# Methods of estimating the eddy deffusivity from the data of velocity fluctuation



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# *Methods of estimating the eddy diffusivity from the data of velocity fluctuation*

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*Abstract*

Dependence of eddy diffusivity on the time-distribution and spatial dimension of cluster is clarified by the statistical theory of diffusion.

The theory of diffusion has been developed by use of the following assumptions: Local time-space correlation of velocity is formulated, as a product of the Lagrangian correlation coefficient and local energy. The local energy in time-space is expressed by an interpolation formula.

Diffusion chart to be used in estimating diffusivity from the time-space dimensions of cluster and data of wind fluctuation is constructed. The relation between diffusivity and scale of phenomenon is discussed.

## 1. Introduction

Turbulent transfer can be approached by the two methods; the mixing length method and the Lagrangian method. Diffusivity in the inhomogeneous turbulence such as shear flow is usually investigated by means of the concept of mixing length, because the application of the Lagrangian method to inhomogeneous turbulence seems to be more difficult. Since a pioneering work by MONIN and OBUKHOV (1954), the transfer theory in the diabatic atmosphere has been investigated by use of the concept of mixing length by many workers as KAZANSKY and MONIN (1956), ELLISON (1957), BUSINGER (1959), RIDEL (1959), YAMAMOTO (1959), PANOFSKY, BLACKADAR, and McVEHIL (1960), TAYLOR (1960), YAMAMOTO and SHIMANUKI (1960), NEUMANN (1961), PANOFSKY (1961), PRIESTLEY (1961), SELLERS (1962), SYONO and HAMURO (1962), YOKOYAMA (1962), DRIMMEL (1963), TAKEUCHI and YOKOYAMA (1963), SWINBANK (1964) and McVEHIL (1964). Some results of the above works show that the vertical diffusivity $K_z$ is proportional to height above the ground in neutral conditions, and its 2-nd order derivative with regard to height is negative at stable, and positive at unstable conditions. The behaviors of horizontal diffusivity $K_y$ were investigated empirically by YAMAMOTO and SHIMANUKI (1964) to be proportional to height, with a constant of proportionality depending on stability. However, it is not clarified by these theories that the diffusivity depends on the scale of phenomenon. The dependence of diffusivity on the scale of phenomenon is very large in the atmosphere, which can be explained by the Lagrangian method.

Lagrangian theory of turbulent diffusion was originated by TAYLOR (1921). He showed that the diffusivity $K$ is

$$K(t)=\int_0^t R(\tau)\,d\tau\,, \tag{1}$$

where $t$ is the floating time from the source, and $R(\tau)$ is the Lagrangian velocity product mean value (usually, called as the Lagrangian velocity correlation function) with lag time $\tau$. From this equation, it is found that diffusivity is a function of the floating time $t$, which is not involved in the mixing-length theory. When the dimension of source is finite, Eq. (1) must be modified, as shown in BRIER (1950), OGURA (1952, 1957, 1959), SHIMANUKI (1961) and SMITH and HAY (1961, 1963). In this case, the value of $K(t)$ largely differs from (1). Recently, the author (SHIMANUKI, 1965) developed a statistical theory of time-space dimensions of clusters of particles, and derived universal equations of diffusion applicable to the discussion in the present paper. The theory is concerned with the diffusivity of particles free from gravity, which is applicable to viscosity and conductivity, with some corrections. The description in section 2 is an outline of the cited paper.

## 2. Equation of diffusion

Let us consider the one-dimensional problem, in which the independent variables are the displacement ($y$-component) and time. The turbulent field is assumed to be homogeneous and stationary. The mean value of a physical property $A(t)$ over all the particles at a floating time $t$ from source is denoted as $\langle A(t)\rangle$. The deviation of velocity $v(t)$ from the mean velocity over a cluster is

$$v(t)=V(t)-\langle V(t)\rangle\,, \tag{2}$$

and the displacement $y(t)$ from the center of mass is

$$y(t)=Y(t)-\langle Y(t)\rangle\,. \tag{3}$$

Assuming that the partial correlation between $y(0)$ and $v(t)$ is zero referred to $v(0)$, we have

$$\overline{\langle y(0)\,v(t)\rangle}=K_0\,\frac{\overline{\langle v(0)\,v(t)\rangle}}{\overline{\langle v(0)^2\rangle}}\,, \tag{4}$$

where

$$K_0=\overline{\langle y(0)\,v(0)\rangle}\,. \tag{5}$$

The detailed discussion of the above assumption will be found in the cited paper. By use of (4), the equation of diffusion becomes

$$\begin{aligned}\frac{d}{dt}\,\overline{\langle y(t)^2\rangle}&=\frac{d}{dt}\,\overline{\langle\{y(t)-y(0)\}^2\rangle}+2\,\overline{\langle y(0)\,v(t)\rangle}\\&=2\int_0^t\overline{\langle v(t)\,v(t')\rangle}\,dt'+2\,K_0\,\frac{\overline{\langle v(0)\,v(t)\rangle}}{\overline{\langle v(0)^2\rangle}}\,.\end{aligned} \tag{6}$$

The second term of the right side of (6) is a correction term, which may be sometimes neglected. Introducing diffusivity $K$ defined by

$$K(t) = \frac{1}{2} \frac{d}{dt} \overline{\langle y(t)^2 \rangle}, \tag{7}$$

we have

$$K(t) = \int_0^t \overline{\langle v(t)\, v(t') \rangle}\, dt' + K(0) \frac{\overline{\langle v(0)\, v(t) \rangle}}{\overline{\langle v(0)^2 \rangle}}, \tag{8}$$

where $K(0)$ is initial diffusivity.

Now let us consider the local correlation $\overline{\langle v(t)v(t') \rangle}$ involved in the right hand side of (8). Consider two particles in fluid indicated by subscripts 1 and 2, respectively. Relative velocity $v(t)$ about the center of mass is

$$v_1(t) = \frac{1}{2} \{V_1(t) - V_2(t)\} = -v_2(t). \tag{9}$$

The operation $\langle \ \ \rangle$ in this case denotes taking the mean over two particles. From (9), local correlation becomes

$$\overline{\langle v(t)\, v(t') \rangle} = \frac{1}{2} \{\overline{V_1(t)\, V_1(t')} - \overline{V_1(t)\, V_2(t')}\}. \tag{10}$$

Assuming that the partial correlation between $V_1(t)$ and $V_2(t')$ is zero referred to $V_2(t)$ as made by BRIER (1950), we have

$$\overline{V_1(t)\, V_2(t')} = \frac{\overline{V_1(t)\, V_2(t)} \cdot \overline{V_2(t)\, V_2(t')}}{\overline{V_2(t)^2}}. \tag{11}$$

Therefore, Eq. (10) becomes

$$\overline{\langle v(t)\, v(t') \rangle} = \frac{\overline{V(t)\, V(t')}}{\overline{V^2}} \overline{\langle v(t)^2 \rangle}. \tag{12}$$

Let us consider that Eq. (12) is valid for a cluster composed of more than two particles. The right side of (12) is a product of the Lagrangian velocity correlation and local energy in the cluster with time-space dimension. In the above discussion, $t$ and $t'$ is commutative, so that (12) is replaced by the mean value of two.

The local energy computed from the sample over the finite observational interval $T$ is

$$\overline{\langle v^2 \rangle_T} = \frac{1}{T^2} \int_0^T (T-\xi)\, D(\xi)\, d\xi \equiv 2E(T), \tag{13}$$

where $D(\xi)$ is the structure function defined by

$$D(\xi) = \overline{\{V(t) - V(t-\xi)\}^2}. \tag{14}$$

The local energy $E(T)$ increases monotonically with $T$ from $E(0)=0$ to $E(\infty)=\overline{V^2}/2$. In order to obtain the local energy in two-dimensional time-space involved in (12), we shall introduce the time-space standard deviation $S(t)$ with time unit, given by

$$S(t)^2 = S_y(t)^2 + S_t(t)^2, \tag{15}$$

where

$$S_y(t)^2 = \frac{1}{u^2} \langle y(t)^2 \rangle , \tag{16}$$

and

$$S_t(t)^2 = S_t(0)^2 = \langle t_0^2 \rangle , \tag{17}$$

where $u$ is the constant of proportionality with the dimension of velocity, and $t_0$ is released time at source. When $\langle y(t)^2 \rangle$ is small, (15) becomes $S(t) \approx \langle t_0^2 \rangle^{1/2}$, and in the case of diffusion from an instantaneous source, becomes $S(t) = \langle y(t)^2 \rangle^{1/2}/u$. If TAYLOR's hypothesis for time- and spatial correlations is valid, the constant $u$ can be replaced by the mean velocity $U$. The property $S(t)$ in these cases stands for the dimension of cluster. In the case where $\langle y(t)^2 \rangle$ and $\langle t_0^2 \rangle$ are both finite, we shall consider that (15) gives an interpolation formula, and $2S(t)$ can be used in place of $T$ in (13).

Using the property $S(t)$, we have

$$\overline{\langle v(t)\, v(t') \rangle} = \{E(2S(t)) + E(2S(t-\tau))\} \left\{1 - \frac{D(\tau)}{D(\infty)}\right\}, \tag{18}$$

from (12) and (13), where $\tau = t - t'$. Consequently, (8) becomes

$$K(t) = \int_0^t \{E(2S(t)) + E(2S(t-\tau))\} \left\{1 - \frac{D(\tau)}{D(\infty)}\right\} d\tau + \frac{K(0)}{2} \left\{1 + \frac{E(2S(t))}{E(2S(0))}\right\} \left\{1 - \frac{D(t)}{D(\infty)}\right\}. \tag{19}$$

Since $S_t$ is independent of $t$, the derivative of $Sy^2$ is equal to that of $S^2$ from (15). Therefore, the relation between the diffusivity $K$ and the time-space dimension $S$ follows from (7) as

$$S(t)^2 = S(0)^2 + \frac{2}{U^2} \int_0^t K(t)\, dt . \tag{20}$$

Eqs. (19) and (20) will give us the informations of diffusion.

## 3. Numerical analysis of the equations

The structure function $D(\tau)$ involved in (19) is the Lagrangian one, and that involved in (13) is the Eulerian one, so that we must know these structure functions. Let us adopt HAY-PASQUILL's hypothesis of the relation between the Lagrangian and Eulerian correlations (HAY and PASQUILL, 1957, 1959). It is assumed that the Lagrangian and Eulerian structure functions have similar shapes but different scales (ratio $\beta$: 1, being taken as $\beta = 4$). Therefore, the Lagrangian structure function $D_L(\tau)$ is

$$D_L(\tau) = D_E(\tau/\beta) , \tag{21}$$

where $D_E(t)$ is the Eulerian structure function. The Eulerian structure function $D_E(\tau)$ should be determined by observations, which may be expressed approximately as follows by use of the five parameters $\tau_0$, $w$, $p$, $q$ and $h$;

$$\left.\begin{aligned} D_E(\tau) &= w^2\left(\beta\frac{\tau}{\tau_0}\right)^p && \text{for} \quad \tau \le \frac{\tau_0}{\beta}, \\ &= w^2\left(\beta\frac{\tau}{\tau_0}\right)^q && \text{for} \quad \frac{\tau_0}{\beta} \le \tau \le h\frac{\tau_0}{\beta}, \\ &= w^2 h^q && \text{for} \quad h\frac{\tau_0}{\beta} \le \tau. \end{aligned}\right\} \tag{22}$$

It should be noted that the parameter $w$ differs from $(\overline{v^2})^{1/2}$. Although the use of more than five parameters enables us to express the observed results more precisely, we use the five parameters in order to avoid the complexity of mathematical expressions.

Introducing the non-dimensional properties denoted by subscript 1 and defined by

$$\left.\begin{aligned} &\frac{t}{t_1} = \frac{\tau}{\tau_1} = \frac{\beta T}{T_1} = \frac{\beta\eta}{\eta_1} = \frac{\beta S(t)}{S_1(t_1)} = \tau_0, \\ &\frac{E(T)}{E_1(T_1)} = \frac{D_E(\tau/\beta)}{D_1(\tau_1)} = \frac{U^2 c_1}{2\beta^2} = w^2, \\ &\frac{K(t)}{K_1(t_1)} = \tau_0 w^2, \end{aligned}\right\} \tag{23}$$

we have non-dimensional equations written as

$$\begin{aligned} K_1(t_1) = &\int_0^{t_1}\{E_1(2S_1(t_1)) + E_1(2S_1(t_1-\tau_1))\}\left\{1 - \frac{D_1(\tau_1)}{D_1(\infty)}\right\}d\tau_1 \\ &+ \frac{K_1(0)}{2}\left\{1 + \frac{E_1(2S_1(t_1))}{E_1(2S_1(0))}\right\}\left\{1 - \frac{D_1(t_1)}{D_1(\infty)}\right\}, \end{aligned} \tag{24}$$

$$E_1(T_1) = \frac{1}{2T_1^2}\int_0^{T_1}(T_1 - \eta_1)D_1(\eta_1)\,d\eta_1, \tag{25}$$

$$S_1(t_1)^2 = S_1(0)^2 + c_1\int_0^{t_1}K_1(t_1)\,dt_1, \tag{26}$$

and

$$\left.\begin{aligned} D_1(\tau_1) &= \tau_1^p && \text{for} \quad \tau_1 \le 1, \\ &= \tau_1^q && \text{for} \quad 1 \le \tau_1 \le h, \\ &= h^q && \text{for} \quad h \le \tau_1. \end{aligned}\right\} \tag{27}$$

The non-dimensional equations (24) and (26) can be solved numerically by use of an electronic computer. The values of the floating time $t_1$, where the values of $K_1$ and $S_1$ are computed may be chosen as,

| | | | | |
|---|---|---|---|---|
| $\Delta$, | $1.2\Delta$, | $1.4\Delta$, | $1.6\Delta$, | $1.8\Delta$, |
| $2.0\Delta$, | $2.2\Delta$, | $2.4\Delta$, | $2.6\Delta$, | $2.8\Delta$, |
| $3.0\Delta$, | $3.4\Delta$, | $3.8\Delta$, | $4.2\Delta$, | $4.6\Delta$, |
| $5.0\Delta$, | $6.0\Delta$, | $7.0\Delta$, | $8.0\Delta$, | $9.0\Delta$, |
| $10\Delta$, | $12\Delta$, | $14\Delta$, | $\cdots\cdots$, | |

where $\Delta$ is an arbitrary small value (e.g. 0.0001). The computations will be carried out along the following line: The first approximation of $K_1(\Delta)$ is evaluated by (24) using $S_1(0)$ in place of $S_1(\Delta)$. The first approximation of $S_1(\Delta)$ is evaluated by (26), using the evaluated value of $K_1(\Delta)$. Next, $K_1(\Delta)$ is corrected by computing (24) by use of the evaluated values of $S_1(\Delta)$, and $S_1(\Delta)$ is also corrected. Sequently, $K_1(1.2\Delta)$, $S_1(1.2\Delta)$, $K_1(1.4\Delta)$, $S_1(1.4\Delta)$.... are computed. Integration of (24) is carried out to the smaller value of $t_1$ and $h$, because the integrand is zero for $\tau_1 \geq h$. The integrand of (24) is computed at 50 points between $\tau_1=0$ and $t_1$(or $h$), and summed up, where the value of $S_1^2$ in each point is obtained by the linear interpolation of the values of known $S_1^2$ at the nearest points.

In order to execute the computation, the values of $p$, $q$, $h$, $c_1$, $S_1(0)$ and $K_1(0)$ must be given, corresponding to each case of diffusion. Properties $p$, $q$, and $h$ are defined by (22). The Eulerian structure function is drawn by use of data obtained from observations, which approximated by three straight lines in log-log graph, from which the values of $p$, $q$ and $h$ are estimated. Properties $c_1$, $S_1(0)$ and $K_1(0)$ are defined by (23). Time-space factor $c_1$ is

$$c_1 = \frac{32\, w^2}{U^2}, \tag{28}$$

where $U$ is mean velocity, and $w$ defined by (22) can be estimated from the graph of structure function. Initial time-space dimension $S_1(0)$ is

$$S_1(0) = \frac{4\,(\langle y(0)^2\rangle + U^2\langle t_0^2\rangle)^{1/2}}{\tau_0\, U}, \tag{29}$$

where $\langle y(0)^2\rangle^{1/2}$ is spatial standard deviation at source, $\langle t_0^2\rangle^{1/2}$ is standard deviation of releasing time, and $\tau_0$ defined by (22) can be estimated from the graph of structure function. Initial diffusivity $K_1(0)$ is

$$K_1(0) = \frac{K(0)}{\tau_0\, w^2}, \tag{30}$$

where $K(0)$ is $K_0$ defined by (5).

## 4. Behavior of diffusivity

With regard to diffusion from the source of which the dimension $S_1(0)$ is small, the solutions near the source can be obtained analytically in the case of $K_1(0)=0$. (See SHIMANUKI, 1965). The Eulerian structure function for inertial subrange is given by the power form as

$$D(\tau) = A\,\tau^{2/3}. \tag{31}$$

Near the source where the increment of the time-space dimension $S_1(t_1)$ is small compared with $S_1(0)$, the relations among $K_1(t_1)$, $S_1(t_1)$ and $t_1$ are

$$c_1^{1/2} K_1 = 0.357\, S_1(0)^{2/3}\,(c_1^{1/2} t_1) \tag{32}$$

$$= 0.845\, S_1(0)^{1/3}\,\{S_1(t_1)^2 - S_1(0)^2\}^{1/2}. \tag{33}$$

If the time-space dimension $S_1(0)$ of the source is very small and the increment of $S_1(t_1)/S_1(0)$ from unity is significant near the source, the solutions become

$$c_1^{1/2} K_1 = 0.0800\,(c_1^{1/2} t_1)^2 \tag{34}$$

$$= 0.897\, S_1\,(t_1)^{4/3}\,. \tag{35}$$

These results are consistent with the power law derived from dimensional analysis. (See BATCHELOR and TOWNSEND, 1956).

The solutions for the other cases can be found by the numerical method as shown in the preceding section. The results giving the relations among $K_1$, $S_1$ and $t_1$ are illustrated in Fig. 1 for $p=2/3$, $q=1/3$, $h=1000$, $c_1=1$ and $K_1(0)=0$. In the left part indicated by a letter A of Fig. 1 is shown the relation between the non-dimensional diffusivity $K_1$ and the non-dimensional floating time $t_1$. In the right part indicated by a letter B is shown the relation between $K_1$ and the non-dimensional space dimension $S_{y1}$, or the spatial standard deviation of the particles, which are illustrated by full and broken lines. The full lines correspond to the case of $S_{y1}(0)=0$, or of the point source, and the broken lines to the case of $S_{t1}(0)=0$, or of the instantaneous source. The indicated parameter is $S_1(0)$, which is equal to $S_{t1}(0)$ for the full lines, and to $S_{y1}(0)$ for the broken lines.

Some behaviors of diffusion are found in Fig. 1 which are written as follows:

1) Fig. 1A. The diffusivity $K_1(t_1)$ is proportional to the floating time $t_1$, at small $t_1$, and to $t_1^2$ at moderate $t_1$ in the case of small values of $S_1(0)$, as shown in (32) and (34); and saturates at large $t_1$, provided that $D(\tau)$ does. The dependence of the diffusivity

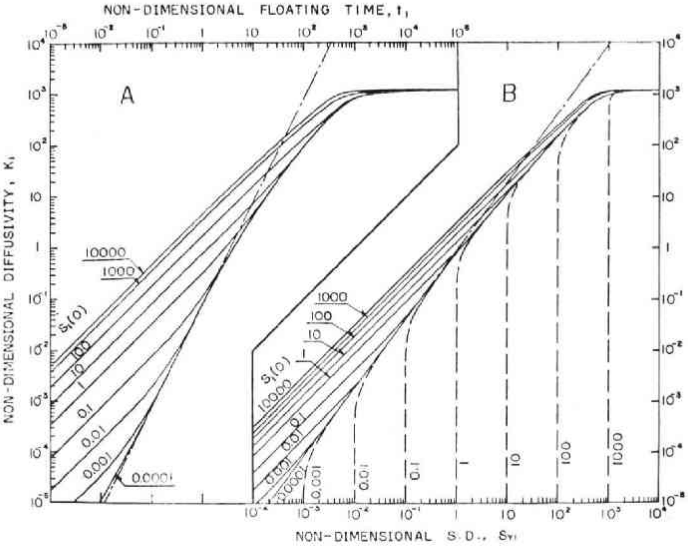


Fig. 1. Diffusivity versus floating time (figure A), and diffusivity versus spatial standard deviation (figure B), under the conditions of $p=2/3$, $q=1/3$, $h=1000$, $c_1=1$ and $K_1(0)=0$. Initial time-space dimension $S_1(0)$ is taken as a parameter. Full and broken lines in the figure B are solutions for point and instantaneous sources, respectively. Figure A involves all types of sources.

$K_1$ on the source dimension $S_1(0)$ is significant at small $t_1$. 2) Fig. 1B. In the problem of the point source (full lines), when the diffusion proceeds, the spatial dimension $S_{y1}(t_1)$ grows to exceed the time dimension $S_{t1}(t_1)$, and the behavior approaches to that of the instantaneous source (broken lines). Therefore, the full and broken lines approach to each other as the spatial standard deviation $S_{y1}$ increases. The main parts of the full lines show that the diffusivity $K_1$ is proportional to the spatial standard deviation $S_{y1}$, as derived from (33). In the confluent parts of the full and the broken lines, the diffusivity $K_1$ is proportional to the 4/3 power of $S_{y1}$, as shown in (35), except for large $S_{y1}$, which corresponds to the 4/3 power law originated by RICHARDSON (1926).

Parts A and B of Fig. 1 correspond to each other. When we want to find the floating time $t_1$ in figure B, we can know it from the corresponding point at the same values of $K_1$ and the other parameters in figure A. For the general types of source except the point and the instantaneous sources, $S_{t1}(0)$ and $S_{y1}(0)$ are both significant. In such the case figure A and the broken lines in figure B are useful, provided that $S_{y1}$ be read as $S_1$. The value of $S_{y1}$ in this case can be calculated by use of (15).

## 5. Diffusion chart

Fig. 1(B) shows the relation between the eddy diffusivity and the size of cluster. We have two groups of curves in Fig. 1 (B), full lines for $S_{y1}(0)=0$ and broken lines for $S_{t1}(0)=0$. For the case of $S_{y1}(0)\neq 0$ and $S_{t1}(0)\neq 0$, we can draw the curves to be $K_1\approx 0$ at $S_{y1}(t_1)\approx S_{y1}(0)$ and to be coincident with full lines at $S_{y1}(t_1)\gg S_{y1}(0)$. The full lines are valid for $S_{y1}(0)\neq 0$, provided that $S_{y1}(t_1)>2S_{y1}(0)$. Let us consider diffusion under the condition of $S_{y1}(t_1)>2S_{y1}(0)$, where $S_{y1}$ is the spatial dimension of cluster. Therefore, we are concerned with the full lines in Fig. 1(B).

From the following inequalities

$$S_{t1}(0)^2 \leq S_1(0)^2 < S_{t1}(0)^2 + S_{y1}(t_1)^2, \tag{36}$$

we have

$$S_1(0)^2 \approx S_{t1}(t_1)^2 \qquad \text{for} \quad S_{t1}(t_1)^2 > S_{y1}(t_1)^2, \tag{37}$$

and

$$S_1(0)^2 < 2S_{y1}(t_1)^2 \qquad \text{for} \quad S_{t1}(t_1)^2 < S_{y1}(t_1)^2. \tag{38}$$

Since the diffusivity $K_1$ is nearly independent of $S_1(0)$ under the condition of $S_1(0)^2 < 2S_{y1}(t_1)^2$ as shown in Fig. 1(B), we can consider that $S_1(0)$ in Fig. 1(B) can be replaced by $S_{t1}(t_1)$. Consequently, we can find the eddy diffusivity from the values of $S_{y1}$ and $S_{t1}$, as a point corresponding to the value of $S_{y1}$ in abscissa, on the full line for $S_1(0)=S_{t1}(t_1)$ in Fig. 1 (B), where $S_{y1}$ and $S_{t1}$ are given by

$$S_{y1}(t_1) = \frac{4}{\tau_0 U} \langle y(t)^2 \rangle^{1/2}, \tag{39}$$

and

$$S_{t1}(t_1) = \frac{4}{\tau_0} \langle t_0{}^2 \rangle^{1/2}, \tag{40}$$

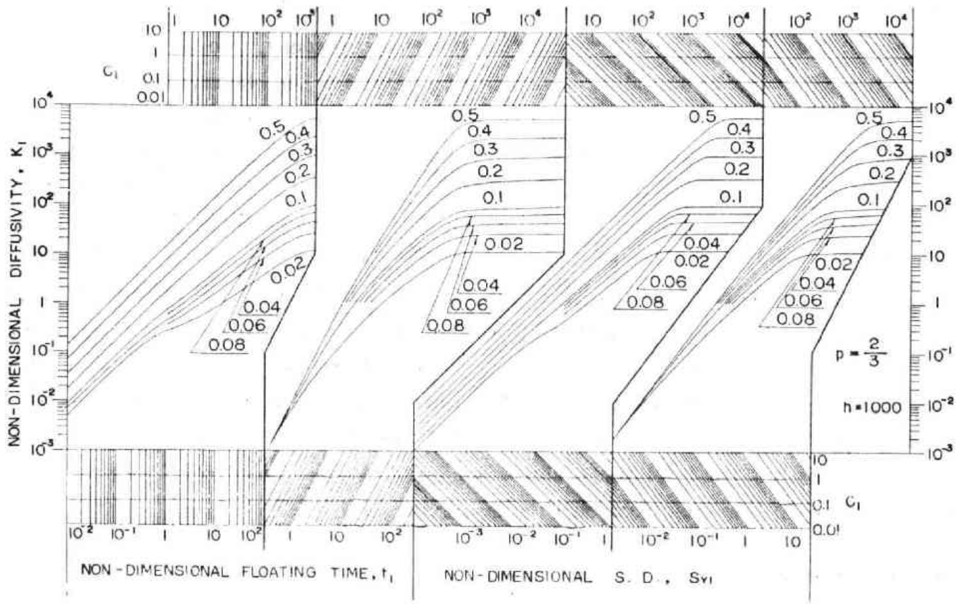


Fig. 2. Diffusion chart for $h=1000$.

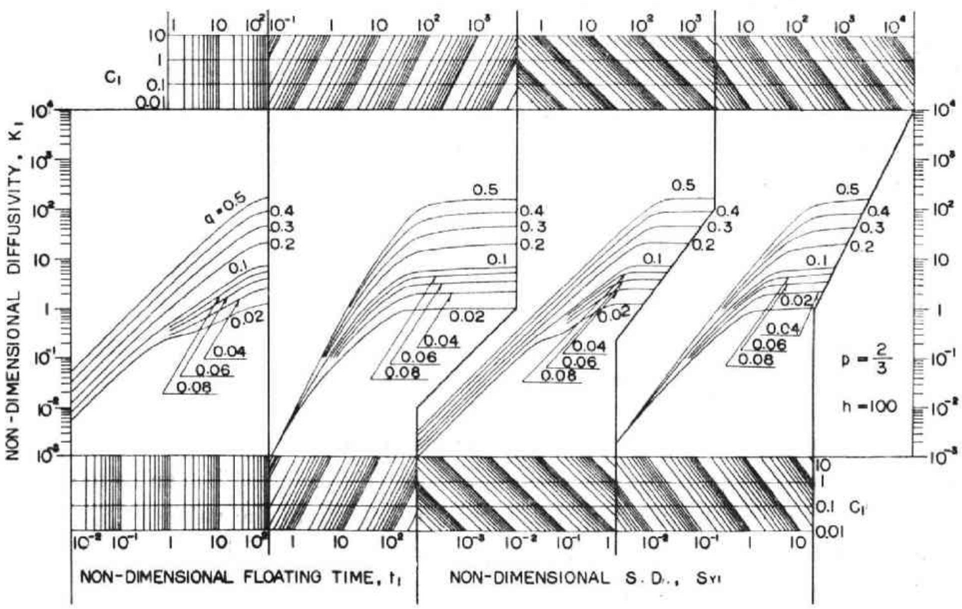


Fig. 3. Diffusion chart for $h=100$.

and $2\langle y(t)^2\rangle^{1/2}$ and $2\langle t_0^2\rangle^{1/2}$ may be considered to be the spatial and time-scale of phenomena, respectively.

However, as Fig. 1 does not cover many conditions of parameters, more generally illustrated figure is necessary. Diffusion charts (Figs. 2 and 3) will be available for this object. Figs. 2 and 3 are drawn to be accurate for $S_1(0)=\infty$ and 0 under the condition of $c_1=1$. For other values of $S_1(0)$, some errors from the solution of our equations are found, which are estimated to be about 10 per cent in the mean value. Correction by $c_1$ is made based on the equations (32) ~ (35). We shall adopt the relation $p=2/3$ from the universal equilibrium theory. Fig. 2 is given for $h=1000$, and Fig. 3 for $h=100$. Parameters $q$, $S_1(0)$, $c_1$ and $h$ must be given for each problem.

We can obtain the diffusion properties from Fig. 2, by the following way, which is explained in Fig. 4, for an example of $q=0.1$, $S_1(0)=10$, $c_1=0.3$ and $h=1000$. The meanings of these parameters are stated in section 3. Points A,B, ······H in Fig. 4

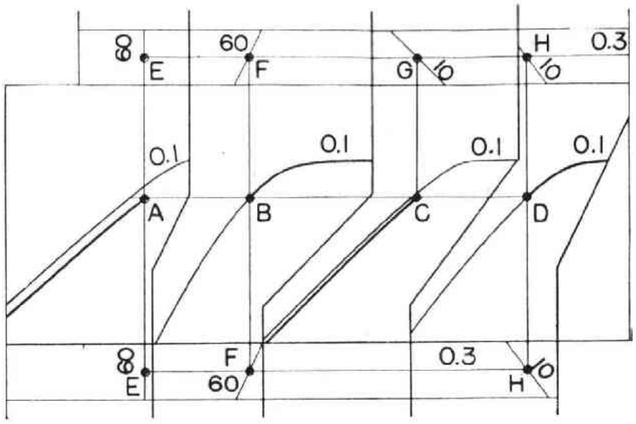

Fig. 4. Diffusion chart for $h=1000$, — for explanation.

are plotted in the following order.

H: Intersection of lines $S_1=10$ and $c_1=0.3$.

G: Intersection of lines $S_1=10$ and $c_1=0.3$.

D: Intersection of the vertical line through H and the curve $q=0.1$.

C: Intersection of the horizontal line through D and the vertical line through G.

B: Intersection of the horizontal line through D and the curve $q=0.1$.

F: Intersection of the line $c_1=0.3$ and the vertical line through B, where $t_1$ is read to be 60.

E: Intersection of the line $t_1=60$ (read at F) and the line $c_1=0.3$.

A: Intersection of the vertical line through E and the horizontal line through D.
Next, curves are drawn in the left hands of A and C, taking the constant differences of $K_1$ between that of curves $q=0.1$; and in the right hands of B and C, along the curves $q=0.1$. The curves drawn through A and C in the left hands and curves through B and D in the right hands show the approximate solution of Eqs. (24) and (26), of which the independent variables $t_1$ and $S_{y1}$ are read on the line $c_1=0.3$.

The conditions in the above example are equivalent to taking as

$$D_E(\tau)=\left(\frac{\tau}{10}\right)^{2/3}\times 10^4\ \mathrm{cm^2\,sec^{-2}} \qquad \text{for} \quad \tau\leq 10\ \mathrm{sec}\,,$$

$$= \left(\frac{\tau}{10}\right)^{0.1} \times 10^4\,\mathrm{cm^2\,sec^{-2}} \qquad \text{for} \quad 10\,\mathrm{sec} \leq \tau \leq 10^4\,\mathrm{sec}\,,$$

$$= 2 \times 10^4\,\mathrm{cm^2\,sec^{-2}} \qquad \text{for} \quad 10^4\,\mathrm{sec} \leq \tau\,,$$

$$U = 10\,\mathrm{m\,sec^{-1}}\,,$$

and

$$\langle t_0{}^2 \rangle^{1/2} = 100\,\mathrm{sec}\,.$$

From these values, we have $\tau_0=40$ sec, $w=1$ m sec$^{-1}$ and $c_1=0.32$ from (22) and (28), and $S_1(0) \approx S_{t1}(t_1)=10$ from (40). From the curve obtained in Fig. 4, we have, for instance,

$$K_1 = 9 \qquad \text{or} \qquad K = 3.6 \times 10^5\,\mathrm{cm^2\,sec^{-1}}$$

$$\text{for} \quad S_{y1}(t_1) = 10 \quad \text{or} \quad \langle y(t)^2 \rangle^{1/2} = 1000\,\mathrm{m}\,,$$

$$K_1 = 1.2 \qquad \text{or} \qquad K = 4.8 \times 10^4\,\mathrm{cm^2\,sec^{-1}}$$

$$\text{for} \quad S_{y1}(t_1) = 1 \quad \text{or} \quad \langle y(t)^2 \rangle^{1/2} = 100\,\mathrm{m}\,,$$

$$K_1 = 0.15 \qquad \text{or} \qquad K = 6.0 \times 10^3\,\mathrm{cm^2\,sec^{-1}}$$

$$\text{for} \quad S_{y1}(t_1) = 0.1 \quad \text{or} \quad \langle y(t)^2 \rangle^{1/2} = 10\,\mathrm{m}\,.$$

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