Effects of Transition Zone and Low Velocity Layer on Frequency Spectra of Plane SH Waves

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Effects of Transition Zone and Low Velocity Layer on Frequency Spectra of Plane SH Waves

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Abstract

Transition zone, which is sometimes assumed to be at the Mohorovicic discontinuity, and low velocity layer within the upper mantle are simulated by two velocity functions, of which the derivatives are continuous and finite. Frequency spectra of reflected and transmitted plane SH waves from each transition zone are theoretically studied.

The Moho transition zone of more than 0.2 km in its effective thickness has a stronger effect on the frequency content in near-vertical reflections than the absorption of energy in the crustal layer with $Q>250$. The low velocity layer amplifies reflected waves, although the effect may be too small to be recognized at the frequencies above 0.1 cps. Frequency dependence of transmitted waves for both types of the transition zones considered is almost negligible.

1 Introduction

The velocity of seismic waves just below the Mohorovicic discontinuity has been studied extensively in explosion seismology, and known to be considerably uniform in the world. Results of explosion studies, however, are not so accurate to solve the problem whether or not there exists a transition at the Moho.

Some geochemists suggest that the Moho is a zone of gradual transition based on the theory of phase transition (e.g., LOVERING, 1958; MACDONALD and NESS, 1960). If this is the actual case, the Moho should affect the frequency contents in reflected, refracted and transmitted waves. SATÔ (1957) studied reflection and transmission of plane SH waves at the transition layer, in which the velocity increases exponentially with depth, for various angles of incidence, and his conclusion is that the layer plays a role of low pass filter for the reflected waves. NAKAMURA and HOWELL, Jr. (1964) made the Fourier analysis of refraction arrivals obtained in the Maine explosion experiment for the frequency range of 2.6 to 25 cps. They showed that the assumption of linear velocity transition at the Moho gave the maximum possible thickness of 1/2 km for the transition layer.

In the present study, a transition zone at the Moho is assumed to be simulated by a downward increase in velocity, of which the derivative is continuous and finite throughout the medium. The low velocity layer, which may possibly be in the upper mantle is represented by a transition zone including a low velocity region. The effects of each transition zone on the frequency spectra of reflected and transmitted plane SH waves are theoretically studied, comparing with the effect of the absorption of energy due to an internal friction within the crustal layer.

2 Velocity Distribution and Reflection and Transmission Coefficients

The rectangular coordinate system is taken in an infinite medium where the density is constant and the shear wave velocity varies continuously along one axis, say z-axis (Fig. 1). Since the displacement v of plane SH waves is assumed to be independent of y-axis, the equation of motion for v is given by

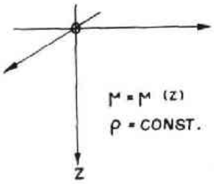

Fig. 1.

$$\rho \frac{\partial^2 v}{\partial t^2} = \mu(z) \frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial z}\left[\mu(z) \frac{\partial v}{\partial z}\right] \qquad (1)$$

where ρ is the density, t the time and μ the rigidity.

If we put the displacement of simple harmonic plane waves as

$$v = \frac{Z(z)}{\sqrt{\mu(z)}} e^{i(\omega t - kx)} \qquad (2)$$

$Z(z)$ is the solution of the equation

$$\frac{d^2 Z}{d z^2} + \left[k_s^2(z) - k^2 + \frac{1}{4\mu^2}\left(\frac{d\mu}{dz}\right)^2 - \frac{1}{2\mu}\frac{d^2\mu}{dz^2}\right] Z = 0 \qquad (3)$$

where $k_s = \omega/v_s$, ω being the angular frequency and v_s the shear wave velocity. As the case where the velocity gradient in the medium is sufficiently smaller than the frequency $f = \omega/2\pi$ is taken into account in the present study, the last two terms in the bracket are negligible compared with the first two terms.

We now assume that the refraction index $n(z)$ in the medium is represented by

$$n^2(z) = 1 - N\frac{e^{mz}}{1 + e^{mz}} - 4M\frac{e^{mz}}{(1 + e^{mz})^2} \qquad (4)$$

where m is a constant, and

$$M = 1 - n^2(0) - N/2$$

$$N = 1 - n^2(+\infty)$$

Any higher derivative of $n(z)$ is finite. The incident wave is assumed to be originated in the homogeneous region far from the transition zone. Considering the reflected and transmitted waves in the far regions, the moduli of the reflection and transmission coefficients, i.e. $|R|$ and $|T|$ respectively, are given as follows, by use of the same procedure of analytic continuation of hypergeometric function as was adopted by BREKHOVSKIKH (1960).

$$|R| = \sqrt{\frac{\cos^2 \pi d \cdot \mathrm{ch}^2\, \pi s A + \sin^2 \pi d \cdot \mathrm{sh}^2\, \pi s A}{\cos^2 \pi d \cdot \mathrm{ch}^2\, \pi s B + \sin^2 \pi d \cdot \mathrm{sh}^2\, \pi s B}} \qquad (5)$$

$$|T|=\sqrt{\frac{(1-N^2)\cos\vartheta_0}{\sqrt{\cos^2\vartheta_0-N}}}\sqrt{\frac{\mathrm{sh}\,(\pi s\cos\vartheta_0)\cdot\mathrm{sh}\,(\pi s\sqrt{\cos^2\vartheta_0-N})}{\cos^2\pi d\cdot\mathrm{ch}^2\pi sB+\sin^2\pi d\cdot\mathrm{sh}^2\pi sB}} \tag{6}$$

where ϑ_0 is the angle of incidence, and

$$2A=\cos\vartheta_0-\sqrt{\cos^2\vartheta_0-N}$$

$$2B=\cos\vartheta_0+\sqrt{\cos^2\vartheta_0-N}$$

$$2d=\sqrt{1-4s^2M}\,,\qquad M\leqq 0$$

$$s=4\pi f/m v_i$$

v_i being the velocity of the incident waves. The negative M is applied to the low velocity layer. For the incident waves with unit amplitude, $|R|$ and $|T|$ are taken as the amplitudes of frequency spectra of reflected and transmitted waves.

3 Transition Zone of Gradually Increasing Velocity

The Moho may be represented by the transition zone in which the velocity increases with z. The refraction index in such a zone is given by

$$n^2(z)=1-N\frac{e^{mz}}{1+e^{mz}} \tag{7}$$

The gradient of $n^2(z)$ is symmetric in z and has the maximum value at $z=0$.

"Effective thickness h" is defined as a measure of the thickness of transition zone so that the gradient of $n^2(z)$ decreases down to a half of its maximum at $z=h/2$. h is independent of the values of the velocities in the homogeneous regions far from the transition zone and is obtained to be $3.526/m$ from (7). The shear velocity distribution for $h=0.5$ km, $v_s(-\infty)=3.8$ km/sec and $v_s(+\infty)=4.6$ km/sec is seen in Fig. 2.

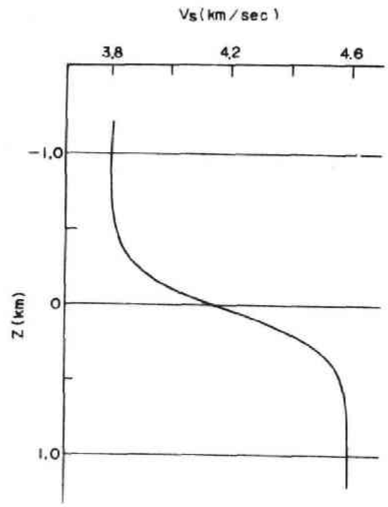

Fig. 2. Shear velocity distribution in a transition zone assumed to be at the Moho.

3.1 Reflection from the Transition Zone

The amplitudes of the reflected waves for the SH wave incidence from the lowest velocity region to the transition zone, are shown in Fig. 3 as the function of frequency, the angle of incidence being taken as parameter. The critical angle for the total reflection is 55°42′. General trend below few cycles per second is relatively flat for intermediate angles. The amplitudes decrease rapidly with frequency at smaller angles. The transition zone plays a role of low pass filter for the reflected waves, as in the case of

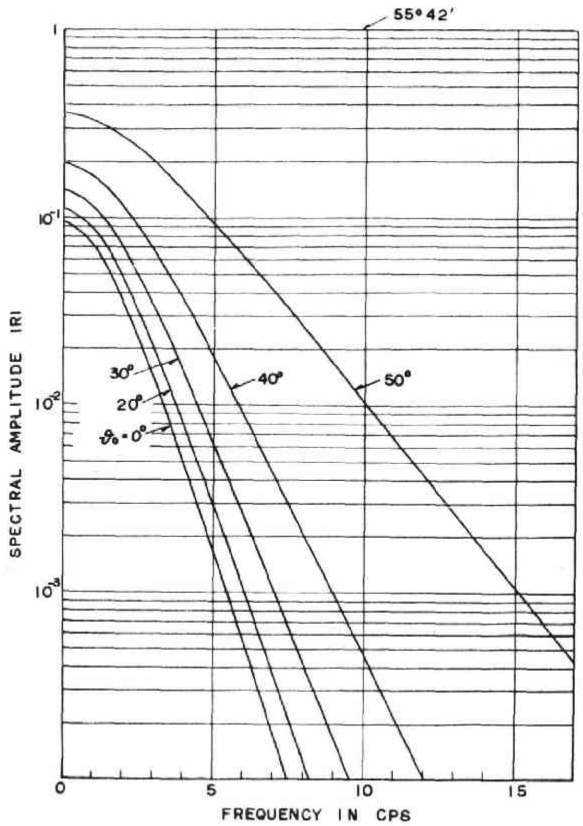

Fig. 3. Spectral amplitudes of reflected waves from the Moho "transition zone" of 0.5 km in effective thickness.

transition layer studied by SATÔ (1957).

The wave length of incident wave at 10 cps is 0.38 km, which is comparable to the effective thickness of 0.50 km. For reference, the ratios of the amplitude at 10 cps to that of reflected waves for the Moho "discontinuity ($h=0$)," are shown in Fig. 4 as the function of the effective thickness. Comparing Figs. 3 and 4, it is seen that the transition zone of 0.15 km or less in its effective thickness does not reduce the amplitude by more than a factor of tenth for frequencies below 10 cps.

The effect of the thickness on the frequency content is compared with that of the absorption of energy due to the internal friction within the crustal layer. The effect of absorption is written in the expression of displacement by the factor of $\exp(-\pi fr/Qv_s)$, where r is the distance along the ray path and $1/Q$ the dissipation function. It is known that Q is almost independent of frequency over the range of frequencies in seismic waves. The value of Q in the crust is estimated to be of the order of 10^2 (PRESS, 1964). The two cases of $Q=250$ and 500 will be considered, although $Q=250$ might be somewhat overestimation of the absorption. We assume the homogeneous crustal layer whose shear velocity and thickness are 3.6 km/sec and 40 km respectively, and an earthquake

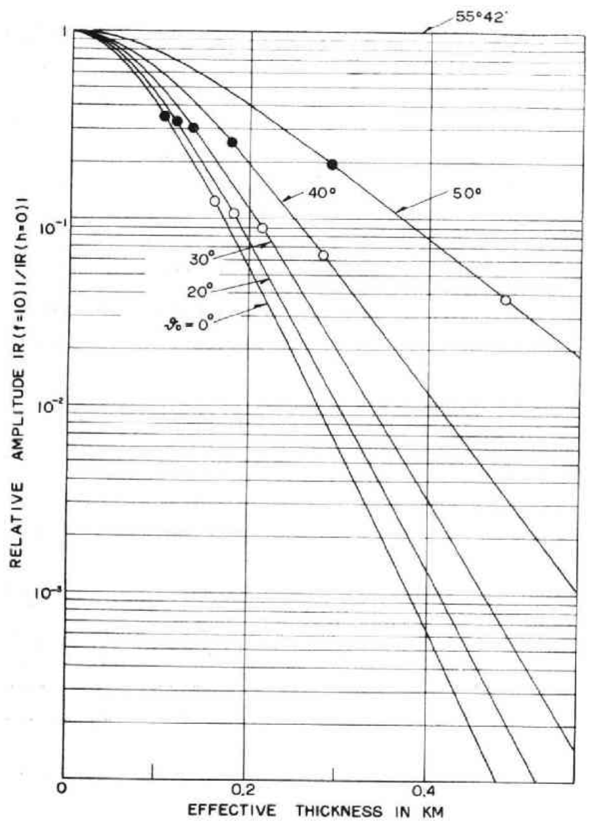

Fig. 4. Amplitude ratios of reflected waves at 10 cps from the Moho "transition zone" and those from "discontinuity ($h=0$)". Open and solid circles show the values of dissipation factor for $Q=250$ and 500 respectively in a situation seen in Fig. 5.

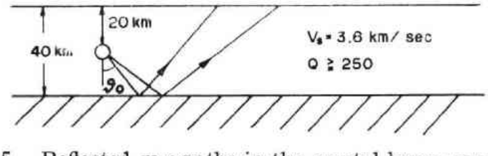

Fig. 5. Reflected ray paths in the crustal layer assumed.

with the focal depth of 20 km is taken into consideration (Fig. 5). The values of dissipation factors at 10 cps for $Q=250$ and 500 are shown by open and solid circles in Fig. 4 respectively, the Moho being here taken as the first order discontinuity ($h=0$).

For the reflection at normal incidence, the effect of the absorption for $Q=250$ is comparable to that of the transition zone of 0.15 km in the effective thickness. The transition zone of more than 0.2 km affects the frequency content in near-vertical reflections much more than the absorption for $Q=250$, which may be too low to be in actual crust. If this is the actual case, it seems to be difficult to identify the near-vertical reflection from the Moho on the seismograms. Little evidence of the reflections from the Moho have been reported (BULLEN, 1963). One of the causes may be the small amplitude in high frequency range in the reflection due to the "transition zone" at the Moho.

3.2 Transmission through the Transition Zone

The frequency dependence of the amplitudes in transmission is seen in Table 1. The spectra of transmitted waves are almost flat in both cases where the incident waves come from the lowest and the highest velocity regions. It may be expected, therefore, that the Moho "transition zone" of the order of 1 km in its effective thickness does not so much affect the transmitted waves.

Table 1. Frequency dependence of transmitted waves for the Moho transition zone $\sqrt{\frac{\sqrt{\cos^2\vartheta_0-N}}{(1-N^2)\cos\vartheta_0}}\,|T|$, for SH wave incidence

	(a) from v_s=3.8 km/sec region			(b) from v_s=4.6 km/sec region		
f	$\vartheta_0=0°$	40°	60°	$\vartheta_0=0°$	40°	60°
0	0.996	0.981	0	0.996	0.990	0.966
1.0	97	85	0	97	94	72
2.0	99	93	0	99	98	84
4.0	1.00	98	0	1.00	99	96
∞	1.00	1.00	0	1.00	1.00	1.00

3.3 The Case of f ≫ 1 cps

For the frequencies far about 1 cps and the angles of incidence below the critical angle, we have

$$|R| \sim \exp\left(-4\pi^2 f\sqrt{\cos^2\vartheta_0 - N}/m\,v_i\right) \tag{8}$$

$$|T| \sim \sqrt{\frac{(1-N^2)\cos\vartheta_0}{\sqrt{\cos^2\vartheta_0 - N}}}\,[1 - O(|R|)] \tag{9}$$

The first term in (9) indicates that the amount of transmitted energy into the highest velocity region passing through the cross-section $\overline{\mathrm{CD}}=a\sqrt{(\cos^2\vartheta_0-N)/(1-N^2)}$ is the same as the energy of the incident wave, which gets into the transition zone through the cross section $\overline{\mathrm{AB}}=a\cos\vartheta_0$, a being a length in the direction parallel to the x-axis (Fig. 6). As almost all energy in the incident waves at higher frequencies are transmitted through the transition zone, it may be expected that the Moho "transition zone" is transparent for short period SH waves.

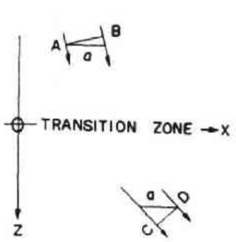

Fig. 6. Ray paths in regions far from the transition zone.
$\overline{\mathrm{AB}}=a\cos\vartheta_0$.
$\overline{\mathrm{CD}}=a\sqrt{(\cos^2\vartheta_0-N)/(1-N^2)}$.

4 Transition Zone Including Low Velocity Layer

The velocity distribution, which corresponds to the low velocity layer in the upper mantle, is assumed as in Fig. 7. The velocity tends to 4.60 km/sec above the

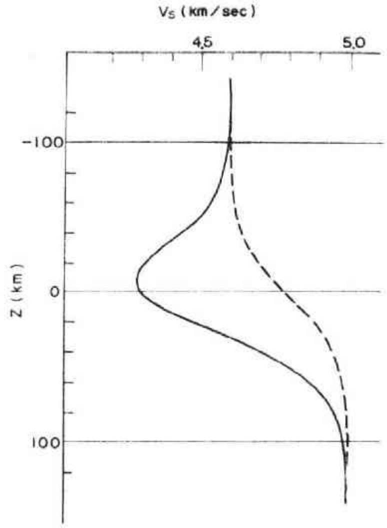

Fig. 7. Shear velocity distributions in transition zones with (full line) and without (broken line) a low velocity layer.

low velocity region and 5.00 km/sec below it. The minimum velocity in the low velocity layer is 4.29 km/sec. Full line shows the case where the low velocity layer exists and broken line the case of absence of the layer.

Fig. 8 shows the amplitudes of reflected waves for the SH wave incidence from the 4.60 km/sec region. Full and broken lines in Fig. 8 correspond to those in Fig. 7. The critical angle for the total reflection is 66°56'. General trend of the broken lines is, of course, similar to that for the Moho "transition zone" considered in the previous section. The spectra demonstrate maxima and minima when a low velocity layer exists, while this feature is not seen in the case of absence of it. The discrepancy

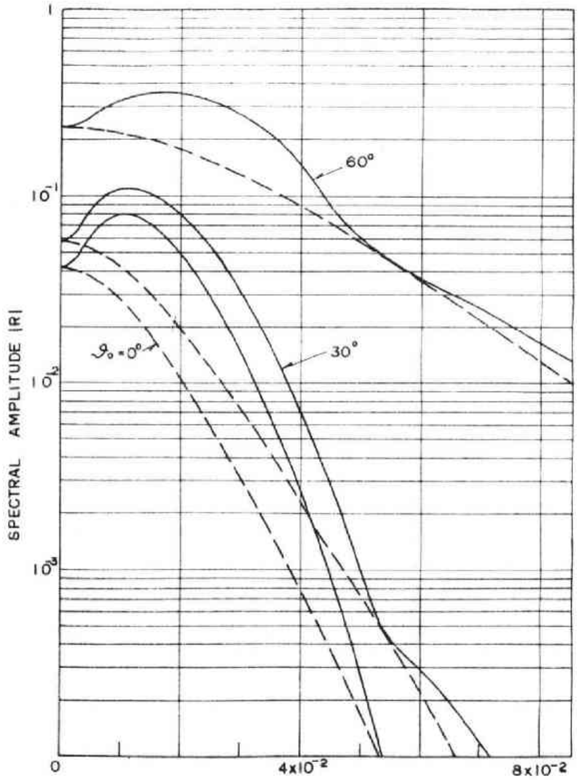

Fig. 8. Spectral amplitudes of reflected waves from transition zones with (full line) and without (broken line) the low velocity layer.

between the full and broken lines decreases with the increasing angle of incidence. The lower the frequency, the larger the amplification of the amplitude by the low velocity layer.

At the frequency

$$f_n = \frac{m\,v_i}{8\pi}\sqrt{\frac{1}{|M|}\left[(2n-1)^2-1\right]} \tag{10}$$

$|R|$ in the case of presence of the low velocity layer has the n th minimum value, which coincides with that in the case of absence of it. f_n is independent of angle of incidence and is given in Table 2. The frequency intervals between the successive minima decrease with n and tend to $m\,v_i/4\pi|M|$ as $n \to \infty$.

Table 2.

n	f_n
1	0 $\times 10^{-2}$
2	5.50
3	9.53
4	13.5
5	17.4
∞	$3.89 \times n$

For the case

$$2\pi^2 f(\cos\vartheta_0 - \sqrt{\cos^2\vartheta_0 - N})/m\,v_i > 1 \tag{11}$$

we have

$$|R| \sim \exp(-4\pi^2 f\sqrt{\cos^2\vartheta_0 - N}/m\,v_i)$$

which is the same factor as that for the absence of the low velocity layer. As the condition (11) corresponds to the frequencies $f > 0.1$ cps, the effect of the low velocity layer may hardly be recognized for the higher frequencies than 0.1 cps.

The frequency dependence of the transmitted waves is shown in Table 3. The difference between the spectra for two types of zone, with and without the low velocity

Table 3. Frequency dependence of transmitted waves $\sqrt{\frac{\sqrt{\cos^2\vartheta_0 - N}}{(1-N^2)\cos\vartheta_0}}\,|T|$, for the transition zones with (A) and without (B) the low velocity layer for SH wave incidence,

(a) from $v_s = 4.6$ km/sec region

f	$\vartheta_0 = 0°$		30°		60°	
	A	B	A	B	A	B
0	0.9992	0.9992	0.9984	0.9984	0.9721	0.9721
1.0×10^{-2}	63	1.000	41	95	452	62
2.0	89	1.000	69	98	335	846
3.0	98	1.000	95	1.000	637	919
4.0	1.000	1.000	97	1.000	907	968
5.0	1.000	1.000	99	1.000	988	988
∞	1.000	1.000	1.000	1.000	1.000	1.000

(b) from $v_s = 5.0$ km/sec region

0	0.9992	0.9992	0.9986	0.9986	0.9906	0.9906
1.0×10^{-2}	65	96	45	90	759	24
2.0	93	1.000	68	92	753	60
3.0	1.000	1.000	91	1.000	914	92
4.0	1.000	1.000	94	1.000	980	94
5.0	1.000	1.000	99	1.000	1.000	1.000
∞	1.000	1.000	1.000	1.000	1.000	1.000

layer, is so little over full ranges of frequency and angle of incidence that it might be impossible to discriminate one type from another.

5 Conclusions

The spectral amplitudes of reflected and transmitted plane SH waves for two types of transition zone, are theoretically studied. A downward increase in seismic wave velocity simulates a transition zone assumed to be at the Mohorovicic discontinuity. A transition zone including a low velocity region represents the low velocity layer, which may possibly be in the upper mantle. The derivatives of two velocity functions are continuous and finite throughout the medium.

The main results of the present study are summarized as follows;

The Moho "transition zone" of 0.15 km or less in its effective thickness does not reduce the amplitude of the reflected waves by more than a factor of tenth for frequencies below 10 cps. The Moho with more than 0.2 km thick affects the frequency content in near-vertical reflections much more than the absorption of energy due to the internal friction in the crustal layer with $Q>250$. The low velocity layer in the upper mantle amplifies the reflected waves, although the effect is too small to be recognized at the frequencies above 0.1 cps. Frequency spectra of the transmitted waves are almost flat for both types of the transition zone considered. The zone is transparent for short period SH waves in both cases.

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