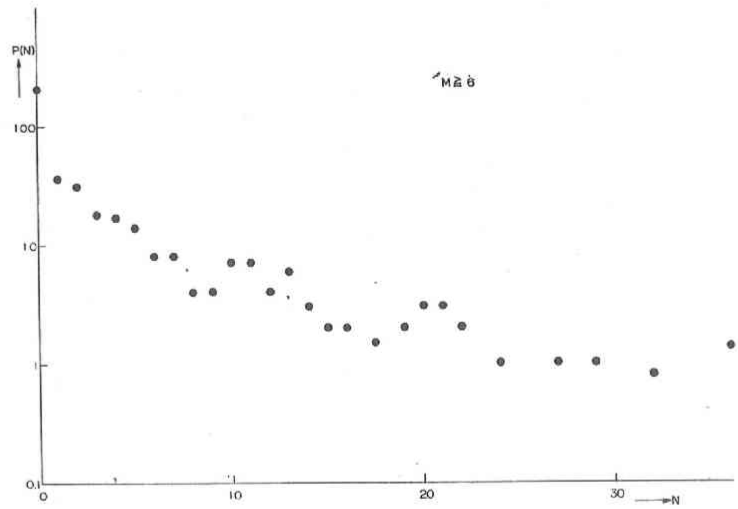

Fig. 2 log $P(N)$ versus N for big earthquakes in the world.

Next, an exponential type function,

$$f(N) = A\,e^{-\alpha N}$$

is examined. A semi-logarithmic diagram, Fig. 2, shows that this distribution does not describe the data so well, especially the number of meshes in which a small number of earthquakes occur. This type, therefore, should be rejected.

The fitteness of geometrical distribution is tested, and this distribution is found to be adequate except the number of meshes corresponding to $N=0$. The discrepancy of the number for $N=0$ was also the case of Tamaki's study. He used then an tentative value for $N=0$ extrapolated from the data with larger N using the function of $f(N)=e^{-\alpha N}$ where α is a constant. In our case, too, this sort of extrapolation gives a value which fits to the geometrical distribution as seen in Table 1.

Finally, the function of power type,

$$P(N) = \gamma\,N^{-\delta} \tag{1}$$

is adopted. The number of meshes, $P(N)$, versus the number of earthquakes, N, is well represented by a straight line in a doubly logarithmic diagram, as shown in Fig. 3.

Thus two distributions, the geometrical and power type, are concluded to be acceptable. From the statistical point of view, neither of them should be rejected. However, we will prefer the power type distribution in this study from the following considerations. As was stated above, the number of meshes for $N=0$ deviate considerably from the geometrical distribution. Although a similar defect exists in the case of power distribution, because $P(N)$ is infinity at $N=0$, we can avoid this difficulty taking the avarage of $P(N)$ at $N=0$ and $N=1$. This average can be represented pretty well by the value of (1) at $N=0.5$. Moreover, this type of distribution is easily connected with the Tomoda type distribution of spatial distance between two neighbouring earthquakes, as will be shown later.

Table 1 Comparison of $P(N)$ and geometrical distribution. The bracketted value at $N=0$ is the extrapolation by an exponential formula. ($M\geqq6$, $q=0.1114$, $p=0.8886$)

N	Observed Value	Theoretical V.	(Ob.-Th.)^2/Th
0	207(42)	27.74	7.33
1	36	24.64	5.23
2	31	21.90	3.78
3	18	19.46	0.11
4	17	17.29	0.00
5	14	15.37	0.12
6	8	13.66	2.35
7	8	12.13	1.41
8	4	10.78	4.26
9	4	9.58	3.25
10	7	8.51	0.27
11	7	7.56	0.04
12	4	6.72	1.10
13	6	5.98	0.00
14	3	5.31	1.00
15	2	4.72	1.57
16	2	4.19	1.14
17	0	3.73	3.73
18	3	3.31	0.03
19	2	2.94	0.30
20	3	2.61	0.06
21	3	2.32	0.20
22	2	2.06	0.00
23	2	1.83	0.02
24	0	1.63	1.63
25	1	1.45	0.14

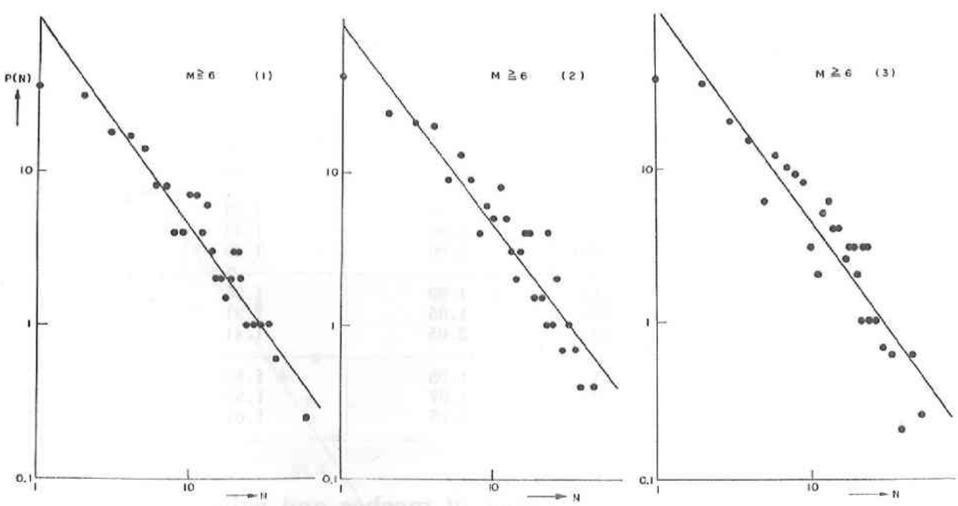

Fig. 3 log $P(N)$ for big earthquakes against log N. (1), (2) and (3) correspond to the cases where meshes are removed.

The power distribution is not a "distribution function" in its exact sense in statistics, because the integration of this function from 0 to ∞ gives an infinitely large value. This defect, however, can be easily avoided taking a finite range of N into consideration. This implies that a finite number of earthquakes, though it is small enough, could occur in every mesh, if an infinite number of shocks is considered in population. Even if this is not the case of actual earthquakes, it is still safely permitted to adopt the power type distribution, because our main purpose in this study is to check the difference, if any, in distribution and numerical values of parameters against magnitude. In such a case, the validity of distribution function itself is not an essential problem, but the adoption of the same function for various range of magnitude is important.

4. Numercal values of δ

The exponent δ and coefficient γ in Eq.(1) are determined by the least square method. In order to check the stability of type of distribution function and numerical values of parameters, the meshes are removed by a half distance of their size in E-W or N-S directions and the distribution in each case is compared. The results are seen in Fig. 3 (1), (2) and (3), the numerical values of δ and $\log \gamma$ being given in Table 2. Any significant difference among three cases cannot be recognzied and, therefore, the distribution function and the values of parameters are concluded to be stable independently of the locality of mesh or the mode of dividing the world.

The change in distribution according to magnitude is examined. The data classified into four ranges of magnitude, say, $M \geqq 7$, $7 > M \geqq 6$, $6 > M$ and the whole range of mangnitdue. The distribution and numerical values in each case are given in Fig. 4 and Table 2 respectively. It is hard to see any significant difference in the numerical value of δ for various ranges of magnitude in consideration.

Table 2. Values of $\log \gamma$ and δ in $P(N)=\gamma N^{-\delta}$.

Magnitude		$\log \gamma$	δ
$M \geqq 7$	(1)	1.73	1.43
	(2)	1.78	1.46
	(3)	1.77	1.45
$7 > M \geqq 6$	(1)	1.94	1.35
	(2)	2.00	1.42
	(3)	2.00	1.40
$M \geqq 6$	(1)	1.99	1.36
	(2)	1.95	1.31
	(3)	2.05	1.41
$6 > M$	(1)	1.98	1.52
	(2)	1.97	1.59
	(3)	2.05	1.61

5. Relation between the total number of meshes and value of δ

The magnitude distribution of earthquakes has been frequently expressed by the formula

$$\log N(M) = a + b\,M \quad \text{(Gutenberg and Richter)} \tag{2}$$

or

$$N(A) = K\,A^{-m} \quad \text{(Japanese seismologists).} \tag{3}$$

Tsuboi found a linear relation between two coefficients, a and b, having considered the results in various areas collectively. Since the above two equations are proved to

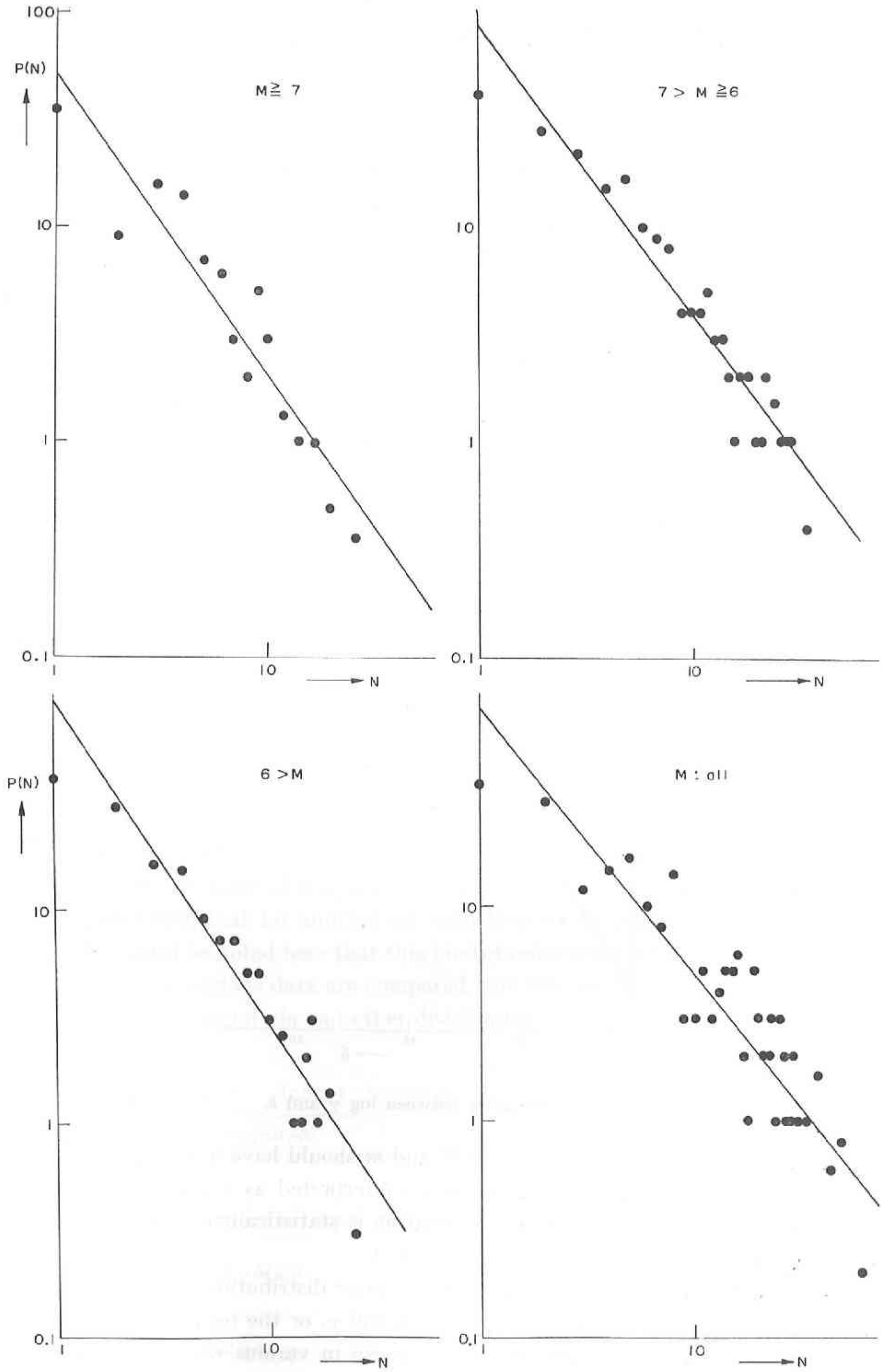

Fig. 4 log $P(N)$ for big earthquakes against log N in various ranges of magnitude.

Table 3 Values of log γ and δ in various sizes of mesh.

size of mesh	log γ	δ	$\sum_{N=1}^{\infty} P(N)$
$\times 4$	1.00	0.82	91
$\times 2$	1.57	1.03	161
	1.46	1.00	148
$\times 1$ (1)	1.93	1.25	232
(2)	2.00	1.31	240
(3)	1.86	1.13	237
$\times \frac{1}{2}$	2.39	1.57	365
	2.30	1.49	346
$\times \frac{1}{4}$	2.71	1.86	513

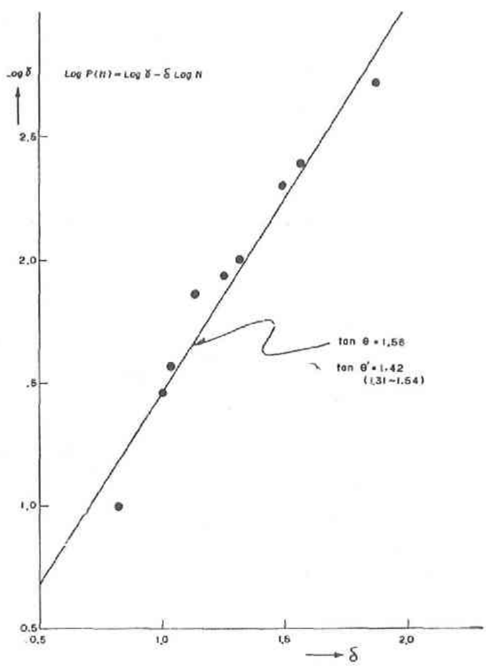

Fig. 5 Relation between log γ and δ.

be identical, this means the values of log K and m should have a similar dependency. But Aki (1961) proved that this relation can be interpreted as a statistical fluctuation, or, in other words, this sort of systematic relation is statistically expected between the coefficients determined by the least square method.

Since a similar expression is adopted for the space distribution in our case, a similar relation must be found between the values of δ and γ, or the total number of meshes. In order to check this, we change the size of mesh in various ways and examine the change in δ with respect to γ. The result is seen in Fig. 5 and Table 3, in which the first row corresponds to double size meshes both in NS and EW direction, the second to double size either in NS or EW direction and so on. Fig. 5 demonstrates a linear relation between the values of δ and $\log \gamma$ in various cases.

According to the similar procedure to Aki's study, the gradient of the line in Fig. 5, $\tan \theta$, is proved to have the value of

$$\tan \theta = \frac{\Sigma (\log N)^2}{\Sigma \log N} . \tag{4}$$

The calculated value of $\tan \theta$ in the present case is 1.42 on an average, while that obtained from Fig. 5 is 1.58. This means that the variation of the values in Table 3 is due to the statistical fluctuation.

In our case, Eq. (1) gives

$$\int_{\infty}^{1} P(N)\, dN = \int_{\infty}^{1} \gamma N^{-\delta}\, dN = \frac{\gamma}{1-\delta} . \tag{5}$$

Therefore

$$\sum_{N=1}^{\infty} P(N) = \frac{\gamma}{\delta - 1} . \tag{6}$$

Since, on the other hand, $\tan \theta$ is defined as

$$\log \gamma = \delta \tan \theta , \tag{7}$$

Eqs. (6) and (7) reduce to

$$\log \sum_{N=1}^{\infty} P(N) = \delta \tan \theta - \log (\delta - 1) . \tag{8}$$

This gives the relation between the total number of meshes and δ. Using Eq. (8), we can reduce the value of δ obtained for a set of data to the standard one, which can be compared with that for another set regardless of the difference in total number of data. It should be noted here that this kind of reduction is important when the values of parameter for various data are compared, not only in the case of space distributions, but also of time, magnitude and other distribution.

Table 4 Values of δ and δ' by Eq. (8).

Magnitude	δ	δ'
$M \geqq 7$	1.43 1.46 1.45	1.05 1.05 1.05
$7 > M \geqq 6$	1.35 1.42 1.40	1.03 1.03 1.04
$M \geqq 6$	1.36 1.31 1.41	1.03 1.03 1.03

Now we provisionally take $\sum_{N=1}^{\infty} n(N)/\sum_{N=1}^{\infty} P(N)=3$ as the standard in this study, i.e., the case where the average number of earthquakes in one mesh is 3, and reduce the value for various magnitudes to those in the standard case, as shown in the third column of Table 4. The result indicates that the space distribution does not vary with mangitude, at least in the range of magnitude treated. The value of δ thus reduced to the standard case is denoted by δ' in this paper.

6. Earthquakes in and near Japan

Although Tamaki (1961) has studied the space distribution of earthquakes occurred in and near Japan during 1926–1956, he has not applied the power type distribution. As seen in Fig. 6, however, the latter distribution fits to the data as well as the geometrical distribution adopted by Tamaki. Therefore, we will discuss the same problem from the standpoint of power distribution. The data of Fig. 6 are based on the Catalogue of Major Shocks published by Japan Meteorological Agency.

The values of δ for various ranges of hypocentral depth are shown in Table 5, and δ' the reduced value to the standard case, is tabulated in the third column of the table, which indicates no systematic change in δ' against hypocentral depth down to 60 km. The comparison of δ' with that for big earthquakes will be made in a later paragraph, together with the value for smaller shocks.

Table 5 Values of δ in various ranges of depth. δ' is the value reduced to standard case.

Depth (km)		δ	δ'
0～10	(1)	1.26	1.07
	(2)	1.23	
	(3)	1.37	
10～20	(1)	1.05	1.09
	(2)	1.24	
	(3)	1.28	
20～30		1.80	1.22
30～40		1.21	1.05
40～50		1.62	1.14
50～60		1.36	1.09
0～60		1.12	1.02

7. Small shocks in the Kanto District

The Earthquake Research Institute, Tokyo University, has published several volumes of seismological report based on the observations at 16 stations in the Kanto District, Japan during 1924–1941. The reported data are studied in similar way, the mesh size being taken to be about 21.5×21.5 km^2. As seen in Fig. 7, $P(N)$ is well represented by the power distribution in this case, too. The stability of this distribution

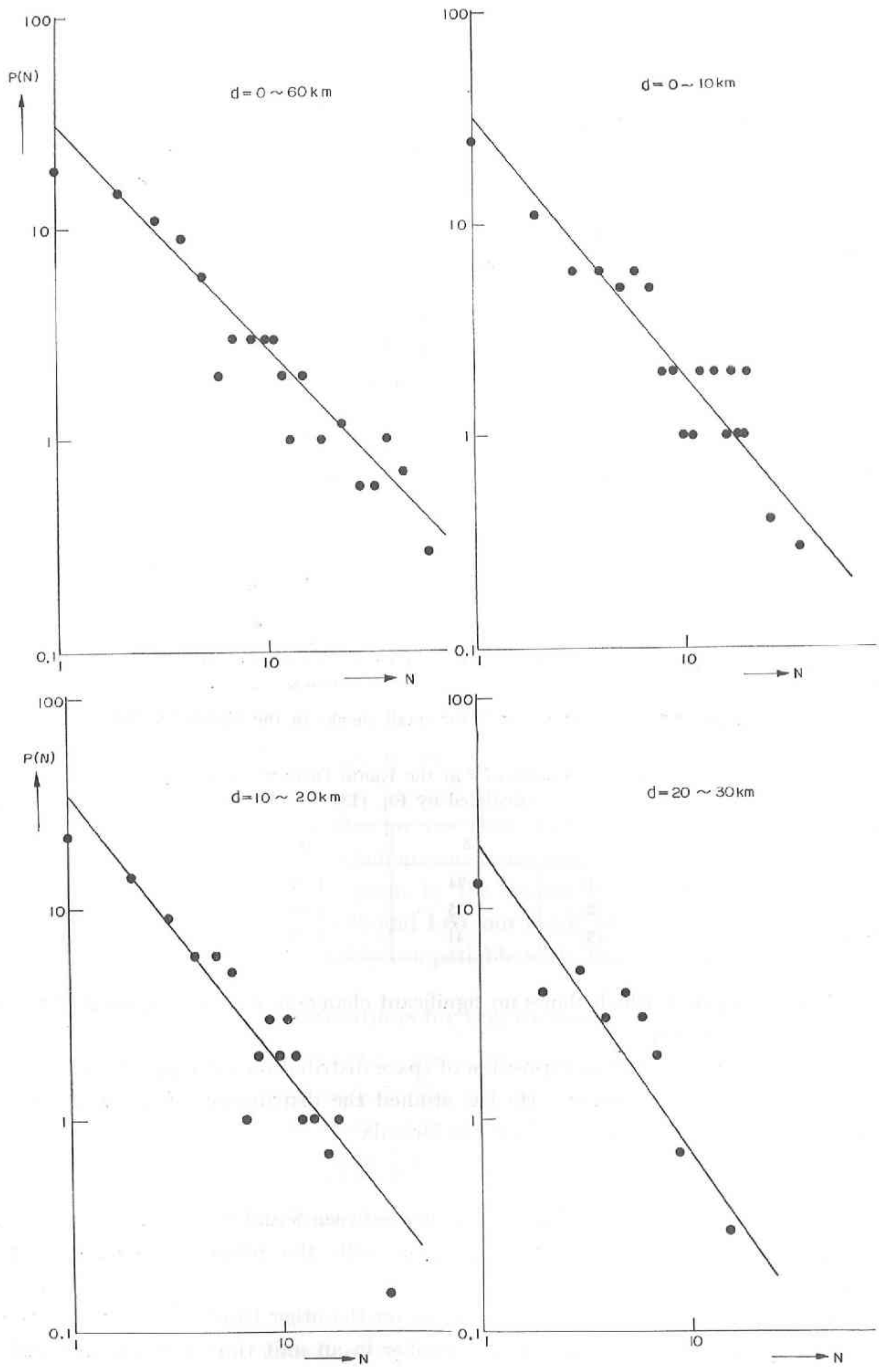

Fig. 6 log $P(N)$ for shocks in Japan against log N in various ranges of hypocentral depth.

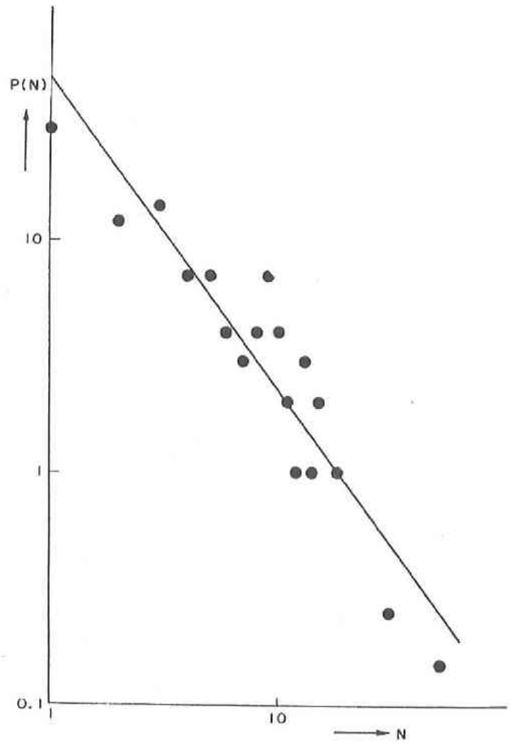

Fig. 7 log $P(N)$ versus log N for small shocks in the Kanto District.

Table 6 Values of δ in the Kanto District. q is calculated by Eq. (13).

	δ	q
(1)	1.34	1.66
(2)	1.35	1.65
(3)	1.41	1.59

is verified by Table 6, which shows no significant change in δ when meshes are removed in NS or EW directions.

Tomoda has discussed an expression of space distribution for small shocks in this district, as mentioned already. He has studied the distribution of distance between two neighbouring shocks and obtained the formula,

$$f(S)\,dS = k\,S^{-q}\,dS\,, \tag{9}$$

where $f(S)$ is the number of distance which lies between S and $S+dS$, and k and q are constants. The connection of this equation with the present expression is now discussed.

For the time distribution of earthquakes, on the other hand, two ways of expression, i.e., the distribution of earthquake number in an unit time duration and that of time interval between two successive shocks have been adopted. Senshu (1959) has discussed the relation between these two distributions. According to a similar consideration, we can deduce the identity of the two expressions of space distribution as follows.

Denoting the linear dimension of mesh by l, the mean distance $\bar{S}$ of neighbouring earthquakes is written as

$$\bar{S} = \frac{l}{N},$$

where N is the number of earthquakes in this mesh. Since the number of distances is the same as the number of earthquakes, the total number $n(N)$ with an average distance $\bar{S}$ is proportional to

$$N \cdot P(N) = \gamma N^{-\delta+1}, \tag{10}$$

taking the distribution (1) of $P(N)$ into consideration. Therefore

$$f(\bar{S})\, d\bar{S} = -\frac{\gamma}{l} N^{-\delta+1} \cdot \frac{dN}{d\bar{S}}\, d\bar{S}, \tag{11}$$

because $f(\bar{S})$ is the distribution for unit distance. This reduces to

$$f(\bar{S})\, d\bar{S} = (\gamma \cdot l^{\delta+1})\, \bar{S}^{\delta-3}\, d\bar{S}, \tag{12}$$

which has the same form as Eq. (9). The relation between q in (9) and δ in (1) is obtained as

$$q = 3 - \delta. \tag{13}$$

Though the above discussion is only for the distribution of average distance between two shocks, the same deduction can be made even when the statistical fluctuation is considered, as was done by Senshu for the time distribution.

According to Eq. (13), we can compare our result with Tomoda's distribution. The values of q calculated from δ are given in the second column of Table 6. As the values of q by Tomoda are 1.54 ,1.59 and 1.63, our result for small earthquakes in the Kanto District is proved to be fairly compatiable with Tomoda's result.

8. Comparison of space distributions for big, medium and small earthquakes

We have treated three cases, i.e., the earthquakes in the whole world, in and near Japan and in the Kanto District. The magnitudes of most earthquakes in these cases are $M \geqq 6$, $6 > M \geqq 4$ and $4 > M > 3$ respectively. The comparison of the three results, therefore, would answer the problem whether or not the space distribution varies for a wide range of magnitude.

Using the relation (8) between δ and the total number of meshes, the results in three cases are reduced to the standard case where the average number of shocks in one mesh is 3. The reduced results are listed in Table 7, for both values of δ' and q calculated from δ'. It is concluded from this table that there is no significant or systematic change in the value of parameter according to the change in magnitude. Therefore, the space distribution, not only in its functional form but also in the value of parameter

Table 7 Values of δ' and q for various magnitude ranges.

Magnitude	δ'	q
$M \geqq 6$	1.03～1.05 (1.04)	1.95～1.97 (1.96)
$6 > M \geqq 4$	1.02～1.22 (1.10)	1.78～1.98 (1.90)
M:3	1.05	1.95

representing the grade of group occurrence, is identical regardless of the magnitude of earthquake. This is a very important fact when a model of seismic filed is constructed.

9. Conclusion

The space distribution of earthquakes is studied from the view point of constructing a functional expression and the following results are obtained.

The frequency of meshes, in which a specified number of earthquakes occur, is expressed as

$$P(N) = \gamma N^{-\delta}$$

for big earthquakes in the world. This function is also the case of medium shocks and small ones in Japan. The numerical value of δ is independent of magnitude. This implies that the earthquakes have a strong tendency of group occurrence of which the grade is constant regardless of earthquake magnitude.

The relation between this distribution and that of distances between two neighbouring shocks is derived. The dependency of value of δ to the total number of data is also discussed. The above comparison of δ in various cases is done, after the value of δ is reduced to a tentative standard cases where the average number of earthquakes in one mesh is 3. The necessity of this sort of reduction should be noted when the results for various data are compared, not only in the case of space distribution but also of other kinds of distributions.

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